Guaranteed Dynamic Localization using Constraints Satisfaction Techniques reaching Global Consistency

A. Gning and Ph. Bonnifait

Heudiasyc UMR 6599
Université de Technologie de Compiègne.
BP 20529, 60205 Compiègne Cedex, France.
{gningelh,philippe.bonnifait}@hds.utc.fr

***Abstract* — This article deals with data fusion using ensemblist tools in general, and Constraints Satisfaction Techniques on real intervals, in particular. Indeed, such an approach seems to be well adapted if the data presents a strong redundancy, if the equations are non linear and if the real time implementation on a computer is a key issue. The contribution of this work is primarily methodological as we propose an original method to reach Global Consistency in a calculable number of arithmetic operations. From the application point of view, we are interested here in the state estimation process of the kinematics state of a car using the measurements of the four ABS sensors, the angle of the driving wheel and a GPS receiver. Experimental results illustrate the performance of such an approach in comparison with the Extended Kalman Filter.**

***Key Words* — Ensemblist State Estimation, guaranteed data fusion, constraints propagation on real intervals.**

I. INTRODUCTION

Mobile robot localization[1, 10] is a well known problem. Usually, the localization process involves dead reckoning sensors (like odometry, gyros, etc.) and absolute sensors (like telemeters, goniometers, GPS, etc.). These two technologies are complementary and the fusion of the sensors measurements brings more precision, availability and integrity. For example, GPS suffers from satellite masks occurring in forests, cities, tunnels, etc. In this case, a dead reckoning technique can still provide an estimation of the pose of the mobile robot. Moreover, by using continuously odometry in a fusion process, one can filter the GPS estimates and eliminate its latency [11].

In some applications, integrity is essential and the ability to guaranty the result is a crucial point. With the assumption that the model and measurement errors are bounded, guaranteed techniques based on ensemblist approaches, can be studied. Some of these ensemblist approaches (also known as "bounded error methods" or "set-membership methods") have the advantage to be independent of the non-linearities of the state space representations thanks to Interval Analysis theory tools [9, 12]. When there is a high redundancy of measurements and equations, propagation techniques on real intervals (like Waltz algorithm [3]) can be used. The main idea of such an approach is to find the consistence of all the data linked by the state space equations which provides what one calls "constraints". A remarkable property of these techniques is that they are very fast and thus they are well adapted to real time considerations.

The objective that we consider in this work is to develop a real time bounded-error approach based on interval analysis, in order to guarantee the vehicle location on a terrestrial reference frame.

The sensors that we consider (i.e. the 4 ABS wheel encoders, a measure of the angle of the driving wheel and a differential GPS receiver) provide redundant data. We propose to study in this paper how satisfaction techniques on real intervals can fuse all this information on the basis of "Consistency Domains".

In order to solve a Constraints Satisfaction Problem (CSP), the first step consists in characterizing the graph of the constraints. If this graph is a tree, a Waltz's algorithm (noted Forward-Backward Propagation (FBP) or Climb-Fall algorithm in the literature [7 6]), solves the problem in an optimal way (the time of calculation is known and the estimated boxes are optimal). Nevertheless, generally in data fusion problems, the graph contains cycles. In this case, Waltz's algorithm can only provide a locally consistent box (satisfying all the constraints taken one by one) with an unknown calculation duration. Our approach based on Consistency domain tries to solve the problem of the cycles: it reaches global consistency (satisfies all the constraints taken together), in a known duration of calculation.

In section II, we present this theoretical concept and we show, in section III, how it efficiently solves a localization problem having high redundancy information. Thanks to data acquired with our experimental car, a comparison of the results of such an approach with the ones of the usual Extended Kalman Filter is given in section IV. Estimation errors are computed by using a centimetre precision GPS obtained thanks to post-process kinematics treatments.

II. CONSISTENCY DOMAINS

This section presents the notion of "consistency domains" and several theorems and properties. Please, refer to [2] for the demonstrations.

A. Local and Global Consistencies

Consider $[x]$ a box of IR^n. To solve the CSP $\mathcal{H} = (x \in [x] \mid F(x)=0)$ means to seek in this box all the inconsistent values with the constraints system $F(x)=0$ and afterwards to eliminate them.

Let denote S, the set of the solutions in [x] satisfying the constraints F:

$$S = \{x \in [x] \mid F(x)=0\}$$

To contract $\mathcal{H}$ means to replace the box $[x]$ by a sub-box $[x']\subset[x]$ which contains all the solutions S, i.e. $S\subset[x']$.
A box $[x_i]$ is locally consistent with $\mathcal{H}$ if it verifies all the constraints taken separately (one by one). A box $[x_i]$ is globally consistent with $\mathcal{H}$ if it verifies all the constraints taken together.

B. Consistency domain associated to a sub-vector

Consider $I=(1,\ \ldots,\ n)$ and $J=(j_1,\ \ldots,\ j_m)$ a subset of I of cardinal m.
For a vector noted $x\in\mathrm{IR}^n$, let us denote x_J a sub-vector of x associated to J. This sub-vector is of dimension m and can be written as: $x_J=(x(j_1),\ \ldots,\ x(j_m))^T$
In the same way, for a box $[x]$ of IR^n, the sub-box of $[x]$ associated to J, noted $[x_J]\in\mathrm{IR}^m$, can be written as $[x_J]=([x(j_1)],\ldots,[x(j_m)])^T$.
Let us call $K=I\setminus J$ the complementary of J in I.
Consider the simple case described by figure 1, where $\mathcal{H}$ is a CSP with 2 variables $(x,\ y)$ and made up by 2 constraints (a band and an ellipse, here). For the value x_0 chosen, the set $\mathrm{D_F}(x_0)$ containing all y such as $(x_0,\ y)$ is globally consistent with $\mathcal{H}$, is represented on figure 2.
In general, consider x_J a sub-vector of x associated to J. We call "consistency domain associated to x_J", the set $\mathrm{D_F}(x_J)$ included in the set of all sub-vectors associated to K.

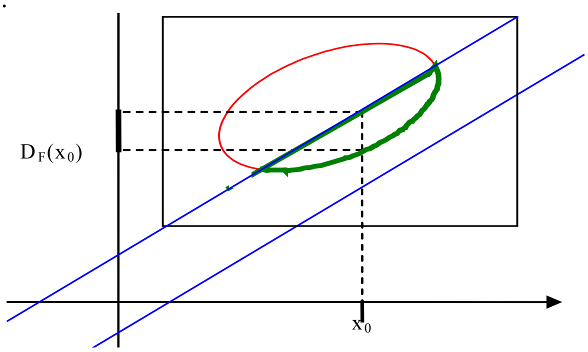

Figure 1. Consistency domain associated to the value x_0.

$\mathrm{D_F}(x_J)$ is empty or verifies the following conditions:

1. Any globally consistent vector having x_J like sub-vector, has its complementary components to x_J included in the consistency domain, i.e. $\forall\ z\in\mathrm{IR}^n$ such as $z_J=x_J$ and z verifies all the constraints F, then $z_K\in\mathrm{D_F}(x_J)$

2. Any vector $\in\mathrm{IR}^n$ made up with sub-vectors $z_K\in\mathrm{D_F}(x_J)$ and x_J is globally consistent, i.e. $\forall\ z_K\in\mathrm{D_F}(x_J)$, the Cartesian product $z_K\times x_J$ in IR^n (in the good order) verifies all the constraints F.

For a sub-vector given x_J, $\mathrm{D_F}(x_J)$ represents thus, the greatest set of sub-vectors such that their Cartesian products with x_J gives a vector in IR^n globally consistent with $\mathcal{H}$.

C. Consistency domain associated to a set of sub-vectors

Now, let us consider the case described by figure 2 where, this time, one seeks to characterize for a given interval $[x]$, the set of y for which $\exists\ x\in[x]$ such as (x,y) is globally consistent with $\mathcal{H}$. This set, noted $\mathrm{D_F}([x])$, is called consistency domain associated to the interval $[x]$.
In the general case, for A_J, a given nonempty set of sub-vectors associated to J, $\mathrm{D_F}(A_J)$ represents the greatest set of sub-vectors verifying: $\forall\ z_K\in\mathrm{D_F}(A_J)$, $\exists\ x_J\in A_J$ such as the Cartesian product with x_J gives a vector in [x] globally consistent.

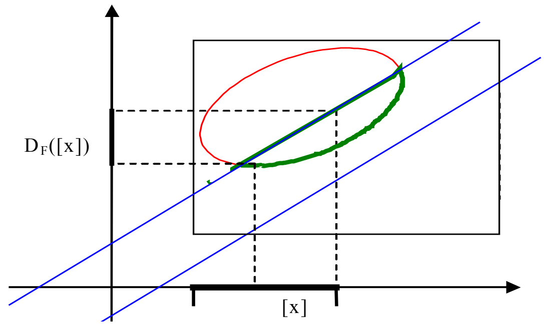

Figure 2. consistency domain of an interval.

Consequently, $\mathrm{D_F}(A_J)$ (included in the set of sub-vectors in [x] associated to K) is empty or verifies:

1. If z is globally consistent and if its components according to J are included in A_J, then the other components, according to K, belong to the consistency domain, i.e. $\forall\ z\in\mathrm{IR}^n$ such as $z_J\in A_J$ and such as z verifies all the constraints F, then $z_K\in\mathrm{D_F}(A_J)$

2. For any sub-vector in $\mathrm{D_F}(A_J)$, one can find a sub-vector in A_J such as the resulting Cartesian product is globally consistent, i.e $\forall\ z_K\in\mathrm{D_F}(A_J)$, $\exists\ z_J\in[A_J]$ such as the Cartesian product of z_K and z_J in IR^n verifies all the constraints F.

Note: if A_J is empty, $\mathrm{D_F}\ (A_J)$ is empty.

D. Properties

Interesting properties are quoted below. They lead to crucial results making possible to evaluate consistency domains.

1. In the particular case of two variables x and y connected by a constraint $x=f(y)$, one has $D_f([y])=f([y])$.
2. Consider $[z]$ and $[y]$ a sub-box of $[x]$ associated to J such as $[y]\subset[z]$ then $\mathrm{D_F}([y])\subset\mathrm{D_F}([z])$
3. The consistency domain associated to a sub-box is the union of the consistency domains of all the sub-vectors of this under-box, i.e.
$$\forall\ [x_J],\ \mathrm{D_F}([x_J])=\bigcup_{x_J\in[x_J]}\mathrm{D_F}(x_J).$$
4. Consider a sub-box $[z]$, and p sub-box $[z_{J,1}],\ \ldots,\ [z_{J,p}]$ associated to J constituting a covering of $[z]$, i.e $[z]=\bigcup_{1\le i\le p}[z_{J,i}]$. Then $\mathrm{D_F}([z])=\bigcup_{1\le i\le p}\mathrm{D_F}([z_{J,i}])$.
5. If a constraints system G (x)=0 is a subset of the system F (x) =0, then for $[x_J]$ sub-box of $[x]$, one has $\mathrm{D_F}([x_J])\subset\mathrm{D_G}([x_J])$.
6. Let us suppose that the system F (x)=0 is equivalent to a system which is written G (x)=0, then for $[x_J]$ under-box of $[x]$, one has $\mathrm{D_F}([x_J])=\mathrm{D_G}([x_J])$.

E. Theorems

Consider a box $[x]$ of IR^n, $(f_i)_{i=1\ldots p}$, p real functions, $F=(f_1, \ldots, f_p)$, $I=\{1\ldots n\}$ and the CSP $\mathcal{H} = (F(x) \mid x \in [x])$.

1) Theorem 1: calculation of the globally consistent box using the consistency domains associated to the complementary of each component

For any $J \in I$, $\Pi_j(S) = [x_j] \cap D_F(\underset{i\in I-\{j\}}{\times} [x_i])$

where $\Pi_j(S)$ is the projection of the solution S according to the j^{th} component. By repeating n times this operation for all the components, one obtains a globally consistent box.

2) Theorem 2: choice of punctual variables and box variables depending of they occurrence, for calculate a consistency domain by considering constraints one by one

Consider $[x_J]$ the sub-box of [x] associated to J. One notes $[x_J^{1-}]$ and $[x_J^{1+}]$ the components of $[x_J]$ which appear respectively once at most and twice at least in the graph of the constraints.

For all x_J^{1+} in $[x_J^{1+}]$,

$D_F([x_J^{1-}] \times x_J^{1+}) = \underset{i=1\ldots p}{\cap} D\, f_i([x_J^{1-}] \times x_J^{1+})$

3) Theorem 3: calculation of a consistency domain thanks to a calculation of a precise consistency domain associated to the set of solution of a "sub-CSP" of $\mathcal{H}$.

For $j \in 1\ldots n$, consider in IR^{n-1}, the CSP noted $\mathcal{H}_{I-\{j\}}$, corresponding to the variables reduced to the $x_i \neq x_j$ and the constraints connecting them i.e. $\mathcal{H}_{I-\{j\}} = (g(y)=0,\ y \in \underset{i\in I-\{j\}}{\times} [x_i])$ where g is the subset of all the constraints in F, independent of the variable x_j. Let us note $S_{I-\{j\}}$ the set of solutions of the CSP $\mathcal{H}_{I-\{j\}}$

One has the result: $D_F(\underset{i\in I-\{j\}}{\times} [x_i]) = D_F(S_{I-\{j\}})$ (1)

<u>Corollary 1</u>

Let us pose $f_{\{j\}}$ the set of the constraints connecting the variable x_j to the other variables in $\mathcal{H}$. One has the following relation, rising from the preceding result:

$$D_F(\underset{i\in I-\{j\}}{\times} [x_i]) = D\, f_{\{j\}}(S_{I-\{j\}}). \quad (2)$$

<u>Corollary 2</u>

Let us suppose moreover, than the set of constraints $f_{\{j\}}$ connects x_j only to q variables whom indices are $j_1,\ldots, j_q$ ($q \leq n$), one has:

$$D_F(\underset{i\in I-\{j\}}{\times} [x_i]) = D\, f_{\{j\}}(\Pi_{j_1 \ldots j_p}(S_{I-\{j\}})) \quad (3)$$

where $\Pi_{j_1 \ldots j_p}$ is the projection on the space of $(x_{j_1} \times \ldots \times x_{j_p})$.

III. Application to the data fusion problem

A. Global architecture of the multi-sensor fusion

The vehicle frame origin M is chosen at the middle of rear axle. The elementary rotation and displacement between two samples can be obtained with good precision uniquely using the measurements of the ABS sensors of the 4 wheels and the driving wheel encoder. Consequently, for a better clearness and more simplicity, let consider two levels of fusion as shown on figure 3. 'δ_θ' and 'δ_s' are given by a static fusion stage which uses the measurements of the ABS sensors and driving wheel encoder. The result is the input of the dynamic module which computes the vehicle location.

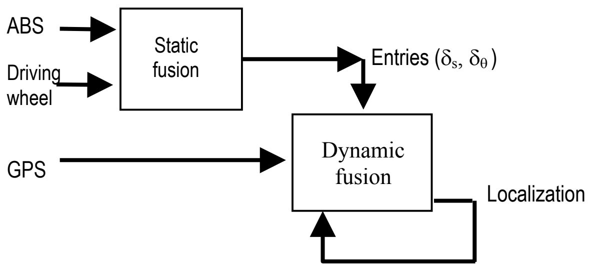

Figure 3. Localizer architecture.

B. Static fusion

Thanks to the Ackerman [5] model (see Fig. 4), the system of constraints can be written as:

$$\begin{cases} \tan(\psi) & = L.\dfrac{\delta_\theta}{\delta_S} = L.\dfrac{e\cdot\delta_\theta}{e\cdot\delta_S} \\ \delta_{RL} & = \delta_S - e.\delta_\theta \\ \delta_{RR} & = \delta_S + e.\delta_\theta \\ \delta_{FL}.\cos(\psi_L) & = \delta_S - e.\delta_\theta \\ \delta_{FR}.\cos(\psi_R) & = \delta_S + e.\delta_\theta \end{cases} \quad (4)$$

Where:

- δ_{RL}, δ_{RR}, δ_{FL}, δ_{FR}, ψ denote the measured variables (the distances travelled between two samples by the different wheels and the angle of a virtual wheel, measured by the driving wheel encoder);
- δ_s, δ_θ, ψ_R, ψ_L are the estimated variables (the elementary distance, the angle between two samples and the angles of the two front wheels);
- L and e are the vehicle parameters (the distance between the axles and the wheel base).

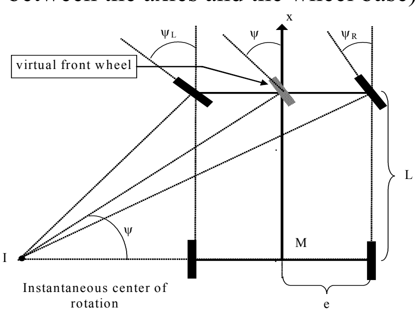

Figure 4. Ackerman model of a car in a turning manoeuvre.

This stage of the fusion is solved thanks to the consistency domains method described previously. For a better clearness, let us build the graph of constraints. To obtain a simplified graph of constraints, one can introduce 4 new intermediate variables called a_1, a_2, a_3 and a_4:

$$a_1 = \frac{e\cdot\tan(\psi)}{L} \qquad a_2 = \delta_{FR}.\cos(\psi_R)$$

$$a_3 = \delta_{FL}.\cos(\psi_L) \qquad a_4 = e\cdot\delta_\theta$$

The system of equations (4) becomes:

$$\begin{cases} a_1 = \dfrac{a_4}{\delta s} \\ \delta_{RL} = \delta s - a_4 \\ \delta_{RR} = \delta s + a_4 \\ a_2 = \delta s - a_4 \\ a_3 = \delta s + a_4 \end{cases}$$

Its graph of constraints is presented on figure 5.

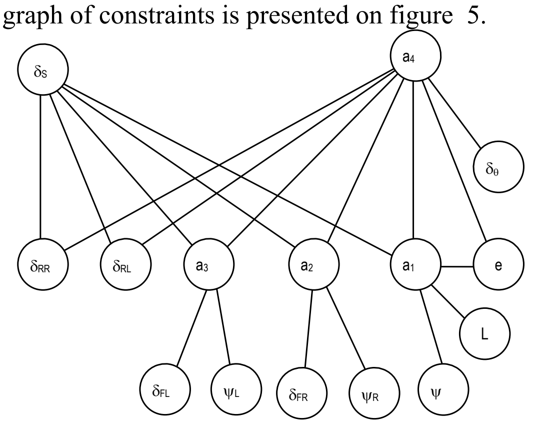

Figure 5. Graph of constraints of the CSP

One takes $x = (\delta s, \delta_\theta, \delta_{RR}, \delta_{RL}, \delta_{FR}, \delta_{FL}, \psi, \psi_R, \psi_L, e, L, a_1, a_2, a_3, a_4)$ gathering the state, the parameters of the model and the auxiliary variables.

Consider I = {1, …, 15} the set of the indices of vector x. The problem is to calculate intervals $[\delta s]$ et $[\delta_\theta]$ solutions thanks to the method based on the consistency domains.

In the continuation, we show how to proceed to calculate the interval $[\delta s]$. The method is quasi-similar for $[\delta_\theta]$.

C. Calculation of $[\delta s]$ globally consistent

1) Step 1

According to theorem 1, one has:

$\Pi_1(S) = [\delta s] \cap D_F(x_{I\backslash\{1\}})$

$= [\delta s] \cap D_F([\delta_\theta], [\delta_{RR}], [\delta_{RL}], [\delta_{FR}], [\delta_{FL}], [\psi], [\psi_R], [\psi_L], [L], [e], [a_1], [a_2], [a_3], [a_4])$

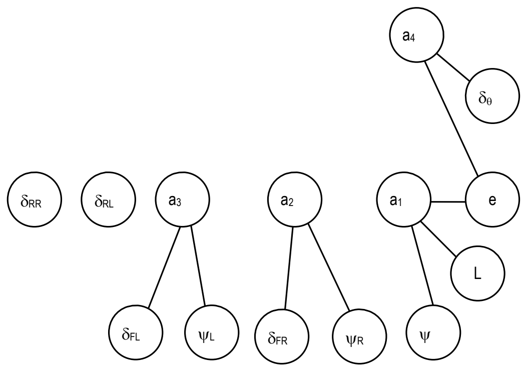

Figure 6. Graph reduced to the variables other than δs

Now, the problem is to calculate $D_F(x_{I\backslash\{1\}})$, knowing that δs is connected only to the variables δ_{RR}, δ_{RL}, e, a_1, a_2, a_3 et a_4 of respective indices 3, 4, 12, 13, 14 and 15 in x. By using corollary 2, one can write:

$$D_F(x_{I\backslash\{1\}}) = D\, f_{\{1\}}(\Pi_{3,4,12,13,14,15}(S_{I-\{1\}}))$$

where $S_{I-\{1\}}$ is the set of the solutions of CSP $\mathcal{H}_{I-\{1\}}$ only made up by the variables other than δs and the constraints other than those involving δs, like illustrated on figure 6.

2) Step 2

Let us calculate projection $\Pi_{3,4,12,13,14,15}(S_{I-\{1\}})$. It turns out that $\mathcal{H}_{I-\{1\}}$ is a set of trees: global consistency can be reached using the algorithm "CLIMB/FALL" [4, 7, 6] for each tree. In addition $\mathcal{H}_{I-\{1\}}$ has a structure such as the variables solutions (δ_{RR}, δ_{RL}, a_2, a_3) are independent between them and independent of others solutions (a_1, a_4) (see figure 6); the fact that δ_θ and a_4 are initialised with $[-\infty, +\infty]$ has the effect to 'inhibit' the constraint $a_4 = e{\cdot}\delta_\theta$ and so leads independency between variable a_1 and a_4. Consequently one has:

$$\Pi_{3,4,12,13,14,15}(S_{I-\{1\}}) = \Pi_3(S_{I-\{1\}}) \times \Pi_4(S_{I-\{1\}}) \times \Pi_{12}(S_{I-\{1\}}) \times \Pi_{13}(S_{I-\{1\}}) \times \Pi_{14}(S_{I-\{1\}}) \times \Pi_{15}(S_{I-\{1\}})$$

The calculation of projections of $S_{I-\{1\}}$ on these six spaces is done by the following calculations:

$\Pi_3(S_{I-\{1\}}) = [\delta_{RR}]$

$\Pi_4(S_{I-\{1\}}) = [\delta_{RL}]$

$\Pi_{12}(S_{I-\{1\}}) = [a_1] = \dfrac{[e]\tan([\psi])}{[L]}$

$\Pi_{13}(S_{I-\{1\}}) = [a_2] = [\delta_{FR}].\cos([\psi_R])$

$\Pi_{14}(S_{I-\{1\}}) = [a_3] = [\delta_{FL}].\cos([\psi_L])$

$\Pi_{15}(S_{I-\{1\}}) = [-\infty, +\infty]$

Finally

$$\Pi_{3,4,12,13,14,15}(S_{I-\{1\}}) = [\delta_{RR}] \times [\delta_{RL}] \times \frac{[e]\tan([\psi])}{[L]} \times [\delta_{FR}].\cos([\psi_R]) \times [\delta_{FL}].\cos([\psi_L]) \times [-\infty, \infty]$$

3) Step 3

It remains to calculate $D\, f_{\{1\}}(\Pi_{3,4,11,12,13,14}(S_{I-\{1\}}))$.

a_4 appearing several times, by applying theorem 2, for a scalar $a_4 \in [a_4]$ and for the intervals $[\delta_{RR}]$, $[\delta_{RL}]$, $[a_1]$, $[a_2]$, $[a_3]$, one has:

$D\, f_{\{1\}}([\delta_{RR}], [\delta_{RL}], [a_1], [a_2], [a_3], a_4) = (a_4/[a_1]) \cap ([\delta_{RR}] - a_4) \cap ([\delta_{RL}] + a_4) \cap ([a_2] - a_4) \cap ([a_3] + a_4)$

Let study the case $a_4 > 0$ (to write $a_4/[a_1]$ boundaries), the other case is similar.

$$D\, f_{\{1\}}([\delta_{RR}], [\delta_{RL}], [a_1], [a_2], [a_3], a_4) = \left(\left[\frac{a_4}{\overline{a}_1}, \frac{a_4}{\underline{a}_1}\right]\right) \cap \left([\underline{\delta}_{RR} - a_4, \overline{\delta}_{RR} - a_4]\right) \cap \left([\underline{\delta}_{RL} + a_4, \overline{\delta}_{RL} + a_4]\right) \cap \left([\underline{a}_2 + a_4, \overline{a}_2 + a_4]\right) \cap \left([\underline{a}_3 - a_4, \overline{a}_3 - a_4]\right)$$

$$= \left[\mathrm{Sup}\left\{\frac{a_4}{\overline{a}_1}, \underline{\delta}_{RR} - a_4, \underline{\delta}_{RL} + a_4, \underline{a}_2 + a_4, \underline{a}_3 - a_4\right\}, \mathrm{Inf}\left\{\frac{a_4}{\underline{a}_1}, \overline{\delta}_{RR} - a_4, \overline{\delta}_{RL} + a_4, \overline{a}_2 + a_4, \overline{a}_3 - a_4\right\}\right]$$

The problem of the research of the min and max according to a_4 is simplified by posing $a_5 = \mathrm{Sup}\{\underline{\delta}_{RL}, \underline{a}_3\}$ and $a_6 = \mathrm{Sup}\{\overline{\delta}_{RR}, \underline{a}_2\}$. One has:

$$\mathrm{Sup}\left\{\frac{a_4}{\overline{a}_1}, \underline{\delta}_{RR} - a_4, \underline{\delta}_{RL} + a_4, \underline{a}_2 + a_4, \underline{a}_3 - a_4\right\} =$$

$$\mathrm{Sup}\left\{\frac{a_4}{\overline{a}_1}, a_5 + a_4, a_6 - a_4\right\}$$

In the same way, by posing a_7 = Inf{ $\overline{\delta}_{RL}$, $\overline{a}_3$ } and a_8 = Inf{ $\overline{\delta}_{RR}$, $\overline{a}_2$ } one has then:

D $f_{\{1\}}$ ([δ_{RR}], [δ_{RL}],[a_1], [a_2], [a_3], a_4) =

[Sup{ $\frac{a_4}{\overline{a}_1}$, a_5 + a_4, a_6 – a_4}, Inf{ $\frac{a_4}{\underline{a}_1}$, a_7 + a_4, a_8 – a_4}]

The study of Sup{ $\frac{a_4}{\overline{a}_1}$, a_5 + a_4, a_6 – a_4} and

Inf{ $\frac{a_4}{\underline{a}_1}$, a_7 + a_4, a_8 – a_4} when a_4 varies in IR, gives a squaring of the space of a_4 where boundaries of D $f_{\{1\}}$ ([δ_{RR}], [δ_{RL}],[a_1], [a_2], [a_3], a_4) are well known. Thanks to a similar reasoning, by comparing the boundaries, one can square the space of a_4 in which

Sup{ $\frac{a_4}{\overline{a}_1}$, $\underline{\delta}_{RR}$ + a_4, $\underline{\delta}_{RL}$ – a_4, $\underline{a}_2$ + a_4, $\underline{a}_3$ – a_4}

≤ Inf{ $\frac{a_4}{\underline{a}_1}$, $\overline{\delta}_{RR}$ + a_4, $\overline{\delta}_{RL}$ – a_4, $\overline{a}_2$ + a_4 , $\overline{a}_3$ – a_4 }

i.e where D $f_{\{1\}}$ ([δ_{RR}], [δ_{RL}],[a_1], [a_2], [a_3], a_4) ≠ ϕ.

4) Step 4

The final stage is the calculation of

D $f_{\{1\}}$ ([δ_{RR}], [δ_{RL}],[a_1],[a_2], [a_3],[a_4])

The property used is:

$$D f_{\{1\}}([\delta_{RR}], [\delta_{RL}],[a_1], [a_2], [a_3],[a_4]) = \bigcup_{a_4 \in [a_4]} D f_{\{1\}}([\delta_{RR}], [\delta_{RL}],[a_1], [a_2], [a_3], a_4)$$

To calculate this union, the monotony of the different functions $\frac{a_4}{\overline{a}_1}$, a_5 + a_4 and a_6 – a_4 is used.

D. Dynamic Localization

In the dynamic fusion stage, we consider an odometric model. The mobile vehicle pose X_k, at time instant k, is calculated thanks to X_{k-1} and to the output of the static fusion process by:

$$\begin{cases} x_{k+1} &= x_k + \delta_{s,k}.\cos(\theta_k + \delta_{\theta,k}/2) \\ y_{k+1} &= y_k + \delta_{s,k}.\sin(\theta_k + \delta_{\theta,k}/2) \\ \theta_{k+1} &= \theta_k + \delta_{\theta,k} \end{cases} \quad (5)$$

where x_k and y_k represent the vehicle position, at time t_k, in the reference frame and θ_k the heading angle.

For every moment k, let consider all the state equations between the time indexes 0 and k. In theory, we have to solve the complete CSP $\mathcal{H}$: (F(x)=0/x∈[x]), where:

$x = (x_k, ..., x_0, y_k, ..., y_0, \theta_k, ..., \theta_0, \delta_{S,k}, ..., \delta_{S,0}, \delta_{\theta,k}, ..., \delta_{\theta,0})$

- [$\delta_{S,i}$] et [$\delta_{\theta,i}$] are provided by the static fusion stage
- The GPS measurement ($x_{gps,mes}$, $y_{gps,mes}$) is used to initialize the intervals [x_k] and [y_k]. The longitude/latitude estimated point is converted to a Cartesian coordinate, in a local frame. The GPS bounded error measurement is obtained as it follows. The GST NMEA sentence is used to characterize the error bound which is taken such as 3 times the estimated standard deviation $\hat{\sigma}$ (computed in real time by the GPS receiver).

$$\begin{aligned} [x_{gps}] &= [x_{gps,mes} - 3\hat{\sigma}_x, x_{gps,mes} + 3\hat{\sigma}_x] \\ [y_{gps}] &= [y_{gps,mes} - 3\hat{\sigma}_y, y_{gps,mes} + 3\hat{\sigma}_y] \end{aligned} \quad (6)$$

- The heading angle [θ_i] =[−∞,+∞] is not measured.
- F represents the system of odometric model (5).

This CSP gives a general solution to the non linear state observation. For a real time implementation of this method, it is unrealistic to consider all the equations from t_0 to t_k. Therefore, we consider a limited horizon '*h'*. Typically, we have used a horizon equal to 3 (i.e. 3 samples) corresponding to the dimension of the state X_k. This induces 18 primitive constraints for each CSP $\mathcal{H}_k$. To solve this complex CSP, we have used Waltz's algorithm, precisely a Forward Backward Propagator (FBP). This is a tractable solution even if it provides only locally consistent boxes.

IV. Experimental results

We have developed two localisers. The first one is a usual EKF that supposes zero mean white noises. The second one uses the constraints satisfaction approach described in this paper.

The results presented in this section were obtained by post-processing real sensors measurements acquired thanks to our experimental car. The data of the ABS sensors were sampled at 100 Hz. The differential GPS receiver (a Trimble AgGPS132 working with Omnistar corrections available on Europe) was used in a synchronized mode at 5 Hz. Thanks to the PPS signal, all the data were resample at 5 Hz and the GPS latency has been compensated. This simplifies the development of the fusion methods.

Let consider first the static fusion stage.

Figure 7 shows guaranteed estimates of the elementary displacement [δ_s] between two samplings (the speed of the vehicle is of approximately *70*km/h). The method based on the consistency domains (curve with "+") gives a more precise estimate than a method based on Waltz's algorithm (curve with "*"). Indeed, the inaccuracy is reduced at least by 25% what is very significant. This result is in conformity with the theory since the method based on the consistency domains reaches global consistency and thus provides more accurate boxes. On the contrary, a method based on local consistency induces additional pessimism.

In term of computing time and to fix the order of magnitude, an iteration of the method takes *280ms* under Matlab and with a PC equipped with a processor INTEL Pentium IV at *1,7Ghz*. For the Waltz's algorithm, the computing time is *35ms* with an iteration count varying generally between 3 and 4, for a selected threshold of contraction of *10^{-3}* order. Regarding the real-time issue, the method based on consistency domains has a known number of computations contrary to the Waltz's method. This result is extremely encouraging for a real-time implementation.

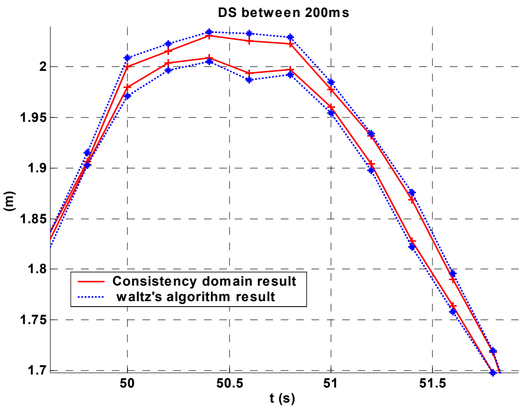

Figure 7. Guaranteed estimates of δ_S *(consistency domain "+" and Waltz's method "*")*

In order to be able to compute estimation errors, we have used a Thales Navigation GPS receiver used in a Post-Processed Kinematic mode working with a local base (a Trimble 7400). This system was able to give positions of reference with a 1 Hz sampling rate. Since the constellation of the satellites was good enough during all the trials (April 2004), all the kinematics ambiguities were fixed. Therefore, we think that a few centimetres accuracy was reached. The synchronization between this reference and the outputs of the dynamic localizers (FBP and EKF) has been made thanks to the GPS timestamps. We have also taken into account the position offsets between the antennas of the two GPS receivers and the origin of the mobile frame.

Figure 8 shows the interval errors of the EKF and of the FBP for the x dimension. In addition to the loss of differential correction, we have simulated 2 complete GPS masks of *30s* duration each, at the instants *7s* and *270s*. During these masks, the speed was about *50kmph*. It appears from these results that the EKF is more accurate since its estimated 3σ errors bounds are smaller than the guaranteed intervals of the FBP.

Nevertheless, on can remark the too much great confidence of the EKF since the reference PPK position is sometimes out of the estimated interval error.

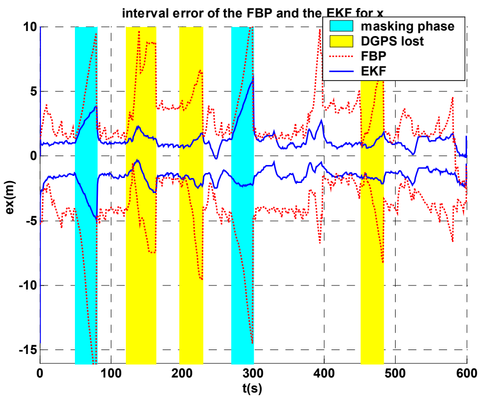

Figure 8. Interval errors of EKF and FBP

V. Conclusion

This paper has presented a new dynamic localization technique based on Constraints Satisfaction on real intervals. This approach guarantees that the real pose of the car is included in the estimated boxes even if the equations are highly non linear. This approach has been compared with the usual technique based on Kalman filtering which relies on linearization and supposes white Gaussian noises.

In the problem treated here, there is a high redundancy in data and equations since we have used 4 ABS encoders, a measurement of the driving wheel angle and the measurements of a GPS receiver. Thanks to this redundancy, constraints satisfaction techniques alone can be used (i.e. without bisection) since the consistence of all the data produces rather precise estimates (i.e. not too pessimistic) as shown by the experimental results. For the dynamic fusion, the contractor presented is based on Forward and Backward Propagation in addition with the use of the Waltz algorithm. This method is well adapted to a real time context since, with a 1.8 GHz Pentium 4 and a Matlab implementation, the FBP treats ten minutes of data in roughly *150s*.

It turned out that the estimates provided by the FBP are more pessimist than the one of a Kalman filter if we consider the 3σ bounds outputted by the filter. It's quite natural since a bounded error approach is always pessimist because its estimates are guaranteed. Moreover, by using consistency domain to solve cycles on the dynamic fusion stage, this precision could be improved. The experimental results indicate that the precision is near to the one of the Kalman filter. Nevertheless, this last can be mistaken in giving sometimes too trustful results or can converge toward local minima.

We think that for applications that need a high level of integrity, the pessimism of the estimates is not handicapping whereas the fact of obtaining guaranteed results is essential.

References

[1] A. Gning and Ph. Bonnifait,"Guaranteed Dynamic Localisation using Constraints Propagation Techniques on Real Intervals". International Conference on Robotic an Automation(ICRA 04.). New Orleans, pp 1499-1504.

[2] A Gning et Ph. Bonnifait,"Fusion de données redondantes avec une technique ensembliste atteignant la consistance globale" Conférence Internationale Francophone d'Automatique (CIFA 2004). Tunisia, Nov. 2004.

[3] D. Waltz. "Generating semantic descriptions from drawings off scenes with shadows", *The psychology off computer Vision*, New York, NY pp. 19-91 (1975).

[4] F. Benhamou, F. Goualard, L. Granvilliers, and J.F. Puget. "Revising hull and box consistency", *International Conference one Logic Programming*, Tired Cruces, Nm, pp.230-244. 1999.

[5] J. Ackerman. Yaw disturbance by robust decoupling off bus steering, *IFAC World Congress, San Francisco*, July 1996.

[6] L. Jaulin, Mr. Kieffer, I.Braems and E. Walter. Guaranteed not linear estimate using constraint propagation one sets. *International Newspaper off Control,* volume 74, No 18, 1772-1782, 2001.

[7] L. Jaulin., Mr. Kieffer, O. Didrit and E. Walter. "Applied Interval Analysis", *Springer-Verlag*, (2001).

[8] Mr. Kieffer "ensemblist Estimate by analysis by intervals, application to the localization of a vehicle". *PhD thesis, University Paris-South*, Orsay, 1999.

[9] Mr. Kieffer, L. Jaulin and E. Walter. "Guaranteed recursive non-linear state estimate using interval Analysis", *International newspaper off adaptive control and signal processing*, 16:193 - 218, 2002.

[10] P. Bouron, D. Meizel, Ph. Bonnifait. "Set-Membership Non-Linear Observers with Application to Vehicle Localization". 6th *European Control Conference*. Porto. Seven 2001.

[11] Ph. Bonnifait, P. Bouron, D. Meizel and P. Crubillé. "Dynamic Localization off Because-like vehicles using Dated Fusion Redundant ABS sensors Off". *The Newspaper Off Navigation,* vol. 56, pp. 1-13. 2003.

[12] R.E. Moore, "Interval Analysis", Prentice-Hall, EnglewoodCliffs, NJ, 1966.