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Brief paper

Constraints propagation techniques on intervals for a guaranteed localization using redundant data☆

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Abstract

In order to estimate continuously the dynamic location of a car, dead reckoning and absolute sensors are usually merged. The models used for this fusion are non-linear and, therefore, classical tools (such as Bayesian estimation) cannot provide a guaranteed estimation. In some applications, integrity is essential and the ability to guaranty the result is a crucial point. There are bounded-error approaches that are insensitive to non-linearity. In this context, the random errors are only modeled by their maximum bounds. This paper presents a new technique to merge the data of redundant sensors with a guaranteed result based on constraints propagation techniques on real intervals. We have thus developed an approach for the fusion of the two ABS wheel encoders of the rear wheels of a car, a fiber optic gyro and a differential GPS receiver in order to estimate the absolute location of a car. Experimental results show that the precision that one can obtain is acceptable, with a guaranteed result, in comparison with an extended Kalman filter. Moreover, constraints propagation techniques are well adapted to a real-time context.

Keywords: Localization; Bounded-error estimation; Data fusion; Interval analysis

1. Introduction

Dynamic localization of a car in outdoor environments is a key issue for many applications (Caltabanio, Muscato, & Russo, 2001), particularly, for those that need absolute positioning information. Usually, the localization process involves dead reckoning sensors (like odometers, gyros, etc.) and absolute sensors (like telemeters, goniometers, vision, GPS, etc.) and tries to exploit their complementary and their redundancy. Indeed, the fusion of sensor's measurements can bring more precision, availability and integrity. For example, GPS suffers from satellite masks occurring in forests, cities, tunnels, etc. . In this case, dead reckoned sensors are able to provide an estimation of the car pose. Moreover, the quality of the positioning depends mainly on the visible satellites configuration. By continuously using dead reckoning sensors in a fusion process, one can filter the GPS estimates which increase the performances of the localizer.

With the assumption that the model and measurement errors are bounded, a class of *bounded-error* approaches proposes to fuse the data in such a way that all the results are guaranteed.

Such an approach proposes a different way to treat the localization process in the sense that human beings usually reasons with points instead of sets like ellipses or boxes. A natural question is therefore what is the relevance of bounded estimate? A first way to answer this question is the fact that an interval can be compared to a point if its size is small regarding the task to perform. On the contrary, if the size of the interval is not negligible, the ambiguity is important and the fact to assimilate the interval to a point can induce an unexpected behavior. This point can be crucial for applications which need high integrity like, for example, a rescue robot or a group of collaborating robots (Farinelli, Iocchi, & Nardi, 2004).

These concept starts to be accepted since several applications rely on this approach. For example, in Bouvet and Garcia (2001), the authors propose to use the set inversion by

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interval analysis (SIVIA) algorithm to find all the possible static 3D locations with an automatic theodolite using indistinguishable landmarks. If only one solution is obtained, then an extended Kalman filter (EKF) is applied. Bounded-error observers based on a predictor/estimator mechanism have also been proposed (Bouron, Meizel, & Bonnifait, 2001; Jaulin, 2002; Kieffer, Jaulin, & Walter, 2002). The same technique has been used to localize a mobile robot with ultrasonic measurements (Meizel, Lévêque, Jaulin, & Walter, 2002). These approaches can reach a high precision with a guaranteed result, but they are not adapted to a real-time implementation since they are very slow and since their computation time is not limited, because of the bisections of the state space used to find the result.

If the measurements and the equations provide redundancy, propagation techniques on real intervals can be used. The main idea of such an approach is to eliminate the *inconsistence* between variables linked by the state space equations which provide what one calls "*constraints*". An interesting property of these techniques is that they are very fast, compared with the bounded-error predictor/estimator observers. Thus, they are well adapted to a real-time processing.

Furthermore, prevalent methods used in the fusion stage often rely on a linear state space representation. Nevertheless, state space representations considered in localization are usually non-linear. The usual solution is to linearize the equations around the previous estimated state and then to apply a linear technique. This is the principle of the EKF in Gaussian noises context. The main drawback of such an approach is that the observer can diverge, which means in this case that the observer can converge towards a local minimum different from the global solution. Another disadvantage of an EKF is that the consistence is difficult to obtain. Often, it underestimates the covariance estimation error, which makes difficult to know the imprecision.

Another objective considered in this work is to compare a real-time bounded-error approach based on forward–backward propagation denoted FBP in the following (Gning & Bonnifait, 2004), and an EKF approach in order to determine the vehicle location on a terrestrial global reference frame (like WGS84). For this purpose, real experiments have been carried out. Moreover, the sensors considered provide redundant data since the two rears ABS wheel encoders of a car, a fiber optic gyro and a differential GPS receiver have been used.

The paper is organized as follows. In Section 2, the concept of state estimation is recalled and bounded-error observers are presented. Section 3 presents the fundamental notions of interval analysis: consistence, constraints propagation and contractors. Then, the FBP contractor algorithm that has been developed is described in Section 4. In Section 5, the fusion problem is decomposed into two stages. For both of them, the solution implemented for the EKF and FBP approaches are explained. Finally, Section 6 presents experimental results carried out with our experimental car with a very precise PPK GPS used for comparison purposes.

2. State estimation in a bounded-error context

The objective of state estimation is to determine an unknown state x thanks to a given evolution model and the knowledge of measurements, with an optimality depending of some desired criteria.

2.1. Notations

Let y_k be the kth measurement at instant k, and $y_{k|k-1}$ the predicted measurement knowing the state at instant k. The error between the measurement and its prediction is: $ey_{k/k-1} = y_k - y_{y/k-1}$. This error is due to the fact that the modeling is always perfectible and the measures suffer from noises or limited precisions of the sensors.

An interval variable is denoted with brackets: for example $[y]$ represents a guaranteed interval of variable y.

2.2. Background

Given a state representation with a dynamic evolution model and measured observations, the original idea of state observation consists in the ability to reconstitute the initial state x_0. Knowing this state, one can deduce the current state at any instant. In practice, a dynamic state observer is often researched since it is characterized by a recursive form well adapted to a real-time implementation.

The usual class of state observers is based on Bayesian filters such as the well-known Kalman filter (Kalman, 1960). The main idea of these methods is to use the error $ey_{k|k-1}$ like a correction of the prediction in an optimal way, according to a criterion. In the linear case and with the hypothesis that all the noises are Gaussian, zero mean and white, the Kalman filter is optimal in the sense that it exactly satisfies a statistical criterion: it provides unbiased estimates and minimum error variance for each step. Unfortunately, the conditions on the noises are rarely verified in practice. In a non-linear case, many state estimators are based on an approximated model of a non-linear system, which is, for example, linearized around the most recent estimate at each iteration step. The linearization can lead to a sub-optimal performance and sometimes can induce a divergence of the filter.

With the only assumption that the model and measurement errors are bounded, a class of bounded-error approaches proposes to fuse the data in such a way that all results are guaranteed. The particularity of these approaches is that they provide only sets which contain all the possible solutions.

In the bounded-error context, there are two great classes: one is based on an ellipsoid modeling (Durieu, Polyak, & Walter, 1996; Fogel & Huang, 1982) and is limited to linear cases. Another one is based on interval analysis which can deal with non-linearity. The algorithm presented in this paper belongs to this class.

In the linear case, a Kalman filter solution based on interval analysis tools has been proposed (Chen, Wang, & Shieh, 1997), and is called interval Kalman filter (IKF). The idea of this filter is to develop an exact and optimal estimation based on an interval condition expectation. It reduces the Gaussian condition only for the measurement noise and supposes that errors of the model are bounded. So, the algorithm has the same structure as the standard Kalman filter with interval variance matrixes.

For state estimation, many bounded-error algorithms are based on the SIVIA algorithm (Jaulin & Walter, 1993). In short, the idea of the SIVIA algorithm is to compute the reciprocal image of a sub-paving by a possibly non-linear function. The principle is to test for each box of a given sub-paving inside a search domain, if its image by a function in the observation model, has an intersection with the measured observation box. A box for which this condition is not verified is excluded, otherwise if the box entirely contained in the observation box, it is memorized. The process is iterated with a bisection of the remaining boxes of the sub-paving until a pre-specified threshold width is reached. The SIVIA algorithm can be applied in two interesting ways for a state estimation problem. The first one implements a natural two-step filter (Jaulin, 2002; Kieffer et al., 2002; Meizel et al., 2002):

- *Prediction*: this step consists in calculating the image interval of the state $[x_{k+1}]$ at instant $k+1$, thanks to the model and the knowledge of both the state $[x_k]$ and the input $[u_k]$ at instant k,
- *Correction*: this step consists in finding the intersection between the current estimated $[x_{k+1}]$ state and the domain consistent with the box given by the SIVIA and the measured observation box.

Another application (Ashokaraj, Tsourdos, Silson, & White, 2004) is a fusion of an unscented Kalman filter (UKF) (Julier & Uhlman, 1997) and the predictor/estimator using the SIVIA described above. This method is able to ameliorate the convergence of the Kalman filter if a great redundancy of sensors exists.

In brief, the prevalent method based on interval analysis adapted to a non-linear equation is the predictor/estimator based on the SIVIA. Unfortunately, the calculation time can explode rapidly and a crucial question about pertinent dimension to bisect is not resolved yet (according to the biggest width? in how many parts? ...).

For real-time implementation issues, another solution is based on constraint propagation on intervals. The calculation time of this method is less explosive than the one based on the SIVIA. The main condition for this kind of approach is to treat problems which present redundant equations and/or measurements, like the localization problem considered in this paper.

3. Interval analysis and constraints propagation

3.1. Basic definitions

A real interval, denoted $[x]$, is defined as a closed and connected sub-set of IR, and a box $[x]$ of IR^n as a Cartesian product of n intervals ($[x] = \times_{i=1,\ldots,n}[x_i]$).

The main idea of interval analysis is to provide efficient tools and different algorithms to guarantee with the least pessimism, the inclusion of all possible solutions of a given bounded-error problem to a box. All elementary arithmetic operations like $\{+, -, \times, /\}$ are extended to the bounded-error context (Moore, 1966). Extensions are also defined for usual operations between sets of IR^n like $\{\cap, \supset, \subset, \text{etc}\ldots\}$.

The image of an interval by a function is not necessarily an interval. Therefore, the inclusion function concept has been developed to calculate efficiently an interval enclosing the image set (Malan, Milanese, Taragna, & Garloff, 1992).

3.2. Constraints satisfaction problem

Constraints satisfaction problems (CSPs) were initially defined for discrete domains where the variables x_i belong to finite sets (Dechter, 2003). Later, CSP were extended to continuous domains.

Let consider a box $[x]$ of IR^n and m relationships representing the constraints and linking the components $[x_i]$ of $[x]$:

$$f_i(x_1, \ldots, x_n) = 0, \quad i = 1, \ldots, m. \tag{1}$$

Let f be the Cartesian product of f_i. Eq. (1) can be rewritten in a vectorized form as $F(x) = 0$.

The problem of finding the minimal $[x]$ box satisfying all the constraints f_i corresponds to a CSP denoted $\mathscr{H}$, which can be formulated as:

$$\mathscr{H} : (F(x) = 0 |\, x \in [x]). \tag{2}$$

The solution set of $\mathscr{H}$ is defined as

$$S = \{x \in [x] |\, F(x) = 0\}. \tag{3}$$

3.3. Consistence

A scalar x_i belonging to $[x_i]$ is *globally* consistent with $\mathscr{H}$, if it is possible to find a vector x within S, and for which x_i is the ith coordinate:

$$\exists\, x_1 \in [x_1], \ldots, x_{i-1} \in [x_{i-1}], x_{i+1} \in [x_{i+1}], \ldots, x_n \in [x_n] |\\ f(x_1, \ldots, x_{i-1}, x_i, x_{i+1}, \ldots, x_n) = 0,$$

x_i is *locally* consistent with $\mathscr{H}$ if for such constraint f_i (taken separately), it is possible to find a vector x consistent with f_i, and for which x_i is the ith coordinate:

$$\forall\, f_i,\ \exists x_1 \in [x_1], \ldots, x_{i-1} \in [x_{i-1}], x_{i+1} \in [x_{i+1}], \ldots, x_n \in [x_n] |\\ f_i(x_1, \ldots, x_n) = 0.$$

An interval $[x]$ is globally (respectively, locally) consistent with $\mathscr{H}$ if $\forall x_i \in [x]$, x_i is globally (respectively, locally) consistent with $\mathscr{H}$. Global consistence implies local consistence.

3.4. Contractors

To contract $\mathscr{H}$ means to substitute the box $[x]$ by a smaller box $[x'] \subset [x]$ containing all the solutions of $\mathscr{H}$ i.e. $S \subset [x']$.

A contractor for $\mathscr{H}$ is defined as an operator used to contract $\mathscr{H}$.

There are different kinds of contractors (Jaulin, Kieffer, Didrit, & Walter, 2001b) developed in order to reduce pessimism of returned box, adapted to different classes of problems. In this paper, a FBP technique based on *primitive* constraints (Benhamou, Goualard, Granvilliers, & Puget, 1999), is used, because of the great redundancy of data and equations, and because of the independency of the method to non-linearities. It is a locally consistent contractor. It consists in two steps. First, the forward propagation step considers the direct forms of the equations. Second, the backward stage uses inversion forms of the functions appearing in the equations. These two steps can be illustrated by the following example.

Let consider the constraint $z = x\sin(y)$. At first, this constraint is decomposed into primitive constraints. A primitive constraint involves only an arithmetic operator or a usual function (cos, exp, etc.).

$$\begin{cases} a = \sin(y), \\ z = x.a, \end{cases} \tag{4}$$

where a is an auxiliary variable initialized by $[a] = [\infty, +\infty]$. Let [sin] and $[\sin^{-1}]$ be inclusion functions for functions sin and $\sin^{-1}$.

The FBP works as follows.

%forward propagation

$[a]=[a]\cap[\sin]([y])$ F1

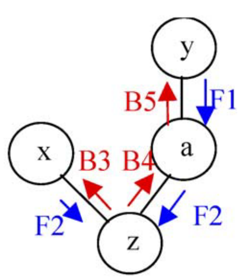
$[z]=[z]\cap[x].[a]$ F2

%backward propagation

$[x]=[x]\cap[z]/[a]$ B3

$[a]=[a]\cap[z]/[x]$ B4

$[y]=[y]\cap[\sin^{-1}]([a])$ B5



Please, notice that the order of the constraints is important. In the considered example, this order is globally optimal as shown by the graph. For more details, please see (Jaulin, Kieffer, Braems, & Walter, 2001a).

Suppose that $x \in [x] = [-1; 2]$, $y \in [y] = [0; \pi/6]$, $z \in [z] = [1; 3]$. In this case, the FPB contractor gives the following results:

$$\text{F1}: [a] = [a] \cap [\sin]([y]) = [-\infty, +\infty] \cap [\sin]([0, \pi/6]) \\ = [0, 0.5],$$

$$\text{F2}: [z] = [z] \cap [x].[a] \\ = [1, 3] \cap [-1, 2].[0, 0.5] = [1, 3] \cap [-0.5\ 1] \\ = [1, 1],$$

$$\text{B3}: [x] = [x] \cap [z]/[a] \\ = [-1, 2] \cap [1, 1]/[0, 0.5] = [-1, 2] \cap [2, +\infty] \\ = [2, 2],$$

$$\text{B4}: [a] = [a] \cap [z]/[x] = [0, 0.5] \cap [1, 1]/[2, 2] \\ = [0, 0.5] \cap [0.5, 0.5] = [0.5, 0.5],$$

$$\text{B5}: [y] = [y] \cap [\sin^{-1}]([a]) \\ = [0, \pi/6] \cap [\sin^{-1}]([0.5, 0.5]) \\ = [0, \pi/6] \cap [\pi/6, \pi/6] = [\pi/6, \pi/6].$$

When there are several redundant constraints, the graph coming from the elementary constraints decomposition contains cycles. In that case, the FBP contractor may not reach global consistence. A solution is then to apply the algorithm of Waltz which provides locally consistent boxes. Its principle is to repeat the propagation until the intervals do not contract any more (Waltz, 1975).

4. Data fusion using the FBP contractor

4.1. Global model for data fusion process

The data fusion problem is solved as a state observation problem with redundant measurements: some of them appear in the evolution model, materialized by the input u_k, and the others appear in the observation equations, materialized by the output y_k. So at any instant k, one tries to reconstitute x_k knowing x_{k-1} and current measurements which give u_k and y_k. The system which describes this process is given by the following equations:

$$\begin{cases} x_k = f(x_{k-1}, u_k, p_k, \alpha_k, \gamma_k) \\ y_k = g(x_k, p_k, \beta_k), \end{cases} \tag{5}$$

where k represents the discretization of time, $u_k \in IR^q$ represents the measured input vector, $y_k \in IR^m$ represents the measured output vector, $x_k \in IR^n$ represents the state vector of the system, $p_k \in IR^p$ is the constant parameters vector, imprecisely known, $\alpha_k \in IR^n$ represents the model noise, $\gamma_k \in IR^q$ represents the measurements noise perturbing the input u_k, $\beta_k \in IR^m$ represents the measurements noise perturbing the output y_k.

4.2. Formalization

System (5) can be viewed at any time index like a CSP $\mathscr{H}_k: (F(x) = 0 | x \in [x])$ where

$x = (x_k, \ldots, x_0, u_k, \ldots, u_0, y_k, \ldots, y_0, p_k, \ldots, p_0)$ and

$$F: x \to F(x) = (x_k - f(x_{k-1}, u_k, p_k), \ldots, x_1 - f(x_0, u_1, p_1), \\ y_k - g(x_k, p_k), \ldots, y_1 - g(x_1, p_1)).$$

The noises α_i, γ_i and β_i do not appear explicitly in $\mathscr{H}_k$ since there are integrated to build the intervals $[x_i]$, $[u_i]$, and $[y_i]$, with the hypothesis that they are additive. For example, if the bounds of β_k are $\underline{b}$ and $\overline{b} \in IR^m$, one can write

$$y_k \in [y_{k,m} - \underline{b}, y_{k,m} + \overline{b}]. \tag{6}$$

Please notice that the parameters p_k which are known with errors are treated as unknown quantities in the FBP.

In order to determine the bounds of the noises, usually several approaches depending on sensors can be used: to use constructor's data sheets, to perform tests in comparison with a reference, or to use an estimated imprecision provided by the sensors in realtime. A crucial point associated with the determination of noises bounds is the incoherencies management. On the one hand, over-estimation induces pessimist estimation boxes, and on the other hand, under-estimation can lead incoherence between boxes.

The goal of the algorithm is to reduce for each step k the box $[x]$ of $\mathscr{H}_k$ with the FBP algorithm. The first step consists in decomposing all the constraints of $\mathscr{H}_k$ in primitive ones. Then, FBPs are applied until that the contractions become smaller than a chosen threshold.

For the CSP $\mathscr{H}_k$ to be solved, variable x include all variable in system (5) from step 0 to k. However, for a real-time implementation, it is unrealistic to consider all the equations from step 0 to step k. Therefore, a limited time windows (denoted "h" for *horizon*) is used. It represents the number of indices (from k to $k-h+1$) which appear in $\mathscr{H}_k$. In this case, $\mathscr{H}_k$ has the expression:

$$(F(x)=0 | x \in [x]),$$

where $x=(x_k,\ldots,x_{k-h+1},u_k,\ldots,u_{k-h+1},y_k,\ldots,y_{k-h+1})$ and $F: x \to F(x)=(x_k-f(x_{k-1},u_k,p_k),\ldots,x_{k-h+2}-f(x_{k-h+1},u_{k-h+2},p_{k-h+2}),\, y_k-g(x_k,p_k),\ldots,y_{k-h+2}-g(x_{k-h+2}p_{k-h+2}))$.

Algorithm FBP

Input: $\{[x_{k-1}], [u_{k-1}], [y_{k-1}], [p_{k-1}], \ldots, [x_{k-h+1}], [u_{k-h+1}], [y_{k-h+1}], [p_{k-h+1}]\}$

Output : $\{[x_k], \ldots, [x_{k-h+1}]\}$

% initialization step

Choose h and η (threshold of insignificant contraction)
Read the input u_k and its error. Deduce $[u_k]$
Read the output y_k and its error. Deduce $[y_k]$
$x_k = ([-\infty, \infty])_{i=1\ldots q}$ % State x_k initialization
$[p_k] = [p_{k-1}]$ % The parameters are supposed to be constant.

While at least one variable "is contracted more than η"

For $i=k-h+1$ **to** k

%forward propagation step

Propagate the constraint $\{x_i - f(x_{i-1}, u_i, p_i)=0\}$
Propagate the constraint $\{y_i - g(x_i, p_i)= 0\}$

%back propagation step

Retro-propagate the constraint $\{x_i - f(x_{i-1}, u_i, p_i)=0\}$
Retro-propagate the constraint $\{y_i - g(x_i, p_i)=0\}$

End
End

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5. Multi-sensor fusion

In this section, the localization of a car is considered as a multi-sensor fusion problem that can be formalized as a state estimation problem and solved thanks to the FBP algorithm.

5.1. Global architecture of the localization process

The vehicle frame origin is chosen at the middle of rear axle and its x-axis is parallel to the longitudinal axle of the car.

Between two sampling instants, elementary rotations of the two rear wheels are integrated by counters which provide the distance traveled by each wheel. The elementary displacement covered by the origin of the frame denoted $\delta_{s,k}$ and the rotation denoted $\delta_{\theta,k}$, at instant k, are given by the following equations:

$$\begin{cases} \delta_{s,k} = \dfrac{\delta_{RR}+\delta_{RL}}{2}, \\ \delta_{\theta,k} = \dfrac{\delta_{RR}-\delta_{RL}}{e}, \end{cases} \tag{7}$$

where $\delta_{RL,k}$, $\delta_{RR,k}$ denote the measured variables (the values counted between two samples), $\delta_{s,k}$, $\delta_{\theta,k}$ are the estimated variables, L and e are the vehicle parameters (the distance between the axles and the wheel base).

Moreover, the gyro provides the rotation between two samples $\delta_{\theta,\text{gyro}}$, which gives

$$\delta_{\theta,k} = \delta_{\theta,\text{gyro}}. \tag{8}$$

The mobile vehicle pose X_k, is calculated, thanks to X_{k-1} and thanks to the output $(\delta_{s,k}, \delta_{\theta,k})$ of the static fusion process:

$$\begin{cases} x_k = x_{k-1} + \delta_{s,k}\cos(\theta_{k-1}+\delta_{\theta,k}/2), \\ y_k = y_{k-1} + \delta_{s,k}\sin(\theta_{k-1}+\delta_{\theta,k}/2), \\ \theta_k = \theta_{k-1} + \delta_{\theta,k}, \end{cases} \tag{9}$$

where x_k and y_k represent the vehicle position, at time t_k, in the reference frame. θ_k is the heading angle.

The GPS antenna has been installed on the vertical of the origin frame, so, the observation model is given by the linear equation:

$$\begin{cases} x_{\text{gps}} = x_k, \\ y_{\text{gps}} = y_k, \end{cases} \tag{10}$$

where x_{gps} and y_{gps} are the GPS measurements.

For a better clearness and more simplicity, let consider two levels of fusion. The elementary rotation 'δ_θ' and elementary displacement 'δ_s' are given by a static fusion stage which uses the measurements of the ABS sensors of the two rears wheels of the car and a yaw fiber optic gyro. The redundancy of the information provided by these sensors should reduce in a significant way, the pessimism of the initial box and therefore should give final boxes for $\delta_{\theta,k}$ and $\delta_{s,k}$ with a good accuracy. The result of this fusion is the input of a dynamic module which computes the vehicle location.

5.2. Static fusion

At the static fusion stage, the FBP returns guaranteed intervals $\delta_{\theta,k}$ and $\delta_{s,k}$ solutions of the CSP

$$\mathscr{H}_k: (F(x)=0 | x \in [x]),$$

where $x=(\delta_s, \delta_{RL}, \delta_{RR}, \delta_q, L, e)$, δ_{RL}, δ_{RR} are the traveled distances of the two rear wheels, δ_θ is the gyro measurement, F represents the system of equations (7).

With the assumption that the vehicle runs without slipping, the maximum distance error between two instants t_{k-1} and t_k, is less than one top of the ABS sensor (denoted δ_{ABS}).

$$[\delta] = [\delta_{\text{mes}} - \delta_{\text{ABS}}, \delta_{\text{mes}} + \delta_{\text{ABS}}]. \tag{11}$$

Thanks to specific static tests, the bound of the error of the gyro has been estimated as

$$\delta_{\theta,\mathrm{gyr}} \approx 3 \times 10^{-3} \text{ degrees} \tag{12}$$

$$[\delta_\theta] = [\delta_{\theta,\mathrm{mes}} - \delta_{\theta,\mathrm{gyr}}, \delta_{\theta,\mathrm{mes}} + \delta_{\theta,\mathrm{gyr}}]. \tag{13}$$

Please notice that to consider the equality between $\delta_{\theta,k}$ and $\delta_{\theta,\mathrm{gyr}}$ as a constraint (Eq. (8)) is equivalent to initialize the variable $[\delta_{\theta,k}]$ with the interval corresponding to the gyro measurement, as it has been proposed above.

$[\delta_S]$ is not measured and so, initialized with unknown value: $[-\infty, +\infty]$, $[e]$ is the wheel base of the vehicle. It is roughly known that is why, in part, it can be treated as an unknown quantity in the FBP.

The estimates of $[\delta_{\theta,k}]$ and $[\delta_{s,k}]$ are obtained thanks to the FBP contractor applied to the CSP $\mathscr{H}_k$. Please notice that between two samples, the CSP $\mathscr{H}_k$ and $\mathscr{H}_{k-1}$ are not connected. In addition, an originality of the method is that all the variables of the CSP can be contracted even the measurements and the parameters used in the model.

5.3. Dynamic fusion

For any time index k, the new CSP to be solved with a chosen horizon equals to 'h', is

$$\mathscr{H}_k : (H(x) = 0 | x \in [x]),$$

where $x = (x_k, \ldots, x_{k-h+1}, y_k, \ldots, y_{k-h+1}, \theta_k, \ldots, \theta_{k-h+1},, \delta_{S,k}, \ldots, \delta_{S,k-h+1}, \delta_{\theta,k}, \ldots, \delta_{\theta,k-h+1})$, $[\delta_{S,i}]$ and $[\delta_{\theta,i}]$ are provided by the static fusion stage, the heading angle $[\theta_i] = [-\infty, +\infty]$ is not measured, H represents the evolution model (9).

The GPS measurement $(x_{\mathrm{gps,mes}}, y_{\mathrm{gps,mes}})$ is used to initialize intervals $[x_k]$ and $[y_k]$. So, each longitude/latitude estimated point is converted in a Cartesian local frame (see Fig. 2) and the GPS bounded-error measurement is obtained thanks to the GST NMEA sentence. The error bounds are supposed to be equal to three times the estimated standard deviation $\hat{\sigma}$ computed in real time by the GPS receiver:

$$[x_{\mathrm{gps}}] = [x_{\mathrm{gps,mes}} - 3\hat{\sigma}_x, x_{\mathrm{gps,mes}} + 3\hat{\sigma}_x],$$

$$[y_{\mathrm{gps}}] = [y_{\mathrm{gps,mes}} - 3\hat{\sigma}_y, y_{\mathrm{gps,mes}} + 3\hat{\sigma}_y]. \tag{14}$$

For more simplicity and like it has been done for the static fusion, constraints corresponding to equalities between variables like those given by GPS measurements, are used as a simple initialization as it has been done for $[x_k]$ and $[y_k]$ above.

This CSP gives a general solution to the non-linear state observation for any instant t_k.

6. Experimental results

The results presented in this section were obtained by post-processing real sensor's measurements acquired thanks to an experimental car (Fig. 1). Data of the two ABS sensors and of the fiber optic gyro (a KVH RD100) were sampled at 100 Hz.

Fig. 1. The experimental car with the Ag132 and Thales GPS receivers.

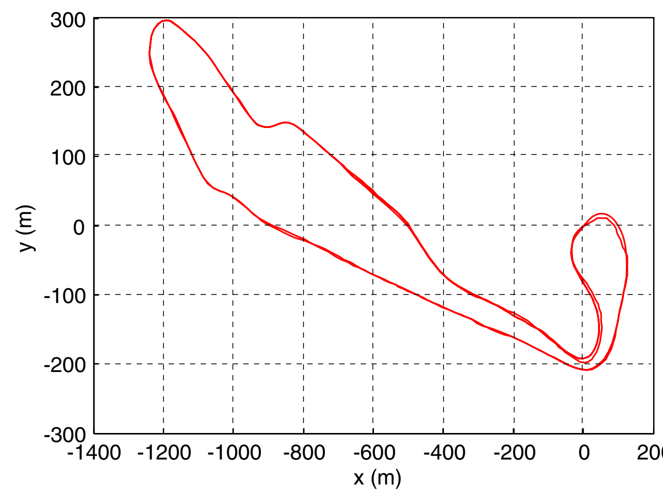

Fig. 2. Overview of the trial in a local frame.

The differential GPS receiver (a Trimble AgGPS132, L1 only, working with Omnistar corrections available in Europe) was used in a synchronized mode at 5 Hz. Using the PPS signal, all the data were resampled at 5 Hz and the GPS latency has been compensated. This simplifies the development of the two fusion methods.

In order to be able to compute estimation errors, a L1/L2 Thales navigation GPS receiver is used in a post-processed kinematic mode working with a local base (a Trimble MSi 7400). This system was able to give positions of reference with a 1 Hz sampling rate. Since the constellation of the satellites was good enough during all the trials (April 2004), all the kinematics ambiguities were fixed. Therefore, it is guaranteed that a few centimeters accuracy was reached. The synchronization between this reference and the outputs of the dynamic localizers (FBP and EKF) has been made thanks to the GPS timestamps. The position offsets between the antennas of the two GPS receivers and the origin of the mobile frame (Fig. 1) have also been taken into account.

Fig. 2 shows a top view of an experiment. It corresponds to two 10 minutes laps. The conditions of the experiment are shown on Fig. 3. The mean speed was 50 kmph and reached

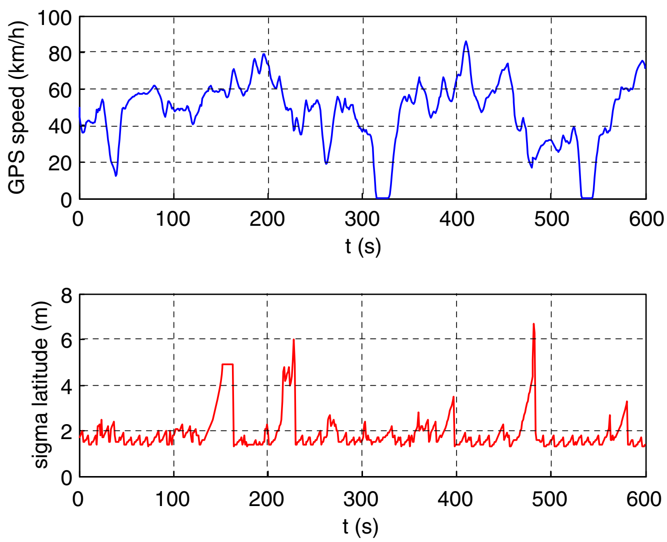

Fig. 3. Speed and standard deviation of the latitude.

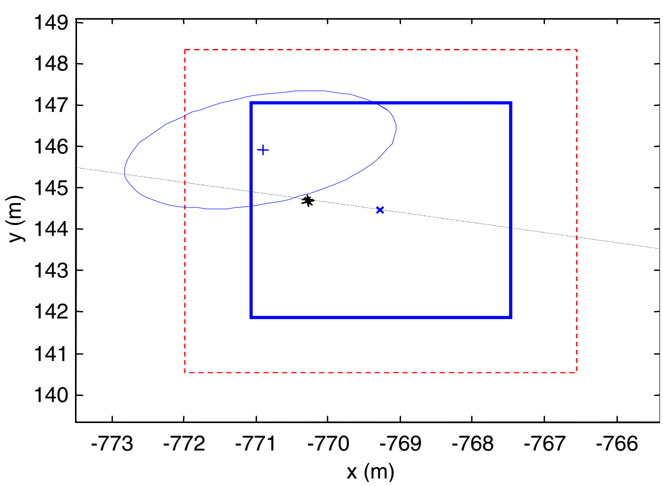

Fig. 4. Illustration of the too great confidence of the EKF.

80 kmph. The standard deviation of the latitude indicates that the differential corrections of the geostationary satellite Omnistar were lost three times because of trees (see Fig. 2). In this case, the Ag132 still propagates an estimation of the correction during 30 s. After that it works in a "natural mode" (in opposite to differential mode) and the precision decreases significantly. Thanks to a quality indicator contained in the GST NMEA sentence, these different modes can be known.

Fig. 4 illustrates the fact that the EKF cannot guarantee a maximum error bound, since the PPK reference position (denoted by the star *) is, at this precise time instant, outside of the Gaussian estimated 99% ellipse. On the contrary, the box obtained by FBP contains the PPK reference. Moreover, one can remark the good contraction of the GPS box provided by the FBP (the GPS box is in dot and the result of the FBP is in bold).

Figs. 5 and 6 show the interval error of the EKF and the FBP, respectively, for the x and y dimensions. In addition to

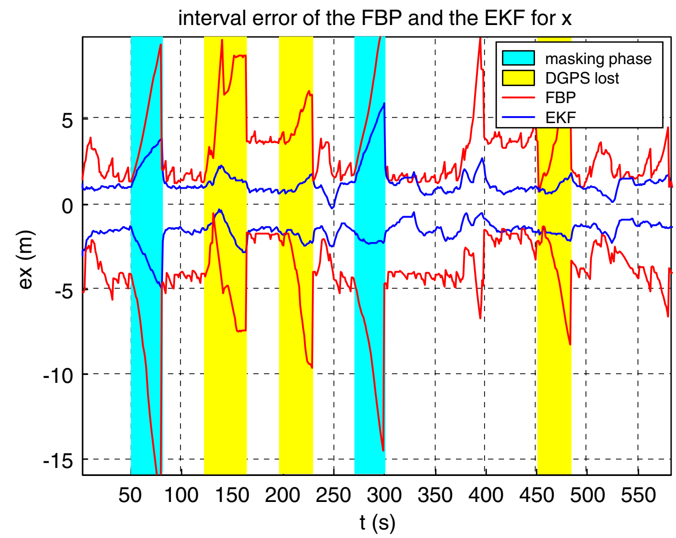

Fig. 5. Comparison between bound errors of EKF and FBP for x.

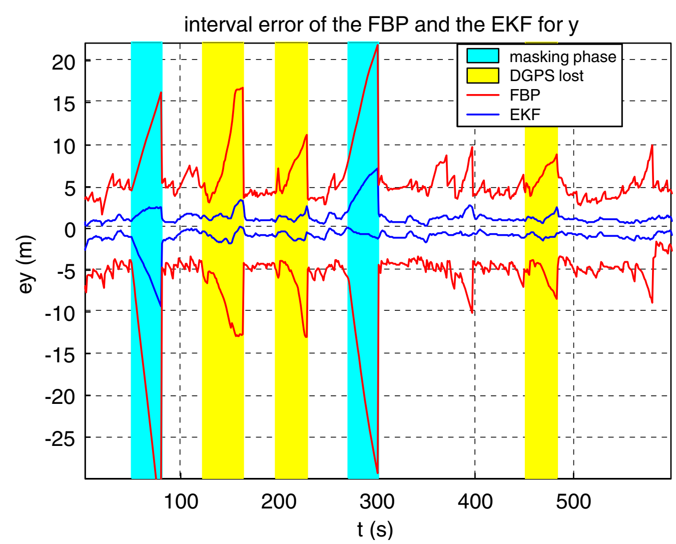

Fig. 6. Comparison between bound errors of EKF and FBP for y.

the loss of differential correction, two complete GPS masks of 30 s duration each, had been simulated, at the instants 7 and 270 s. During these masks, the speed was about 50 kmph (cf. Fig. 3).

It appears from these results that the EKF is more accurate since its estimated 3σ errors bounds are smaller than the guaranteed intervals of the FBP. This is due in part to the fact that the FBP provides locally consistent estimated intervals. Nevertheless, one can notice that the values are of the same order of magnitude. This proves the feasibility of a bounded-error approach.

It can be seen that the value "0" is always included in the FBP estimated intervals, which confirms the fact that the results are guaranteed: to contain "0" is equivalent to say that the box contains the PPK's point. On the contrary, the EKF cannot guarantee a maximum error and sometimes underestimates its

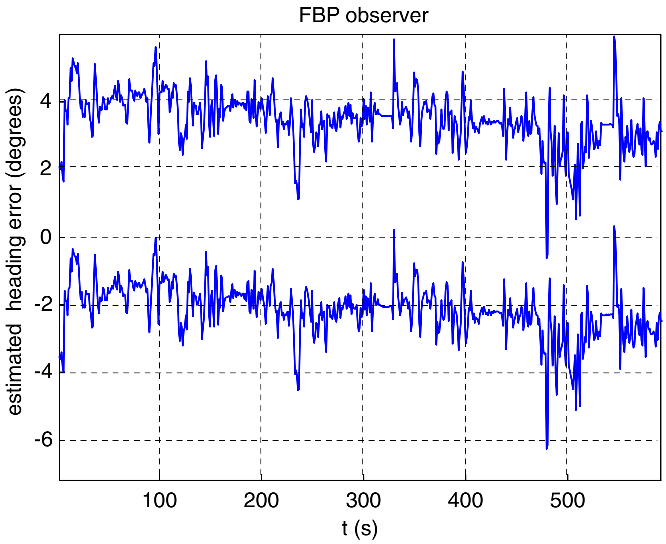

Fig. 7. Observation error of the heading angle estimated by the FBP (in degrees).

error. Indeed, several real points are out of the 3 sigma-bound as shown by Figs. 5 (time ≈ 250 s) and 6 (time ≈ 120 s, 400 s).

Fig. 7 plots the heading estimated error of the FBP and illustrates the fact that this observer is able to reconstruct a none directly measured variable. The three times where the value “0” does not make part of the estimated interval are due to a noise affecting the reference heading angle since it has been built manually from the PPK measurements. One can notice the very good heading estimation provided by the FBP.

Finally, the processing has never detected any inconsistency in data used in this trail which indicates a good tuning of the FBP.

7. Conclusion

This paper has presented a new dynamic localization technique based on constraints propagation on real intervals. This approach guarantees that the real pose of the car is included in the estimated boxes even if the equations are highly non-linear. This approach has been compared with the usual technique based on Kalman filtering which relies on linearization and zero-mean white Gaussian assumptions for the noises.

In the problem treated in this paper, there is a high redundancy in data and equations since two encoders, a gyro and a DGPS receiver have been used. Thanks to this redundancy, contractors alone can be used (i.e. without bisection) since the consistence of all the data produces rather precise estimates (i.e. not too pessimistic) as shown by the experimental results. The contractor presented is based on forward and backward propagation (FBP). With a 1.8 GHz Pentium 4 and a Matlab implementation, the FBP treats 10 minutes of data in roughly 150 s which is approximately 10 times the calculation duration of the EKF. Even if the FBP needs more computations than the EKF, one can conclude that FBP is well adapted to a real-time context.

The major implementation problem of such an approach is to determine correctly the bounds of the noises. Indeed, if these bounds are underestimated, the contractor may lead to no solution. On the contrary, if the bounds are overestimated, the estimated boxes can be very large (the estimates are then very pessimistic). A methodological way to treat the tuning of these parameters represents the main perspective of this research.

Moreover, it turned out that the estimates provided by the FBP are more pessimist than the one of a Kalman filter, if we consider the 3σ bounds of the filter output. It is quite natural since a bounded-error approach is always pessimist because its estimates are guaranteed. Nevertheless, the experimental results indicate that the precision is near to the one of the Kalman filter. The latter can be mistaken in giving sometimes too trustful results or can converge toward local minima.

We think that for applications that need a high level of integrity, the pessimism of the estimates is not handicapping whereas the fact of obtaining guaranteed results is essential.

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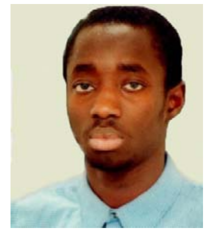

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