Dynamic Vehicle Localization using Constraints Propagation Techniques on Intervals A comparison with Kalman Filtering

Amadou Gning and Philippe Bonnifait

Heudiasyc UMR 6599 CNRS
Université de Technologie de Compiègne.
BP 20529, 60205 Compiègne Cedex, France.
{gningelh,philippe.bonnifait}@hds.utc.fr

Abstract **-In order to implement a continuous and robust dynamic localization of a mobile robot, the fusion of dead reckoning and absolute sensors is often used. Depending on the objectives of precision or integrity, the choice of an algorithm could be crucial. For example, if the models used for the fusion are non linear, classical tools (such as a Kalman filter) cannot guarantee maximum error estimation. There are bounded error approaches that are insensitive to non linearity. In this context, the random errors are only modeled by their maximum bound. This paper compares a technique based on constraints propagation on intervals, with the usual Extended Kalman Filter for the data fusion of redundant sensors. We have thus developed both techniques and we consider the fusion of wheel encoders, a gyro and a differential GPS receiver. Experimental results show that the precision of a constraints propagation technique can be very good with guaranteed estimations. Moreover, such an approach is well adapted to a real time implementation.**

Index Terms – Outdoor Localization, Sensor Fusion, Bounded-error State Observation, Kalman filtering, GPS.

I. INTRODUCTION

Mobile robot localization in outdoor environments is a key issue for many applications [5], particularly, for those that need absolute positioning information. Usually, the localization process involves dead reckoning sensors (like odometers, gyros, etc.) and absolute sensors (like telemeters, goniometers, vision, GPS, etc.) and tries to exploit their complementary and their redundancy. Indeed, the fusion of sensors measurements can bring more precision, availability and integrity. For example, GPS suffers from satellite masks occurring in forests, cities, tunnels, etc... In this case, dead reckoned sensors are able to provide an estimation of the mobile robot pose. Moreover, the quality of the positioning depends mainly on the visible satellites configuration. By using continuously dead reckoning sensors in a fusion process, one can filter the GPS estimates and increase the performances of the localizer.

With the assumption that the model and measurement errors are bounded, a class of “bounded error methods” or “set-membership methods” propose to fuse the data in such a way that all results are guaranteed.

For example, in [4], the authors propose to use SIVIA algorithm (Set Inversion by Interval Analysis) to find all the possible static 3D locations with an automatic theodolite using indistinguishable landmarks. If only one solution is obtained, then an EKF is applied.As bounded error method, observers based on a predictor/estimator mechanism have also been proposed [8, 10, 3]. The same technique has been used in [12] to localize a mobile robot with ultrasonic measurements. These approaches can reach a high precision with a guaranteed result, but they are not adapted to a real time implementation since they are very slow and since their computation time is not limited, because of the bisections of the state space used to find the result.

If the measurements and the equations provide redundancy, propagation techniques on real intervals can be used. The main idea of such an approach is to eliminate the *inconsistence* between variables linked by the state space equations which provide what one calls "*constraints*". An interesting property of these techniques is that they are very fast, compared with the bounded error predictor/estimator observers. Thus, they are well adapted to a real time running.

Furthermore, prevalent methods used in the fusion stage often rely on a state space representation (the most popular is Kalman filtering). Nevertheless, the state space representations considered in robotics are usually non linear (both for the evolution and observation models). The usual solution is to linearize the equations around the previous estimated state and then to apply a linear technique. This is the principle of the Extended Kalman Filter (EKF) in Gaussian perturbations context. The main drawback of such an approach is that the convergence of the observer cannot be guaranteed. In practice, the observer can converge towards a local minimum different of the real solution.

The objective we consider in this work is to compare a real time bounded-error approach based on forward-backward propagation [7] (denote FBP in the following), and an EKF approach in order to determine the vehicle location on a terrestrial global reference frame (like WGS84).

The sensors that we consider in this work provide redundant data since we use two rears ABS wheel encoders, a gyro and a differential GPS receiver.

The paper is organized as follows. Part II presents the fundamental notions of interval analysis: consistence, constraints propagation and contractors. Then, the FBP contractor we have developed is described. In section III, the equations of the discrete and non linear state space are provided. We have decomposed the fusion problem into two stages. We explain for both of them the solution which we implemented for the EKF and FBP approaches. Finally, section IV presents experimental results carried out with our experimental car with a very precise PPK GPS used for comparison purposes.

II. Interval Analysis and Constraints Propagation Techniques

In this section, we briefly present interval analysis and we describe the constraints propagation technique (also called consistence technique in the literature) that we use for the multisensor fusion process.

A. Basic definitions

A real interval, denoted $[x]$, is defined as a closed and connected subset of IR, and a box $[x]$ of IR^n as a Cartesian product of n intervals ($[x] = \times_{i=1...n} [x_i]$).

The goal of interval analysis is to provide efficient tools and algorithms to guarantee the inclusion of all solutions of an bounded error problem to a given box with the least pessimism. A natural idea is to extend all elementary arithmetic operations like $\{+, -, \times, / \text{ etc}\ldots\}$ to the bounded error context [13] and to extend also usual operations between sets of IR^n like $\{\cap, \supset, \subset, \text{etc}\ldots\}$.

Moreover, the objective of working with interval leads the introduction of inclusion function [11], defined such that image of an interval by a function is an interval, and calculate such that the interval enclosing the image set is optimal.

Different algorithms exist in order to reduce the size of boxes [9] enclosing the solutions. For the fusion problem considered, we have chosen to use constraints propagation techniques, because of the great redundancy of data and equations.

B. Notion of Constraints Satisfaction Problem (CSP)

Constraints Satisfaction Problems (CSP) was initially defined for discrete domains i.e. the values x_i belong to finite sets [6]. Later, CSP were extended to continuous domains (box of IR^n in particular) [8].

Consider a box $[x]$ of IR^n. Let consider m relationships representing the constraints and linking the components $[x_i]$ of $[x]$:

$$f_i\,(x_1,\ \ldots,\ x_n) = 0,\ \ i = 1\ \ldots m \qquad (1)$$

Let F be the Cartesian product of f_i. Equation (1) can be rewritten in a vectorized form as $F(x) = 0$

The problem of searching the minimal box of $[x]$ satisfying all the constraints f_i corresponds to a CSP denoted $\mathcal{H}$, which can be formulated as:

$$\mathcal{H}:(F(x) = 0 \mid x \in [x]) \qquad (2)$$

The solution set of $\mathcal{H}$ is defined as

$$S = \{x \in [x] \mid F(x) = 0\} \qquad (3)$$

One can notice that this formulation of a CSP is an adaptation of discrete CSP problems, where constraints are defined by two sets [6], to continuous intervals.

C. Notion of consistence

A scalar x_i belonging to the i[th] component of $[x]$ is *globally* consistent with $\mathcal{H}$, if it is possible to find a vector x within S having it like i^{th} coordinate.

A scalar x_i belonging to the i[th] component of $[x]$ is *locally* consistent with $\mathcal{H}$ if for such constraint f_i (taken separately), it is possible to find a vector x consistent with f_i having it like i^{th} coordinate

An interval $[x]$ is globally (respectively locally) consistent with $\mathcal{H}$ if $\forall\ x_i \in [x]$, x_i is globally (respectively locally) consistent with $\mathcal{H}$. Consequently, global consistence implies local consistence.

D. Contractors and the FBP contractor

To contract $\mathcal{H}$ means to substitute the box $[x]$ by a smaller box $[x'] \subset [x]$ containing all the solution of $\mathcal{H}$ i.e. $S \subset [x']$.

A contractor for $\mathcal{H}$ is defined as an operator used to contract $\mathcal{H}$. There are different kinds of contractors [9].

In this paper, we use a forward backward propagation technique based on *primitive* constraints [1]. A primitive constraint only involves an arithmetic operator or a usual function (cos, exp, etc.). The FBP uses Waltz algorithm in order to optimise the contraction (the principle is to repeat the propagation until the intervals do not contract any more). It is a locally consistent contractor called FBP in the following. Please refer to 7 for more details.

III. Data Fusion Using FBP Contractor and EKF

In this section, we present the global architecture of the localization problem that we consider. The same scheme will be used by the FBP and an EKF.

A. Global architecture of the multi-sensor fusion

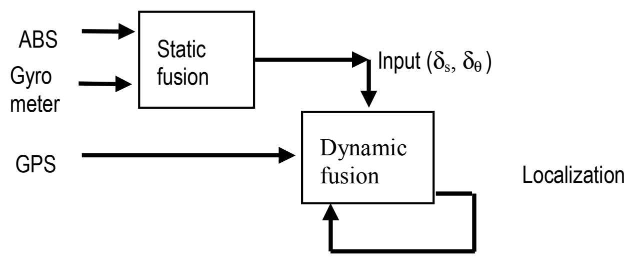

Fig. 1. Localizer architecture.

The vehicle frame origin M is chosen at the middle of rear axle. The elementary rotation and displacement between two samples can be obtained with good precision uniquely using the gyrometer and the two rears wheels. Consequently, for a better clearness and more simplicity, let consider two levels of fusion as shown on Figure 1. 'δ_θ' and 'δ_s' are given by a static fusion stage which uses the measurements of the ABS sensors, the two wheels and the gyro. The result is the input of the dynamic module which computes the vehicle location.

B. Models used

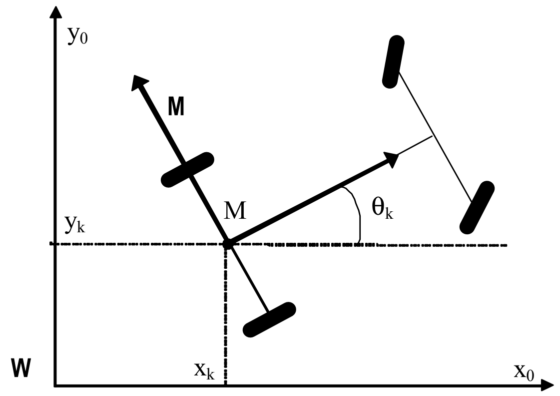

Fig. *2*. Definition of the frames.

Between two sampling instants, elementary rotations of the two rear wheels are integrated by counters. These values allow calculating the distances travelled between two samples by the rear wheels. The elementary displacement covered by M denoted $\delta_{s,k}$ and the rotation denoted $\delta_{\theta,k}$ (see fig (2), at instant k, are given by the following equations:

$$\begin{cases} \delta_{s,k} = \dfrac{\delta_{RR} + \delta_{RL}}{2} \\ \delta_{\theta,k} = \dfrac{\delta_{RR} - \delta_{RL}}{e} \end{cases} \qquad (4)$$

Where:

- $\delta_{RL,k}$, $\delta_{RR,k}$ denote the measured variables (the values counted between two samples)
- $\delta_{s,k}$, $\delta_{\theta,k}$, are the estimated variables
- *L* and *e* are the vehicle parameters (the distance between the axles and the wheel base).

In the dynamic fusion stage, the mobile vehicle pose X_k, at instant *k*, is calculated thanks to X_{k-1} and thanks to the output ($\delta_{s,k}$, $\delta_{\theta,k}$) of the static fusion process:

$$\begin{cases} x_{k+1} &= x_k + \delta_{s,k}.\cos(\theta_k + \delta_{\theta,k}/2) \\ y_{k+1} &= y_k + \delta_{s,k}.\sin(\theta_k + \delta_{\theta,k}/2) \\ \theta_{k+1} &= \theta_k + \delta_{\theta,k} \end{cases} \qquad (5)$$

Where x_k and y_k represent the vehicle position, at time t_k, in the reference frame. θ_k is the heading angle.

C. Static fusion

1) FBP solution

At the static fusion stage, the FBP returns guaranteed intervals for $\delta_{\theta,k}$ et $\delta_{s,k}$. At any moment t_k, we have to solve the CSP $\mathcal{H}_k$: (F(x) = 0 / x $\in$ [x]), where:

- $x = (\delta_S, \delta_{RL}, \delta_{RR}, \delta_\theta, L, e)$
- F represents the system of equations (4)
- The 2 rear wheels provide [δ_{RL}], [δ_{RR}]. For intervals borders, we suppose that the covered distance error between two instants t_{k-1} and t_k, is less than the covered distance corresponding to one top of the ABS sensor counter (denoted δ_{ABS}), with the assumption that the vehicle rolls without slipping

$$[\delta] = [\delta_{mes} - \delta_{ABS}, \delta_{mes} + \delta_{ABS}] \qquad (6)$$

- [δ_θ] come from the gyro measurement. Thanks to specific static tests, we estimated that the maximum of this error is $\delta_{\theta,gyr} \approx 3.10^{-3}$degrees

$$[\delta_\theta] = [\delta_{\theta,mes} - \delta_{\theta,gyr}, \delta_{\theta,mes} + \delta_{\theta,gyr}] \qquad (7)$$

- [δ_S] is not measured and so, initialized with unknown value: [−∞,+∞]
- [L], [e] are vehicle parameters. They are roughly known that's why, in part, they are treated as unknown quantities in the FBP.

The estimations of [$\delta_{\theta,k}$] et [$\delta_{s,k}$] are obtained thanks to the FBP contractor 7 applied to the CSP $\mathcal{H}_k$. Please notice that between two samples the CSP are independent. In addition, an originality of the method is that all the variables of the CSP can be contracted even the measurements and the parameters used in the model.

2) EKF solution

The EKF static fusion estimates of $\delta_{\theta,k}$ and $\delta_{s,k}$ and their variances are obtained by a straightforward computation which is statistically optimal here because the equations are linear.

D. Dynamic fusion

1) FBP solution

For any time index *k*, let consider all the state equations between the time indexes 0 and *k*. theoretically, we have to solve the complete CSP $\mathcal{H}_k$: (F(x)=0/x$\in$[x]), where $x = (x_k, \ldots, x_0, y_k, \ldots, y_0, \theta_k, \ldots, \theta_0, \delta_{S,k}, \ldots, \delta_{S,0}, \delta_{\theta,k}, \ldots, \delta_{\theta,0})$

- [$\delta_{S,i}$] et [$\delta_{\theta,i}$] are provided by the static fusion stage
- The GPS measurement ($x_{gps,mes}$, $y_{gps,mes}$) is used to initialize intervals [x_k] and [y_k]. So, the longitude/latitude estimated point is converted in a Cartesian local frame (see Fig. (5)) and the GPS bounded error measurement is obtained thanks to the GST NMEA sentence. The error bounds are supposed to be equal to 3 times the estimated standard deviation $\hat{\sigma}$ computed in real time by the GPS receiver

$$\begin{aligned} [x_{gps}] &= [x_{gps,mes} - 3\hat{\sigma}_x, x_{gps,mes} + 3\hat{\sigma}_x] \\ [y_{gps}] &= [y_{gps,mes} - 3\hat{\sigma}_y, y_{gps,mes} + 3\hat{\sigma}_y] \end{aligned} \qquad (8)$$

- The heading angle [θ_i] =[−∞,+∞] is not measured
- F represents the odometric model (5).

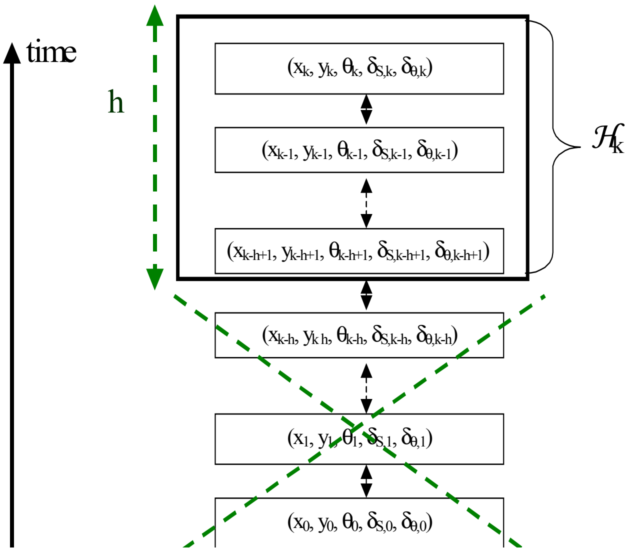

Fig. 3. Graphical interpretation of the horizon limited CSP $\mathcal{H}$.

This CSP gives a general solution to the non linear state observation [8] for any instant t_k. For a real time implementation, it is unrealistic to consider all the equations from t_0 to t_k. Therefore, we consider a limited time windows denoted "*h*" for *horizon*. Typically, we have used a horizon equal to 3 samples i.e. the dimension of the state X_k. This induces 27 primitive constraints for each CSP $\mathcal{H}_k$.

2) *EKF solution*

The EKF localization algorithm computes at the same frequency (5 Hz), the predictive phase, using the expected pose and using the result of the static fusion of gyro and ABS. The estimation phase uses the GPS measurement.

The model (5) we consider can be written simply:

$$\begin{cases} X_{k+1} = f(X_k, U_k) \\ Z_k = H \cdot X_k \end{cases} \quad (9)$$

where

- $X_k = (x_k\ y_k\ \theta_k)$ is the state
- $U_k = (\delta_{\theta,k}\ \delta_{S,k})$ the input of the system
- $Z_k = (x_{GPS},\ y_{GPS})$ is the GPS navigation solutions projected into a French Lambert coordinates system
- H verifies

$$H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad (10)$$

We suppose that the reader is familiar with Kalman filtering and we don't detail the equations.

IV. Experimental Results

The results presented in this section were obtained by post-processing real sensors measurements acquired thanks to our experimental car (Fig. 4). The data of the two ABS sensors and of the optical gyro KVH were sampled at 100 Hz. The differential GPS receiver (a Trimble AgGPS132 working with Omnistar corrections available on Europe) was used in a synchronized mode at 5 Hz. Thanks to the PPS signal, all the data were resampled at 5 Hz and the GPS latency has been compensated. This simplifies the development of the two fusion methods.

In order to be able to compute estimation errors, we have used a Thales Navigation GPS receiver used in a Post-Processed Kinematic mode working with a local base (a Trimble 7400). This system was able to give positions of reference with a 1 Hz sampling rate. Since the constellation of the satellites was good enough during all the trials (April 2004), all the kinematics ambiguities were fixed. Therefore, we think that a few centimeters accuracy was reached. The synchronization between this reference and the outputs of the dynamic localizers (FBP and EKF) has been made thanks to the GPS timestamps. We have also taken into the position offsets between the antennas of the two GPS receivers and the origin of the mobile frame (Fig. 2).

Figure 5 shows a top view of an experiment. It corresponds to two ten minutes laps. The conditions of the experiment are shown on Figure 6. The mean speed was 50 kmph and reached 80 kmph. The standard deviation of the latitude (Fig. 6) indicates that the differential corrections of the geostationary satellite Omnistar were lost three times because of trees. In this case, the Ag132 still propagates an estimation of the correction during 30 s. After this duration, it works in an autonomous mode and the precision decreases significantly.

Fig. 4. The experimental car with the Ag132 and Thales GPS receivers

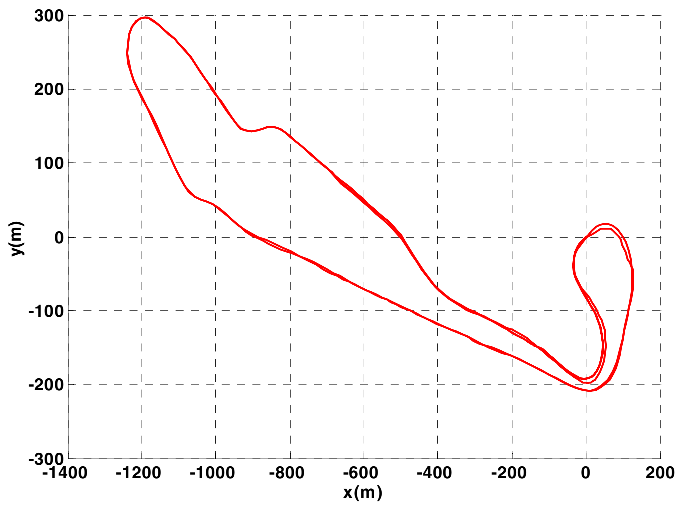

Fig. 5. Overview of the trial in a local frame.

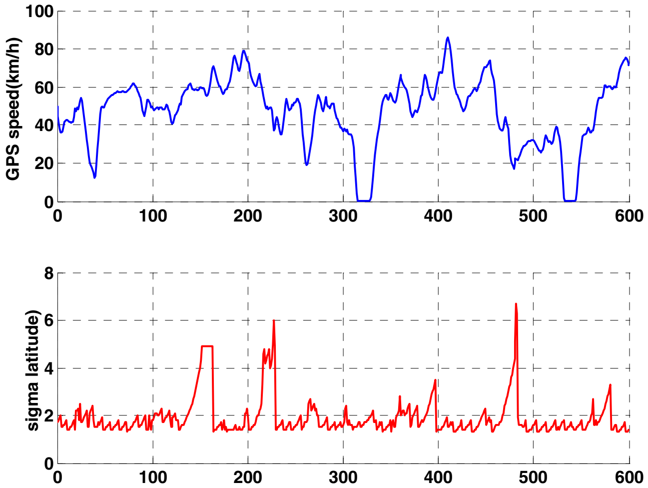

Fig. 6. Speed and standard deviation of the latitude during the test.

Figures 7 and 8 show the interval error for the EKF and the FBP respectively for the dimension x and y. In addition to the loss of differential correction, we have simulated two complete GPS masks of 30 s duration each, at the instants 7 s and 270 s. During these masks, the speed was about 50 kmph (c.f. Fig 6).

It appears from these results that the EKF is more accurate since its estimated 3σ errors bounds are smaller than the guaranteed intervals of the FBP. This is due in part to the fact that the FBP provides locally consistent estimated intervals.

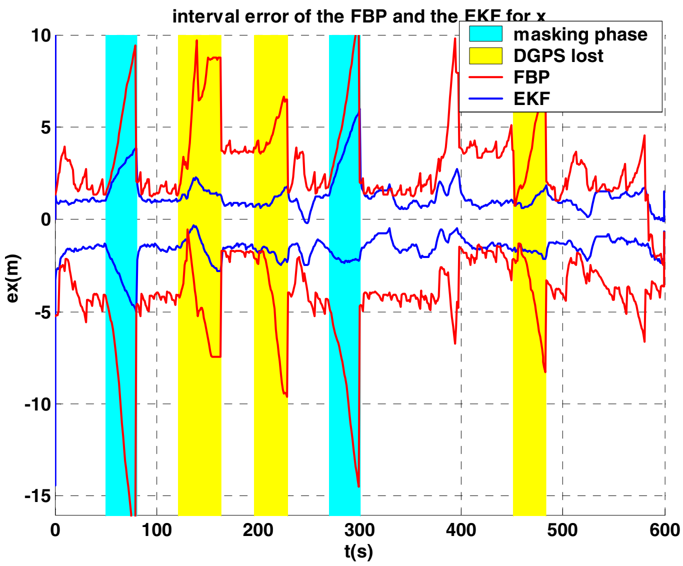

Fig. 7. Comparison between bound errors of EKF and FBP for x.

On can notice that the value "0" always makes part of the FBP estimated interval which confirms the fact that the results are guaranteed. On the contrary, the EKF can be wrong since the real position can be out of the estimated ellipse has shown by Figure 7 (time≈250 s) and Figure 8 (time≈120 s, 400 s).

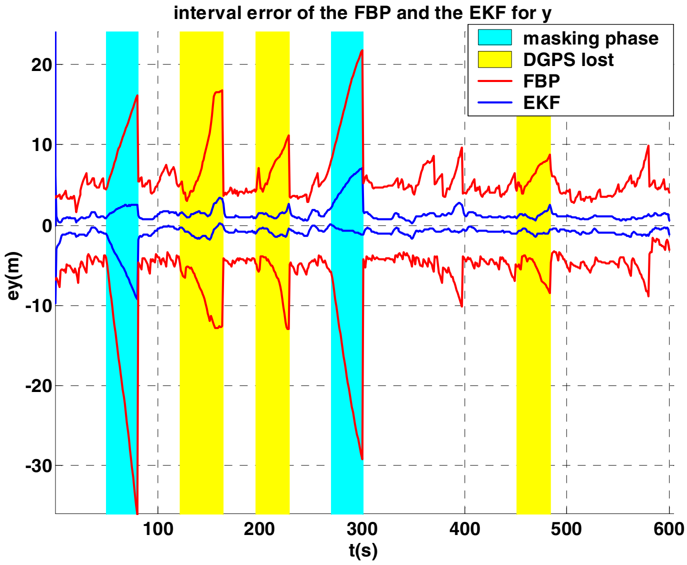

Fig. 8. Comparison between bound errors of EKF and FBP for y.

Figure 9 gives another example of the too much great confidence of the EKF since the reference PPK position is out of the 99% ellipse. One can remark the good contraction of the GPS box provided by the FBP.

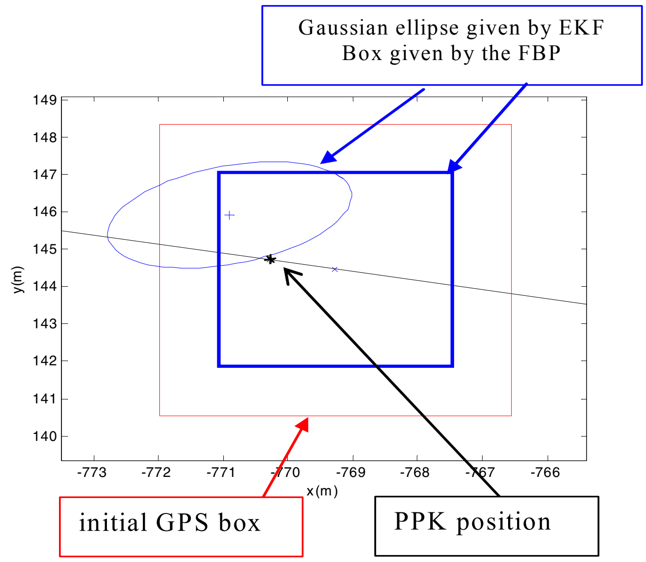

Fig. 9. Illustration of the too much great confidence of the EKF

Figure 10 plots the heading estimated error of the FBP and illustrates the fact that this observer is able to reconstruct a non directly measured variable. The three times where the value "0" does not make part of the estimated interval are due to a noise affecting the reference heading angle since it has been build manually from the PPK measurements. On can notice the very good heading estimation provided by the FBP.

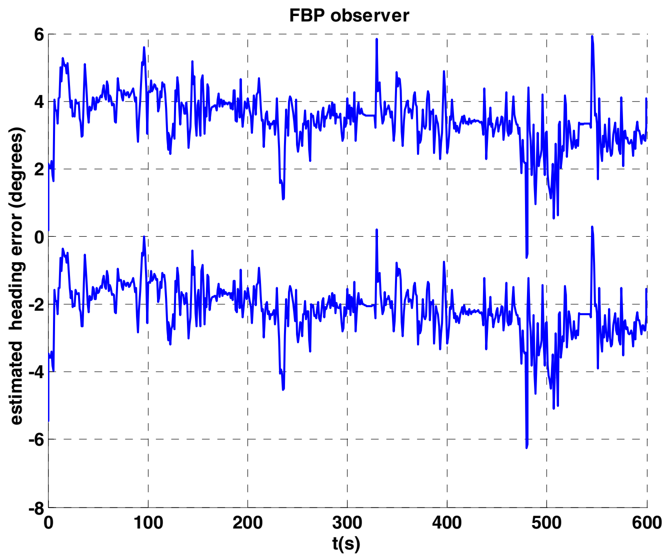

Fig. 10. Observation error of the heading angle by the FBP (in degrees).

We also noticed that the post-processing has never detected any inconsistence for all the data used in this trail. As a matter of fact, one knows that bounded error methods are sensitive to outliers. This indicates a good tuning of the FBP.

V. Conclusion

This paper has presented a new dynamic localization technique based on constraints propagation on real intervals. This approach guarantees that the real pose of the car is included in the estimated boxes even if the equations are highly non linear. This approach has been compared with the usual technique based on Kalman filtering which relies on linearization and supposes white Gaussian noises.

In the problem treated here, there is a high redundancy in data and equations since we have used two encoders, a gyro and the measurements of a DGPS receiver. Thanks to this redundancy, contractors alone can be used (i.e. without bisection) since the consistence of all the data produces rather precise estimates (i.e. not too pessimistic) as shown by the experimental results. The contractor presented is based on Forward and Backward Propagation (FBP) in addition with the use of the Waltz algorithm. This method is well adapted to a real time context since, with a 1.8 GHz Pentium 4 and a Matlab implementation, the FBP treats ten minutes of data in roughly 150s.

It turned out that the estimates provided by the FBP are more pessimist than the one of a Kalman filter if we consider the 3σ bounds outputted by the filter. It's quite natural since a bounded error approach is always pessimist because its estimates are guaranteed. Nevertheless, the experimental results indicate that the precision is near to the one of the Kalman filter. Moreover, this last can be mistaken in giving sometimes too trustful results or can converge toward local minima.

We think that for applications that need a high level of integrity, the pessimism of the estimates is not handicapping whereas the fact of obtaining guaranteed results is essential.

Acknowledgment

This research has been carried out in the Framework of the French PREDIT project "ARCOS". The authors wish to thank Mr. D. Bétaille, C Lemaire and F. Peyret for their support for the experiments.

References

[1] F. Benhamou, F. Goualard, L. Granvilliers, and J.F. Puget. "Revising hull and box consistency", Proceedings of the Intervational Conference on Logic Programming, Las Cruces, NM, pp.230-244. 1999.

[2] Ph. Bonnifait, P. Bouron, P. Crubillé, D. Meizel. "Data Fusion of Four ABS Sensors and GPS for an Enhanced Localization of Car-like Vehicles", IEEE International Conference on Robotics and Automation (ICRA 01). pp. 1597-1602. Séoul. May 2001.

[3] P. Bouron, D. Meizel, Ph. Bonnifait. "Set-Membership Non-Linear Observers with Application to Vehicle Localisation". 6th European Control Conference. Porto. Sept 2001. pp 1255-1260

[4] D. Bouvet and G. Garcia. "Guaranteed 3-D mobile robot localization using an odometer, an automatic theodolite and indistinguishable landmarks". ICRA 01. pp 3612-3617.

[5] D. Caltabanio, G. Muscato, F. Russo,"Localization and Self Calibration of a Robot for Volcano Exploration". ICRA 04. New Orleans, pp 586-591.

[6] R. Dechter "Constraint Processing" Published by Morgan Kaufmann, Mai 2003

[7] A. Gning and Ph. Bonnifait,"Guaranteed Dynamic Localisation using Constraints Propagation Techniques on Real Intervals". ICRA 04. New Orleans, pp 1499-1504.

[8] L. Jaulin. "Nonlinear bounded-error state estimation of continuous-time systems". Automatica, 38, 1079-1082. 2002.

[9] L. Jaulin., M. Kieffer, O. Didrit and E. Walter. "Applied Interval Analysis", Springer-Verlag, 2001.

[10] M. Kieffer, L. Jaulin and E. Walter. "Guaranteed recursive non-linear state estimation using interval Analysis", International journal of adaptative control and signal processing, 16:193-218, 2002.

[11] S.A Malan, M. Milanese, M. Taragna, And J Garloff. "B3 Algorithm for Robust Performance Analysis in Presence of Mixed Parametric and Dynamic Perturbations", Proceedings of the 31st IEEE Conference on Decision and control, Tucson, 1992. AZ, pp.128-133

[12] D. Meizel, O. Lévêque, L. Jaulin and E. Walter. "Initial Localization by Set Inversion". IEEE transactions on robotics and Automation. Volume 18, Number 6, pages 966-971. 2002.

[13] R. E. Moore, "Interval Analysis", Prentice-Hall, Englewood Cliffs, NJ, 1966.

[14] S. Sukkarieh, E. M. Nebot, HF. Durrant-Whyte. "Achieving Integrity in an INS/GPS Navigation Loop for Autonomous Land Vehicle Applications". IEEE International Conference on Robotics and Automation. ICRA98 pp 3437-3442

[15] D. Waltz. "Generating semantic descriptions from drawings of scenes with shadows", The psychology of computer Vision, New York, NY pp. 19-91.1975.