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Road Tracking for Multi-Hypothesis Localization on Navigable Maps

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Abstract – **Efficient and reliable map matching algorithms are essential for *Advanced Driver Assistance Systems*. While most of the existing solutions fail to provide trustworthy outputs when the situation is ambiguous (road intersections, roundabouts, parallel roads …), we present in this paper a new map-matching method based on a multi-hypothesis road tracking that takes advantage of the geographical database road connectedness to provide a reliable road-matching solution with a confidence indicator.**

Index Terms – GNSS-based Localization, Map-Matching, Multi-Hypothesis Tracking

I. INTRODUCTION

Map-matching (MM), using GNSS positioning and navigable maps, is a data association problem which consists in selecting the most likely road that corresponds to the current position of the mobile. Unfortunately, as a result of inaccuracies in the map or because of large estimation errors, map-matching often has several solutions, i.e. several segments are declared candidates with good confidence. These segments can belong to the same road or to different roads (ambiguous situation).

To solve this problem, we propose in this paper a multi-hypothesis road tracking method that attempts to exploit data pertaining to road-connectedness. This approach belongs to the class of dynamic state observers, and therefore makes use of multisensor fusion capabilities.

Tracking techniques [7] allow a system to observe and follow the state of a mobile target by filtering noisy observations. They have very efficient implementations since they often rely on first order Markov assumption, which means that all the information can be captured in the current state estimation. Therefore, it is unnecessary to keep in memory a window of data; by using a recursive scheme, previous states can be forgotten.

For localization purposes, tracking the pose (position and attitude) of a mobile is very useful since it allows fusing sequentially redundant data, once the initial global localisation stage has been solved. Indeed, in practice model equations are non linear, and an arbitrary initialization can conduct to a wrong convergence.

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The spatial road network data can be also used to improve the positioning accuracy, for instance when GPS is not available. Indeed, the road network can be used to constraint the localization space (geometry) and to predict the next future (connectivity). Therefore, a problem is to integrate such navigable map information in the localization tracker.

Map-matching induces unavoidable ambiguity situations for instance at junctions or with parallel roads, or when GPS suffers from outages. By applying a mono-hypothesis approach, the risk is to choose a wrong solution. When the system will detect this mistake, it will need time to recover the good solution and the tracking will be reset. A multi-hypotheses approach, on the contrary, will maintain all the possible solutions in case of ambiguity; each hypothesis lives in its own world ignoring the other ones. Hypotheses that become unlikely are removed as time and travelled distance evolve. Using a Bayesian framework, it is possible to quantify the probability of the hypotheses. So, at each step, the most probable hypothesis can be output. The main advantage of Multi-Hypotheses Map-Matching (MHMM) over a Mono-Hypothesis approach is that the true solution is tracked with a high probability: if the current solution is declared incorrect, the system can output immediately a new solution without any transient phase.

In general, algorithmic complexity of MHMM is exponential since each hypothesis can generate at each step new ones. In this paper, we propose to use the road connectedness information of the navigable map to solve this issue in order to create new hypotheses only when necessary. This is one of this paper's contributions. We present a MHMM based on a Gaussian mixture that consists in associating a Electronic Horizon (of the global road-map) to each hypothesis that performs a Gaussian filter. The associate sub-map is a set of two roads that the hypothesis is supposed to follow. A weight (called also score) is associated to each hypothesis for the management of the hypotheses set. It indicates the probability of each hypothesis with respect to the others.

In this paper organized in 4 sections, we present the different elements of this strategy and propose finally a new Map-Matching integrity criterion that has been tested under real conditions using GPS, a gyrometer, an odometer and a NavTeQ database. Experimental results illustrate the performance of this approach.

II. Localization using an a priori map information

Suppose that a map information source is available. This map provides *a priori* information that constraints the localization space. For example, a car has a better chance of being on a road, and is unlikely to go through a building. The cartographic information considered here is a set of roads described by nodes connected to each other. Each road is made of a begin node and an end node, with several intermediate points.

In this section, we formalize the problem of using *a priori* cartographic information in a probabilistic localization process. We will show that the map can be used as an observation (like any exteroceptive measurement) in a state observation process.

Suppose that s_k represents the mobile state vector at time k; z_k is an exteroceptive sensor observation (a GPS for example).

$$\begin{cases} s_k = f(s_{k-1}, u_k) + \alpha_k \\ z_k = h(s_k) + \beta_k \end{cases} \quad (1)$$

The localization problem consists in estimating the probability $p(s_k \mid z^k, g, u^k)$, knowing the set of observations $z^k = \{z_k, \ldots, z_1\}$, and the a priori geographical information g. u^k represents the proprioceptive sensors used as input.

Let's see how this geographical information can be used in order to estimate this probability density.

$$p(s_k \mid z^k, g, u^k) = p(s_k \mid z_k, z^{k-1}, g, u^k) \quad (2)$$

Using Bayes theorem, eq. (2) can be also written like:

$$p(s_k \mid z_k, z^{k-1}, g, u^k) = \frac{p(z_k \mid s_k, z^{k-1}, g, u^k).p(s_k \mid z^{k-1}, g, u^k)}{p(z_k \mid z^{k-1}, g, u^k)} \quad (3)$$

The denominator p(zk | zk-1 ,g, uk) is independent of sk. It can be considered as a normalization term η.

Let's consider now each of the two expressions of this product.

The observation z_k at time k is independent of all the previous z^{k-1}, the observation noise being a white one. By remarking also that the exteroceptive sensor noise is independent of the map g, we can write:

$$p(z_k \mid s_k, z^{k-1}, g, u^k) = p(z_k \mid s_k) \quad (4)$$

Let's consider now the second term of the product and let's make the density a priori $p(s_k \mid z^{k-1}, g, u^k)$ appear by using the total probabilities and Bayes' theorems:

$$p(s_k \mid z^{k-1}, g, u^k) = \int p(s_k, s_{k-1} \mid z^{k-1}, g, u^k).ds_{k-1} \quad (5)$$

$$= \int p(s_k \mid s_{k-1}, z^{k-1}, g, u^k).p(s_{k-1} \mid z^{k-1}, g, u^k).ds_{k-1} \quad (6)$$

However, $p(s_k \mid s_{k-1}, z^{k-1}, g, u^k)$ represents the evolution model. It is independent from the observations z^k and under the assumption of a 1st order Markov process, it depends only on the current entry u_k.

$$p(s_k \mid z^{k-1}, g, u^k) = \int p(s_k \mid s_{k-1}, g, u_k).p(s_{k-1} \mid z^{k-1}, g, u^{k-1}).ds_{k-1} \quad (7)$$

Let's substitute (4) and (7) into equation (3):

$$\underbrace{p(s_k \mid z^k, g, u^k)}_{Localization\ at\ k} = \eta.p(z_k \mid s_k)\ldots \int p(s_k \mid s_{k-1}, g, u_k).\underbrace{p(s_{k-1} \mid z^{k-1}, g, u^{k-1})ds_{k-1}}_{Localization\ at\ k-1} \quad (8)$$

Let's consider now the term $p(s_k \mid s_{k-1}, g, u_k)$ that expresses the influence of the *a priori* information in the localization process: it can be used in the prediction step [8], [9], or considered as an observation as we proposed. Using Bayes' theorem, one can write:

$$p(s_k \mid s_{k-1}, g, u_k) = \frac{p(g \mid s_k, s_{k-1}).p(s_k \mid s_{k-1}, u_k)}{p(g \mid s_{k-1})} \quad (9)$$

By supposing that the cartographic observation depends only on the current pose, we have:

$$p(g \mid s_k, s_{k-1}) = p(g \mid s_k) \text{ and } p(g \mid s_{k-1}) = p(g) \quad (10)$$

To make these two assumptions valid, it is necessary that the vehicle moves relatively to the map ($s_k \neq s_{k-1}$). By making substitutions in the equation (9), we obtain:

$$p(s_k \mid s_{k-1}, g, u_k) = \frac{p(g \mid s_k).p(s_k \mid s_{k-1}, u_k)}{p(g)} \quad (11)$$

By introducing $\eta' = \frac{\eta}{p(g)}$, Eq. (3) can thus be written as:

$$p(s_k \mid z^k, g, u^k) = \eta'.p(z_k \mid s_k).p(g \mid s_k)\ldots .\int p(s_k \mid s_{k-1}, u_k).p(s_{k-1} \mid z^{k-1}, g, u^{k-1}).ds_{k-1} \quad (12)$$

In this expression $p(z_k \mid s_k)$ and $p(g \mid s_k)$ represent respectively the likelihood of the exteroceptive observation and the one of the map g relatively to the predicted position s_k. The map thus is well considered here as an observation.

As an example, let us consider a map made of with one segment representing a road on which the vehicle moves. This case is represented by Fig. 1 which illustrates $p(g \mid s_k)$, with g being the map. Let's consider the line Δ passing by $h(s_k)$ (where $h(s_k)$ is the projection of the state s_k in 2D map observation space) and perpendicular to the segment under consideration. Let us suppose that along Δ the probability density function (pdf) $p(g \mid s_k)$ is Gaussian. The likelihood is obtained by calculating the innovation μ (which is the deviation with the road here) and by using it in a Gaussian pdf.

One can note that the pdf is not necessarily Gaussian; it can have any shape and be multimodal because of several poly-lines close to the prediction. It's interesting to notice that every pdf can be approximated by a mixture of Gaussians. Let $p_A(s)$ be the approximation of the pdf $p(s)$ associated with a random vector. $p_A(s)$ is defined by:

$$p_A(s) \equiv \sum_{i=1}^{N} w_i . N(s - \bar{s}_i, P_i) \quad (13)$$

$$\sum_{i=1}^{N} w_i = 1 \quad and \quad 0 \le w_i \le 1 \quad (14)$$

Where w_i are the weights associated to each Gaussian i. As N increases, $p_A(s)$ tends toward $p(s)$. In the multiple hypothesis filtering presented in the next section, the road tracking will allow to avoiding the use of Gaussian mixturing..

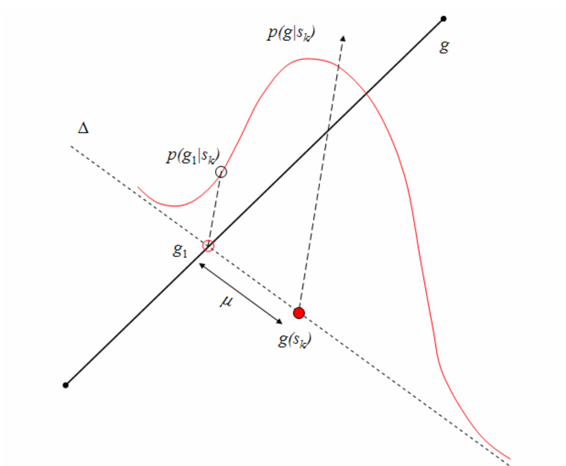

Fig. 1 Map pdf modelled by a Gaussian pdf.

III. Using a Map-Matching Method Based on a Multi-Hypothesis Approach

In this section, we define the concept of localization hypothesis in relation to MHMM. We show in details the management policy of the hypotheses by associating to each Gaussian filter an elementary electronic horizon (noted EH) made of with 2 roads.

A. Hypothesis definition

The EH associated with each Gaussian filter consists of two roads: the road from the mother hypothesis (the original hypothesis which has generated the current hypothesis) and a road that is connected to this hypothesis in the travelling direction. The EH has a depth equals to 1.

A hypothesis F_i at time k is defined as being composed of the elements shown in table 1.

Fi : **Localization Hypothesis**
★ a state: a state vector $s_{i,k}$ and its associated covariance matrix $P_{i,k}$,
★ An electronic horizon g_i that includes the road of the mother-hypothesis R_{idm} and an upcoming one R_{idf}, connected to it $g_i = \{R_{idm,i} , R_{idf,i}\}$
★ A weight (score) $w_{i,k}$, corresponding to the probability of the hypothese with respect to the others
★ An Normalized Innovatin Squared (NIS) $vr_{i,k}$ quantifying to consistence of this hypothesis.

***Table** 1. Definition of a localization hypothesis*

B. Hypothesis Creation

An important issue is to consider an efficient strategy when a hypothesis comes to the end of its road.

Let's suppose that a hypothesis approaches the end of a road-segment, and let's assume that the current road is connected to 2 upcoming roads. A first idea is to duplicate the actual hypothesis into two others: each one corresponding to one of the upcoming roads. The EH associated with each hypothesis includes the actual road and one of the two upcoming ones. Please note that at the time of duplication the Gaussians have the same weight. Another idea is to clone the current hypothesis with anticipation. This is essential in order to take into account the map and estimation errors.

More generally, let's suppose that at a time k, a hypothesis i designated by $F_{i,k}$ ($s_{i,k}$, $P_{i,k}$, g_i, $w_{i,k}$, $vr_{i,k}$) arrives at a distance Δ from the end of its EH gi. The hypothesis $F_{i,k}$ is divided into a number of new hypotheses. The information on the number of roads connected to the end of the actual one n_c, is stored in the map structure: the number of created hypotheses is equal to the number of roads connected n_c. For j = 1 to n_c, each new hypothesis j gets the same weight as the mother-hypothesis i and the same state at the time of creation (ie state vector $s_{i,k}$ and covariance matrix $P_{i,k}$). The new EH g_j associated with each new hypothesis j contains a road from the EH $R_{idf,j}$ (roads connected to the end of the current segment) and the road associated to the mother runway $R_{idf,i}$, road on which the hypothesis F_i was evolving (One could write $R_{idm,,j} = R_{idf,i}$). Please, note that the new hypotheses don't keep the road $R_{idm,i}$ of their mother hypothesis because the size of each EH g_j would then increase endlessly. After transmitting its characteristics to the new created hypotheses, the mother hypothesis $F_{i,k}$ is eliminated. A normalizing step for the weights $\boldsymbol{w_k}$ is then carried out.

To illustrate the EH management associated with the hypotheses, consider the case of a simple intersection of 3 roads as shown on Fig. 2. Let's consider that the hypothesis F_i,, associated with the EH {ID_0, ID_1} has reached the threshold distance Δ to the end of road ID_1. Two hypotheses F_m and F_n as created from the properties of F_i. The EH associated with F_m and F_n, will be respectively composed of $\{ID_1\ ID_2\}$ and $\{ID_1\ ID_3\}$. F_i is right after eliminated.

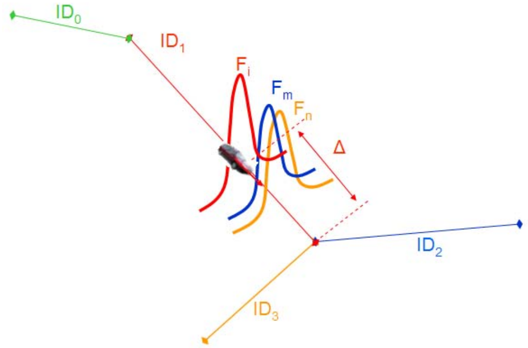

Fig. *2 Illustration of a 3 roads junction situation*

The weights w_i, the NIS vr_i, and the filters' estimates are updated by 2 exteroceptive sources: an hybridized GPS location (loosely coupled, noted h-GPS) and a map observation. The weights of the filters are updated by the h-GPS. The weight of the wrong hypotheses will decrease step after step. The NIS will increase but more rapidly than the weights (which are cumulative normalized probabilities). The NIS (eq. 15) can be interpreted as an indicator of the overall consistency of the system, since they correspond to a priori normalized residual quantities.

$$vr_{i,k} = N(z_k ; h_k(s_{i,k|k-1}), H_k.P_{i,k|k-1}.H_k^T + R_k) \quad (15)$$

Where R_k is the covariance matrix of the observation error β_k and H_k the Jacobian relating the observation z_k to s_k.

C. Hypothesis Deletion

As soon as a hypothesis' weight falls below a fixed deletion threshold ξ_{el}, we consider that it is no more credible and is eliminated. To avoid the deletion of a credible hypothesis F_i whose instant likelihood $vr_{i,k}$ may decrease excessively at time k, because of an inappropriate observation, for example, that will make its weight $w_{i,k}$ falls below the deletion threshold ξ_{el}, we propose to filter the computing weight $w_{i,k}$:

$$w_{i,k} = vr_{i,k}.w_{i,k-1} + L_{mem}.w_{i,k-1} \quad (16)$$

Where L_{mem} is a forget factor that quantifies the part of the former w_{k-1} that is injected in w_k. L_{mem} must verify $0<L_{mem}<1$ (typically $L_{mem} = 0.1$). Please note that threshold ξ_{el} is a parameter that is tuned respectively to the map offset.

D. Detecting the tracking divergence

The tracking divergence can occur when all hypotheses are mistaken and become far away from the observations that update these hypotheses.

In usual conditions, if a hypothesis F_i moves away from the updating observations, its NIS vr_i will decrease in the update stage, and thus its weight w_i will also decrease. In the case where all the hypotheses move away from the updating observations, their weights will decrease, but as a normalizing step follows, the weights diminution will be no longer effective.

So, in order to detect a system divergence, a non-normalized sum of weights over a Δt time interval must be done, and the decision of the system detection divergence must be undertaken based on this computed sum. If the non-normalized computed sum stays below a fixed threshold δ_{div} during Δt, a re-initialization of the system is undertaken with the first valid GPS data. Please note that the system re-initialisation is a case that occurs rarely. It can be due to significant offsets of the digital maps in some places or connectedness errors.

E. Estimating the vehicle location from the different hypotheses

Several solutions can be proposed to achieve the estimated map-matching from the different hypotheses at time k. We propose to select a set of credible hypotheses as output: The ratio of the weight $w_{j,k}$ with respect to the maximum of the weights is the metric that is compared to a threshold δ_{imp} to characterize the probable output hypotheses.

$$(\overline{X}_k, \overline{P}_k) = \left\{ (X_{j,k}, P_{j,k}) \,\middle|\, \frac{w_{j,k}}{\max\limits_{i=1:N}(w_{i,k})} \geq \delta_{imp} \right\} \quad (17)$$

The threshold δ_{imp} must be chosen in some optimal way. If δ_{imp} is too small, an important number of hypotheses (including unlikely ones) will be proposed as outputs. On the contrary, a high δ_{imp} will reduce the number of likely hypotheses, to zero, one or two. In the particle filters literature, the notion of "effective particles" is often used to trig a new process of particles resampling. In [3], the number of effective particles is defined as:

$$N_{EFF_k} = \frac{1}{\sum_{i=1}^{N} w_{i,k}^2} \quad (18)$$

[3] proposes that if N_{Eff} becomes less than two-thirds of the total number of particles N, then the particles must be resampled. Using this idea, we linked the δ_{imp} with N_{Eff}:

$$\delta_{imp} = 1/(2.N_{Eff}) \quad (19)$$

F. Update step

We have 2 separate sources of exteroceptive observations: the GPS and the map observations. To compute efficiently the weights, the update steps are serialized. So, in the update step, the hypotheses' state is corrected by each observation (here GPS and map), and thus the weight of each hypothesis $w_{i,k}$ is also updated and normalized as many times as there are observations.

We prefer using a hybridized GPS instead of a standalone GPS to overcome the problems of GPS jumps and especially to the low availability of GPS in urban areas. If there is a masking and thanks to the navigation using the dead-reckoning prediction, h-GPS continues to provide exteroceptive observation to the MHMM system and the different hypotheses continue to be updated in terms of weight and state. It's important to note that the map data of the EH is always coherent with its hypothesis. However, a hypothesis can rapidly become inconsistent with the h-GPS (simply because it's a wrong hypothesis). Thus, we implemented a Chi-2 test with the h-GPS before correcting the hypothesis pose in case of any inconsistency with the GPS. Nevertheless, the weights are always updated.

Suppose now that the system is running under normal tracking operation (after the initialization stage). If we keep all the hypotheses, their number will increase without bound, given that, at the end of each road-segment, each hypothesis will be divided into at least two. So, we set a maximum allowed number of hypotheses (denoted N_{pmax}).

G. *Segment selection*

Each hypothesis has its EH composed of 2 roads. The road-matching method consists in selecting the nearest segment the direction of which is coherent with the vehicle heading. The orthogonal projection is considered as the map-matched position and used as an exteroceptive observation by the corresponding hypothesis filter.

H. *Integrity Monitoring*

Integrity of a localization system is the measure of confidence that can be accorded to the exactitude of the positioning delivered by this system. MM Integrity is a sub-product of MHMM. When the outputs of a localization system contain the true (but unknown) solution, the integrity of the system is reached. Therefore, since MHMM is able in theory to explore all the hypotheses, the true solution makes part of this set (under the hypothesis that the map contains all the roads). In this paper, our proposal is to declare the MM confident when the number of hypotheses equals one, and when a Chi-2 test on the NIS of the most likely hypothesis is passed. The Chi-2 test is done between the h-GPS and the most likely hypothesis, on the distance and heading variables.

IV. Experiments

Experiments have been performed in Compiègne using a KVH fibre optic gyro, an odometer input and a Trimble AgGPS 132 (L1-only receiver). The GIS used by the map-matching module is based on a Software Development Kit (SDK) developed by BeNomad [6].

A. *Results*

To illustrate the MHMM mechanism at intersections, Fig. 3 shows a real case. An initial hypothesis (dotted, light blue lines) arrives at the threshold distance Δ (here Δ=7m) from the end of the road. This road-end is connected to 2 different roads. The initial hypothesis is therefore divided into 2 new ones (one is shown in dark blue 'x' and the other in green '◇'). The h-GPS is shown in green '*'. The evolution of the weight *w* and the NIS of the two created hypotheses *vr* are shown respectively in the curves on the lower left and lower right.

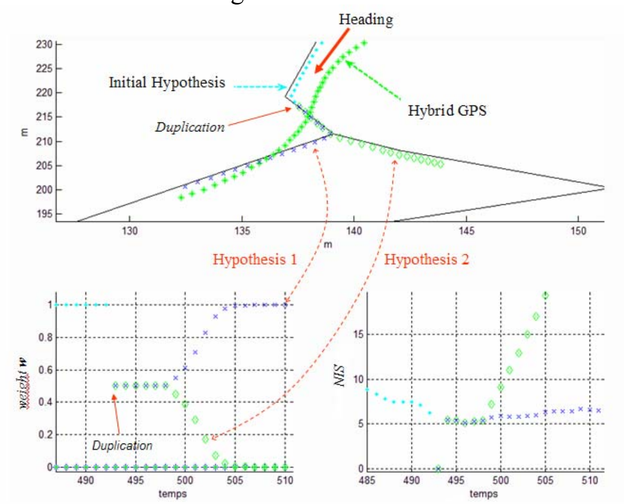

Fig. 3. *Hypothesis creation at a road-intersection*

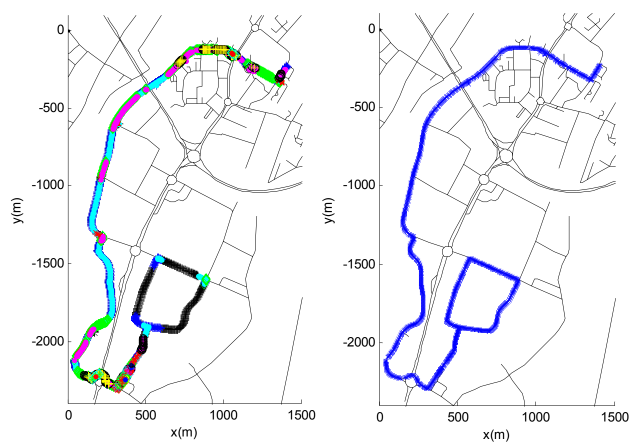

Fig. 4. *Hypotheses and the most likely one during an on-road trial*

Fig. 4 shows, on the left part, all the hypotheses during an on-road test in Compiègne. The travelled distance in this test is 5.7 Km. On the right, the most likely hypothesis is shown at each moment of this trial. The percentage that the most likely hypothesis corresponds effectively to the real position of the vehicle obtained during this test is 97% of good matches. We checked that the wrong matches correspond to ambiguous situations correctly detected by the MHMM.

Fig. 5 shows the result of the number of efficient hypotheses N_{Eff} during an acquisition test. Different values of N_{Eff} have been matched up with the driving situations: N_{Eff} = 1 is often obtained for the case where the vehicle is running on segment, far from an intersection, with the associated runway having a fairly large weight. N_{Eff} = 2, 3, 4 is generally obtained when approaching an intersection, with, respectively, 2, 3, 4 roads in the upcoming intersection.

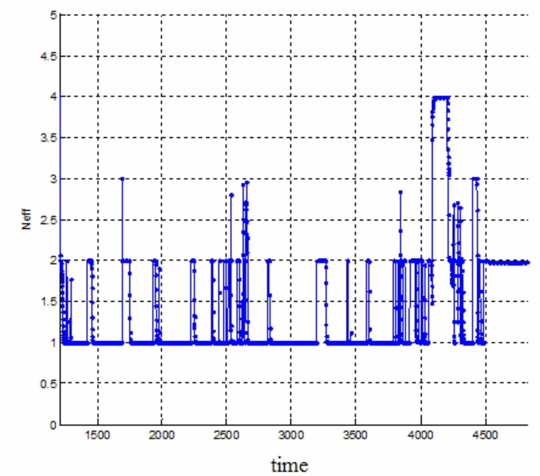

Fig. *5. Number of effective filter during the road test*

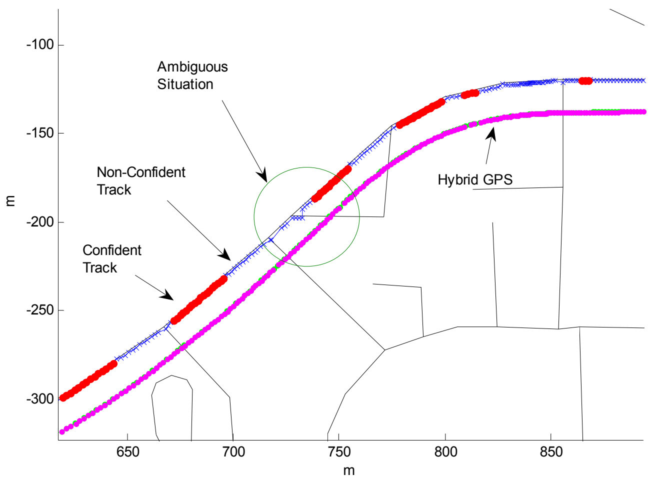

Fig. *6 Most likely hypothesis, integrity and hybrid GPS*

Fig. 6 shows the integrity computation result on the most difficult part of the trial (upper part of Fig. 4). The most likely hypothesis (denoted by MLH) is shown in blue, the h-GPS in magenta. We can clearly see that the map offset with respect to the GPS. When the MLH is shown in bold red, it means that it is considered confident. The integrity indicator is here the bold red. Please look at the ambiguous situation pointed by the circle. Because of the map offset, the MLH is not the appropriate one. Nevertheless, the confidence indicator clearly indicates that the output is not likely. This correctly corresponds to ground truth.

V. CONCLUSION

This paper has presented a map-matching method that relies on multi-hypothesis tracking for on-road vehicles. This method fuses proprioceptive sensors with GPS and map information. The main idea behind this approach is to associate a hypothesis to each newly encountered road after an intersection or a roundabout. The likelihood of each available hypothesis is evaluated by computing a recursive weight or score through an instantaneous likelihood that updates the hypotheses' weight. An integrity indicator is also calculated on the most likely hypothesis to determine whether this choice is a coherent one. The decision rule we proposed considers the estimated location consistency with the map and the probability of the hypotheses with respect to the others to handle ambiguity zones. Real tests were undertaken on road and results showed the validity of this approach.

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