# Decentralized Collaborative Localization with Map Update using Schmidt-Kalman Filter

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***Abstract*—This paper presents a new decentralized approach for collaborative localization and map update relying on landmarks measurements performed by the robots themselves. The method uses a modified version of the Kalman filter, namely Schmidt Kalman filter that approaches the performance of the optimal centralized Kalman filter without the need to update each robot pose. To deal with data incest and limited communication, the computation of cross-covariance errors between robots must be well managed. Each robot individually updates its own map, the map fusion is performed by using the unweighted Kullback-Leibler Average to keep estimation consistency. The performance of the approach is evaluated in a simulation environment where robots are equipped with odometry and a lidar for exteroceptive perception. The results show that collaboration improves the localization of the robots and the estimation of the map while maintaining consistency.**

***Keywords*—Map aided localization, Schmidt-Kalman filter, Map update, Decentralized architecture, Kullback-Leibler Average, Consistency.**

## I. Introduction

Localization is a key component for the navigation of mobile robots or autonomous vehicles. For a team of multiple robots, Collaborative Localization (CL) can be applied to improve and robustify the localization performance [1], [2]. Classically, CL is achieved based on inter-robot relative measurements that correlate the pose estimates [3], [4]. However, robots have to be in the same field of view. To cope with this problem, other CL methods use robots-to-infrastructure information, through the observation of landmarks.

For autonomous vehicles navigating in urban environments, landmark-based localization has gained significant attention in the last years [5], [6]. The information associated to the landmarks can be retrieved from a prior map (e.g. High Definition Maps, OpenStreetMap) or can be constructed through Simultaneous Localization and Mapping (SLAM) methods [7]. Collaborative SLAM uses the sensor readings of multiple robots to improve the poses of the landmarks and of the different robots [8], [9]. In this paper, we aim to localize a group of robots using an uncertain prior map containing landmarks.

The localization task can be performed through different architectures. The centralized architecture relies on a central unit that gathers the data from all the robots. This architecture is easy to implement, but introduces a high computational cost, and a heavy dependency on the central unit and on the communication network. On the other hand, the decentralized architecture allows a direct communication of information between the robots. Therefore, they are instantly aware of local changes. However, this architecture is delicate to implement and suffers from data incest (double-counting) [10]. The inconsistency resulting from the reuse of the same information and the communication problems should be well taken into account. The Covariance Intersection Filter (CIF) is well adapted to this problem, but leads to a pessimistic solution in terms of covariance estimation [11].

In this paper, we present a fully decentralized architecture for CL with map update using robots-to-landmarks measurements. The communication between robots is only allowed when they observe a common landmark. To do so, the landmarks are considered to be discernible. The method is based on the use of the Schmidt Kalman Filter (SKF) that allows obtaining a consistent and non-pessimistic solution, which is a necessary condition for the navigation of autonomous vehicles with strong safety requirements [12], [13]. Indeed, the SKF is derived from the Kalman Filter (KF) and is well adapted to include the contributions of non-estimated states into the covariances (through cross-covariances between robots involved and not involved in a landmark observation).

The main contributions of this paper are:

- The proposition of a decentralized architecture for collaborative localization using indirect measurements under restricted communication,
- The use of SKF for decentralized data fusion,
- The map update using the Kullback Leibler average,
- The analysis, with simulation, of the collaboration on the accuracy and uncertainty of the state estimation.

This paper is organized as follows: a discussion on related works and the problem statement are given in section II, then the CL filter is presented in a centralized manner in section III before explaining the core method which is the Schmidt-Kalman filter for decentralized collaborative localization in section IV. Finally, simulation results are presented in section V to evaluate its performance.

## II. Related Works

This work is part of the decentralized multi-robot SLAM in an existing map [7] with a particular attention to collaborative localization based on landmarks.

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Decentralized SLAM has been investigated in different works [14]–[16]. It refers to the case where a scalable number of robots cooperate to produce a common map while simultaneously performing localization relative to that map [17].

The multi-robot SLAM in [18] uses a sparse Extended Information Filter (EIF) to perform the update in constant time, independently of the number of landmarks. However, the collaborative aspect is mostly focused on the construction of a single map by aligning the maps of the different robots without dealing with CL. Indeed, every robot operates independently using its own measurements, and with the sparse hypothesis, the robots do not need to communicate all the time.

In [19], the multi-robot collaboration is also done through the map update. The map is represented as a Probability Hypothesis Distribution (PHD), making it easy to add and remove landmarks collaboratively by a simple addition. The localization is done using a particle filter.

A decentralized SLAM has been addressed in [20] where each robot has its own state (pose and map). The global map can be obtained by each robot by fusing the received states. However, no correlated state should be exchanged which can lead to loss of information.

In addition to filter based methods, optimization based methods are also used for decentralized collaborative SLAM. In [21], locally consistent submaps are constructed and exchanged. A pose-graph approach is used to estimate and optimize the robots trajectories. This method requires a fully connected network.

Regarding the decentralized CL, it has also received a lot of attention apart from the SLAM problem.

In [22], Information Filter (IF) is coupled to a Cholesky decomposition to allow robots to use a queued network where each robot only depends on the downstream robots. Likewise, the Channel Filter is presented in [23]. It propagates the information on a tree and converges to the global estimate in a decentralized way, while allowing interrupted communications. However, the topology communication graph has to be constant, and each robot should estimate the entire state, leading to high computation loads.

In [24], the Interlaced Extended Kalman Filter (IEKF) is used for fully decentralized CL with inter-robot and landmarks measurements. The robots exchange their state when they observe each other. However, using this method, the cross-correlations are not taken into account and the robots are considered as decoupled, which may lead to overconfident solutions.

The cross-correlations are the main problem that appears during decentralized data fusion. Even if the classical KF deals with the correlation terms, it is not adapted to decentralized architecture, since the computation of the off-diagonal covariance elements requires a permanent communication among the robots [25]. Different real scenarios need that robots communicate occasionally. The CIF is well adapted to treat estimates with unknown correlations [26]. Likewise, the split version (SCIF) [27] handles independent and unknown correlations. However, the CIF and SCIF lead, in general, to pessimistic solutions.

Luft et al. [28] present a method for decentralized CL using direct measurements under restricted communication. The method relies, in part, on the use of the SKF. The SKF allows partitioning the state of the KF into two parts: the part to be estimated, and the parameter part that contains the components of the state that do not need to be estimated but whose influence on the estimated part must be determined. Likewise, in their paper, they propose a method to approximate the cross-covariances terms.

In this paper, CL is based on indirect measurements and is formulated when two robots observe the same landmark. At this instant, they will communicate to exchange their observations, poses and covariances and update them. The other robots poses will not be updated, but to keep consistency, we seek to determine their effect on the covariance, which is equivalent to using the SKF. The landmarks' poses (the map) must be estimated at the same time according to a decentralized architecture. Therefore, each robot will have its own version of the map, which is updated using its filter.

## III. Centralized Collaborative localization

Let consider the localization of $N$ robots navigating in an environment with landmarks referenced with uncertainty in the map. Collaboration between the robots is used to improve the accuracy of localization while keeping integrity, as well as to refine the map estimate. The collaborative localization problem is formulated when two robots observe the same landmark.

The landmarks are represented in the form of poses: $X_l = [x, y, \theta]_l^T$. The variables $x$ and $y$ represent the planar cartesian coordinates, and $\theta$ the orientation of the landmark. Likewise, the robot pose is considered to be the position and the yaw of the robot $X_v = [x, y, \theta]_v^T$.

In the rest of this paper, we denote the set of robots $V$ with elements $v_i$, $i = \{1, ..., N\}$. Likewise, the set of landmarks is denoted $L$ with elements $l_j$, $j = \{1, ..., M\}$ and $M$ is the number of landmarks.

Before proposing a decentralized architecture using the SKF, a centralized Extended Kalman Filter (EKF) for collaborative localization is presented: all the observations are sent to a central unit which does the fusion.

### A. Prediction step

The joint state vector is considered to be the robots and the landmarks poses

$$X = \begin{bmatrix} (X_v)_{v \in V} \\ (X_l)_{l \in L} \end{bmatrix}. \quad (1)$$

The propagation equations of the robots at instant $k$ are obtained using odometry, where the input vector provides the elementary displacement and rotation of robot $i$, $u_{i,k} = \begin{bmatrix} \Delta_i & \Omega_i \end{bmatrix}_k^T$

$$\begin{cases} x_{i,k+1|k} &= x_{i,k|k} + \Delta_{i,k} \cos\left(\theta_{i,k|k} + \Omega_{i,k}/2\right) \\ y_{i,k+1|k} &= y_{i,k|k} + \Delta_{i,k} \sin\left(\theta_{i,k|k} + \Omega_{i,k}/2\right) \\ \theta_{i,k+1|k} &= \theta_{i,k|k} + \Omega_{i,k} \end{cases}. \quad (2)$$

The predicted covariance matrix associated to the robot $i$ is then

$$P_{i,k+1|k} = F_{i,k} P_{i,k|k} F_{i,k}^T + B_{i,k} Q_{u_i} B_{i,k}^T + Q_i, \tag{3}$$

with $F_{i,k}$ and $B_{i,k}$ the Jacobian matrices associated to the state and input vector, $Q_{u_i}$ the covariance of the input noise, and $Q_i$ the covariance of the model noise.

The prediction of the cross-covariances between robots $i$ and $j$ is

$$P_{ij,k+1|k} = F_{i,k} P_{ij,k|k} F_{j,k}^T. \tag{4}$$

Regarding the landmarks, the evolution model is motionless. The prediction of the cross-covariances between a robot $i$ and a landmark $j$ is done similarly to equation (4), with $F_{j,k} = I$.

### B. Update step

In order to correct odometry drift, localization updates with landmarks is studied. The observation vector is considered to be the pose of the observed landmarks in the robot frame. When robot $v_i$ observes landmark $l_j$, the observation vector is written as

$$Z_{v_i}^{l_j} = \underbrace{\begin{pmatrix} \cos\theta_{v_i} & \sin\theta_{v_i} & 0 \\ -\sin\theta_{v_i} & \cos\theta_{v_i} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{T_{v_i}} \begin{pmatrix} x_{l_j} - x_{v_i} \\ y_{l_j} - y_{v_i} \\ \theta_{l_j} - \theta_{v_i} \end{pmatrix} + \beta_{v_i}^{l_j} \tag{5}$$

$$Z_{v_i}^{l_j} = T_{v_i}(X_{l_j} - X_{v_i}) + \beta_{v_i}^{l_j}, \tag{6}$$

where $\beta_{v_i}^{l_j}$ is the measurement noise considered as white Gaussian with covariance $R_{v_i}^{l_j}$.

It should be noted that for the centralized version, all the landmarks observed by the $N$ robots are part of the observation vector. Therefore, the observation vector is the concatenation of all the observations at instant $k$ and is denoted $Z_k$.

The observation model is nonlinear, a linearization around the predicted estimation is done leading to a Jacobian matrix $H$ that can be written as

$$H = \begin{bmatrix} H_v & H_l \end{bmatrix}, \tag{7}$$

where

$$H_v = \begin{bmatrix} (H_v)_{v_i}^{l_j} \\ \vdots \end{bmatrix}. \tag{8}$$

Each bloc $(H_v)_{v_i}^{l_j}$ is defined by:

$$(H_v)_{v_i}^{l_j} = \begin{cases} T_{v_i}\Gamma_{v_i}^{l_j} & \text{if column block index is } i \\ \mathbb{0}_{3\times 3} & \text{otherwise} \end{cases}, \tag{9}$$

where

$$\Gamma_{v_i}^{l_j} = \begin{pmatrix} -1 & 0 & y_{l_j} - y_{v_i} \\ 0 & -1 & -(x_{l_j} - x_{v_i}) \\ 0 & 0 & -1 \end{pmatrix}. \tag{10}$$

Likewise, for the landmarks part

$$H_l = \begin{bmatrix} (H_l)_{v_i}^{l_j} \\ \vdots \end{bmatrix}. \tag{11}$$

Each bloc $(H_l)_{v_i}^{l_j}$ is defined by

$$(H_l)_{v_i}^{l_j} = \begin{cases} T_{v_i} & \text{if column block index is } j \\ \mathbb{0}_{3\times 3} & \text{otherwise} \end{cases}. \tag{12}$$

Taking the example of three robots $v_1$, $v_2$ and $v_3$ and two landmarks $l_1$ and $l_2$. At instant $k$, $v_1$ observes $l_1$ and $v_2$ observes $l_1$ and $l_2$, $H_k$ would be as follows:

$$H_k = \begin{bmatrix} T_{v_1}\Gamma_{v_1}^{l_1} & \mathbb{0} & \mathbb{0} & T_{v_1} & \mathbb{0} \\ \mathbb{0} & T_{v_2}\Gamma_{v_2}^{l_1} & \mathbb{0} & T_{v_2} & \mathbb{0} \\ \mathbb{0} & T_{v_2}\Gamma_{v_2}^{l_2} & \mathbb{0} & \mathbb{0} & T_{v_2} \end{bmatrix}. \tag{13}$$

The measurements are supposed to be uncorrelated, so the covariance matrix of the measurement noise $R_k$ is bloc-diagonal, composed of the $R_{v_i}^{l_j}$ of each measurement.

The update step is then given by

$$X_{k|k} = X_{k|k-1} + K_k(Z_k - h(X_{k|k-1})), \tag{14}$$
$$P_{k|k} = (I - K_k H_k) P_{k|k-1}. \tag{15}$$

where $K_k$ is the standard Kalman gain

$$K_k = P_{k|k-1} H_k^T (\underbrace{H_k P_{k|k-1} H_k^T + R_k}_{W_k})^{-1}. \tag{16}$$

After the update step, correlation terms that appear in the covariance matrix can be written as:

$$P_{k|k} = \begin{bmatrix} P_V & P_{V,L} \\ P_{L,V} & P_L \end{bmatrix}_{k|k}, \tag{17}$$

where $P_V$ (respectively $P_L$) is a matrix containing all the robots (respectively landmarks) covariances and cross-covariances and $P_{L,V}$ is a matrix that represents the covariance between landmarks and robots.

The problem being stated for the centralized EKF, we will discuss the adaptation to the decentralized architecture, using a Schmidt-Kalman Filter.

## IV. Schmidt Kalman Filter for decentralized collaborative localization

The centralized architecture creates a lot of dependence on the central unit and is prone to its failure. Likewise, this architecture increases the computational complexity, in particular when the number of observations to be sent to the central unit is high.

In this section, a decentralized architecture for collaborative localization and map update is presented. When two robots observe the same landmark, at almost the same time, they communicate to collaborate and exchange their observations, poses and covariance matrices.

Therefore, each robot has its own filter to estimate its pose and map. It does the prediction step on its own, and the update step using its observations concatenated with observations of some other robots. To do so, when a robot observes a landmark, it broadcasts it to its neighbors along with the required information (pose, covariance, cross-covariances and map) to reconstruct the covariance matrix. When the other robots receive the information, if they have observed the same landmark at almost the same time, they perform a collaborative update. In other words, they take their own observation of this landmark and add the received ones to their observation vector. Otherwise, they perform a standalone update.

### A. Schmidt Kalman filter

Consider a robot $v_i$ and a subset $V_i$ of robots that observe at least one landmark in common with $v_i$, $v_i \in V_i$. The state vector can be divided into two parts. The first part is composed of the robots poses involved in the collaborative localization (the robots in $V_i$) in addition to the landmarks' poses. The elements of this part are denoted $s$. The second part includes the robots poses not involved in the collaborative localization. They are considered as parameters which are not to be estimated but whose impact on the covariance must be determined. They are denoted $p$.

The state vector given in equation (1) may be rearranged as

$$X = \left[ \begin{array}{cc} X_s & X_p \end{array} \right]^T, \tag{18}$$

where the part relative to $s$ is given by

$$X_s = \left[ \begin{array}{c} (X_v)_{v \in V_i} \\ (X_l)_{l \in L} \end{array} \right], \tag{19}$$

and that relative to $p$ is

$$X_p = (X_v)_{v \notin V_i} = (X_v)_{v \in \bar{V}_i}. \tag{20}$$

A choice was made to consider all landmark poses in the $s$ part. Indeed, landmarks are not intelligent and do not have computational capabilities. By considering the map in the $s$ part, the robots can access all the data they need to perform the map update and it removes some complexity on future operations.

Likewise, the covariance matrix can be reorganized into

$$P = \left[ \begin{array}{cc} P_{ss} & P_{sp} \\ P_{ps} & P_{pp} \end{array} \right], \tag{21}$$

where the elements are

$$P_{ss} = \left[ \begin{array}{cc} P_{V_i} & P_{V_i,L} \\ P_{L,V_i} & P_L \end{array} \right], \tag{22}$$

$$P_{sp} = \left[ \begin{array}{c} P_{V_i,\bar{V}_i} \\ P_{L,\bar{V}_i} \end{array} \right], \tag{23}$$

$$P_{pp} = P_{\bar{V}_i}. \tag{24}$$

For example, $P_{V_i,L}$ is the cross-covariance between the robots in $V_i$ and the landmarks $L$.

The observation vector $Z$ is limited to the measurements between the robots in $V_i$ and the landmarks observed by $v_i$. The Jacobian matrix $H$ defined in equation (7) can be reorganized as $H = \left[ \begin{array}{cc} H_s & H_p \end{array} \right]$ where it occurs that all the elements of $H_p$ are null. If we take the same example as in equation (13), it can be seen that $H_s$ is the first and last two columns, and $H_p$ is the middle column (zero elements).

Since the aim is to estimate only the pose of the robots in $V_i$ (i.e. robots that are currently collaborating), the updated state and covariance of the robots in $\bar{V}_i$ can be written as [13]:

$$X_{p,k|k} = X_{p,k|k-1}, \tag{25}$$

$$P_{pp,k|k} = P_{pp,k|k-1}, \tag{26}$$

which results directly from the SKF.

Regarding the $s$ part, the optimal Kalman gain is [13]

$$K_{s,k} = \left( P_{ss,k|k-1} H_{s,k}^T + P_{sp,k|k-1} H_{p,k}^T \right) W_k^{-1}. \tag{27}$$

However, $H_{p,k}$ is a zero matrix. The optimal Kalman gain is then expressed as

$$K_{s,k} = P_{ss,k|k-1} H_{s,k}^T W_k^{-1}, \tag{28}$$

with

$$W_k = H_{s,k} P_{ss,k|k-1} H_{s,k}^T + R_k. \tag{29}$$

Given that $H_{p,k} = 0$, the update step is obtained in a simple form

$$X_{s,k|k} = X_{s,k|k-1} + K_{s,k}(Z_k - h(X_{s,k|k-1})), \tag{30}$$

$$P_{ss,k|k} = (I - K_{s,k} H_{s,k}) P_{ss,k|k-1}, \tag{31}$$

$$P_{sp,k|k} = (I - K_{s,k} H_{s,k}) P_{sp,k|k-1}. \tag{32}$$

In the following, $H_{s,k}$ and $K_{s,k}$ can be written as:

$$H_{s,k} = \left[ \begin{array}{cccc} H_{s_1,k} & H_{s_2,k} & \ldots & H_{s_n,k} \end{array} \right], \tag{33}$$

where $n$ is the number of elements (robots and landmarks) in $s$. Likewise, the Kalman gain can be written as

$$K_{s,k} = \left[ \begin{array}{c} K_{s_1,k} \\ K_{s_2,k} \\ \vdots \\ K_{s_n,k} \end{array} \right]. \tag{34}$$

### B. Decentralized issues

Given the limited communication where the robots only communicate when observing the same landmarks, it is impossible to reproduce the exact terms of the off-diagonal covariances between robots. To solve this problem, the cross-correlation terms of the covariance matrix are decomposed as [25]

$$P_{ij} = \sigma_{ij} \left( \sigma_{ji} \right)^T, \tag{35}$$

with $i$ and $j$ two robots. Each part will be kept by a different robot ($\sigma_{ij}$ by robot $i$ and $\sigma_{ji}$ by robot $j$). Beware, $\sigma_{ij}$ and $\sigma_{ji}$ can take any value, while satisfying equation (35). In particular, after an update step, one of the two robots will keep the full covariance $\sigma_{ij,k|k} = P_{ij,k|k}$ whereas the other will set its covariance part to the identity $\sigma_{ji,k|k} = I$.

The prediction step of robots $i$ and $j$ can be written as

$$\sigma_{ij,k+1|k} = F_{i,k} \sigma_{ij,k|k}, \tag{36}$$

$$\sigma_{ji,k+1|k} = F_{j,k} \sigma_{ji,k|k}, \tag{37}$$

with $F_{i,k}$ (respectively $F_{j,k}$) the Jacobian matrix of the prediction model of robot $i$ (respectively $j$) as used in equation (4).

Then, when the two robots communicate, they can exchange their parts and reconstitute the full cross-covariance before doing the update:

$$P_{ij,k+1|k} = \sigma_{ij,k+1|k} \left( \sigma_{ji,k+1|k} \right)^T, \tag{38}$$

$$= F_{i,k} \sigma_{ij,k|k} \left( \sigma_{ji,k|k} \right)^T F_{j,k}^T, \tag{39}$$

$$= F_{i,k} P_{ij,k|k} F_{j,k}^T. \tag{40}$$

This reconstruction is possible thanks to the use of only left products (equations (36) and (37)) and is equivalent to the prediction step of the cross-covariance given in equation (4). The robots are now ready for the update step.

In the update step, in the case of communicating robots, no decomposition is needed as the full cross-covariance can be reconstructed (equation (38)). Therefore, there is no need to update only the part belonging to $i$. However, it is not the case for the update of the cross-covariances between non-communicating robots: a robot in $s$ (denoted $s_i$) and a robot in $p$ (denoted $p_j$).

Using equation (32), the cross-covariance term between these two robots can be written as

$$\begin{aligned} P_{s_ip_j,k|k} &= (I - K_{s_i,k}H_{s_i,k})P_{s_ip_j,k|k-1} \\ &\quad - \sum_{r=1,r\neq i}^{n} K_{s_i,k}H_{s_r,k}P_{s_rp_j,k|k-1}. \end{aligned} \tag{41}$$

$P_{s_ip_j,k|k-1} = \sigma_{s_ip_j,k|k-1}\left(\sigma_{p_js_i,k|k-1}\right)^T$ and $P_{s_rp_j,k|k-1} = \sigma_{s_rp_j,k|k-1}\left(\sigma_{p_js_r,k|k-1}\right)^T$ cannot be reproduced because the robots in $s$ are not communicating with $p_j$. Likewise, equation (41) cannot be written in the form of a matrix multiplied by $P_{s_ip_j,k|k-1}$, a form that allows the decomposition of $P_{s_ip_j,k|k}$.

Therefore, to keep a track of the update of the cross-covariance and be able to reconstitute it during the next communication, an approximation is needed as proposed in [28]:

$$P_{s_ip_j,k|k} \approx P_{s_is_i,k|k}(P_{s_is_i,k|k-1})^{-1}P_{s_ip_j,k|k-1}, \tag{42}$$

where the part owned by $s_i$ is

$$\sigma_{s_ip_j,k|k} \approx P_{s_is_i,k|k}(P_{s_is_i,k|k-1})^{-1}\sigma_{s_ip_j,k|k-1}. \tag{43}$$

Notice that this approximation is exact if the elements of $s$ are totally uncorrelated or strongly correlated, as proven in [28].

When the two robots are in $s$, the cross-covariances can be reconstructed and the update step can be done. Then, one of the $\sigma_{ij}$ is set to the updated $P_{s_ip_j}$ and the other to the identity.

The cross-covariances between the robots and the map are managed in another way. Each robot stores its local map and the cross-covariances with each landmark, then update them when needed. These cross-covariances are not decomposed, and are fully transmitted in the case of a collaborative update. In this way, the other robots are able to reconstitute $P_{ss}$.

### C. Map fusion

To be able to make a consistent estimation of the poses, the robots need to work on very similar map versions when they perform CL. Hence, the fusion of $m$ maps coming from $m$ robots involved in CL is done using the unweighted Kullback-Leibler Average (KLA) [29]. For Gaussian distributions, it is equivalent to

$$\bar{P}^{-1}_{L,k|k-1} = \frac{1}{m}\sum_{i=1}^{m}(P^i_{L,k|k-1})^{-1}, \tag{44}$$

$$\bar{X}_{L,k|k-1} = \bar{P}_{L,k|k-1}\frac{1}{m}\sum_{i=1}^{m}(P^i_{L,k|k-1})^{-1}X^i_{L,k|k-1}, \tag{45}$$

with $P^i_L$ and $X^i_L$ the landmarks covariance and poses as given by robot $v_i$.

Notice that this method is equivalent to the use of the CI filter [11] which provides pessimistic solutions compared to the optimal KF. However, the KLA is only applied to the map part. Nevertheless, this approximation provide good results, as we will see.

Consider two matrices $P_1$ and $P_2$ composed of block matrices associated to robots $V$ and map $L$

$$P_i = \begin{bmatrix} P_{i,VV} & P_{i,VL} \\ P_{i,LV} & P_{i,LL} \end{bmatrix}, \quad i = \{1,2\}. \tag{46}$$

Inverses of these covariance matrices (namely information matrices) are given by

$$Y_i = \begin{bmatrix} Y_{i,VV} & Y_{i,VL} \\ Y_{i,LV} & Y_{i,LL} \end{bmatrix}. \tag{47}$$

The average of all the elements leads to a matrix $\frac{Y_1+Y_2}{2}$ where the elements of the block $L$ are

$$\begin{aligned} Y^{avg} &= \frac{1}{2}(P_{1,LL} - P_{1,LV}P^{-1}_{1,VV}P_{1,VL})^{-1} \\ &\quad + \frac{1}{2}(P_{2,LL} - P_{2,LV}P^{-1}_{2,VV}P_{2,VL})^{-1}. \end{aligned} \tag{48}$$

The average of the part relating only to the map gives an information matrix

$$Y = \frac{P^{-1}_{1,LL} + P^{-1}_{2,LL}}{2}. \tag{49}$$

The independent data fusion leads to

$$Y^{ind} = P^{-1}_{1,LL} + P^{-1}_{2,LL}. \tag{50}$$

It can be seen that $Y^{-1} > (Y^{avg})^{-1} > (Y^{ind})^{-1}$. The consistency is therefore preserved using the KLA.

## V. Simulation Results

In this section, we evaluate the approach in a simulation environment with three robots navigating in a map composed of four landmarks. The robots are equipped with odometry and a lidar perception system. The simulation is done using the Robot Operating System (ROS). The trajectories of the three robots are presented in Fig. 1, with the landmarks detection ranges and the collaboration areas between robots. Initially, all robots have the same inaccurate map. Landmark 1 is assumed to be perfectly located, while a small offset is added to the others. The landmarks form a square of 10m side, and the robots can perceive the landmarks up to a distance of six meters.

These trajectories allow studying several collaborative situations. First, robot 3 observes alone the landmark 4 then the landmark 2. It starts to collaborate with robot 1 by observing landmark 1 at $t = 11$s. Starting from $t = 20$s, robots 1 and 3 collaborate for a long time by observing landmark 2. At $t = 36$s, robot 2, which was operating alone since the beginning, by observing landmark 3, joins the collaborative localization of robots 1 and 3 by observing landmark 4. Then robot 3 observes landmark 2 again, and robots 1 and

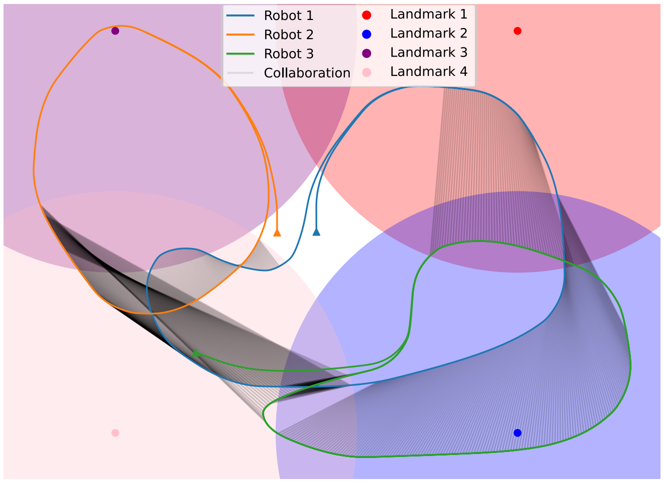



Fig. 1: Robots trajectories with landmarks detection ranges of 6m (circles) and linked poses when collaboration occurs, which is indicated by thin lines between the trajectories.

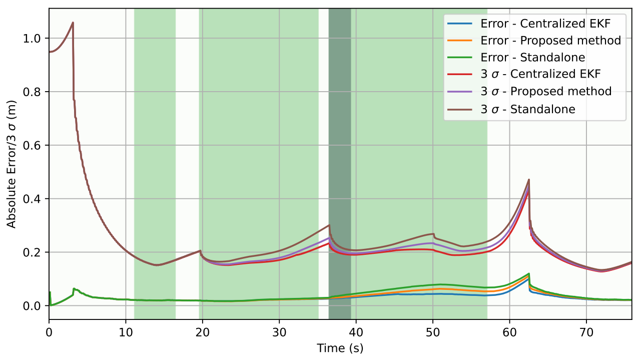



Fig. 2: Errors and uncertainties of robot 1. The colors represent the number of robots involved in the collaboration (light green for two robots and dark green for three robots).

2 observe landmark 3 for a short moment. Finally, around $t = 57$s, robot 1 does not observe anything and only works with odometry data, before observing landmark 1 on its own at $t = 63$s. The collaboration periods are represented as green areas on Fig. 2 to Fig. 4, with light green when two robots collaborate and darker green when the three robots collaborate.

The results are presented on 100 simulation runs where Fig. 2 to Fig. 6 show averaged errors and covariances.

The extraction of landmarks in the form of a pose and the identification when multiple robots observe the same landmark are supposed to be done.

In the following, the performance of the proposed decentralized approach is compared to the standalone case where each robot performs localization without communication and the centralized EKF where a communication with a central unit is always done (the optimal filter).

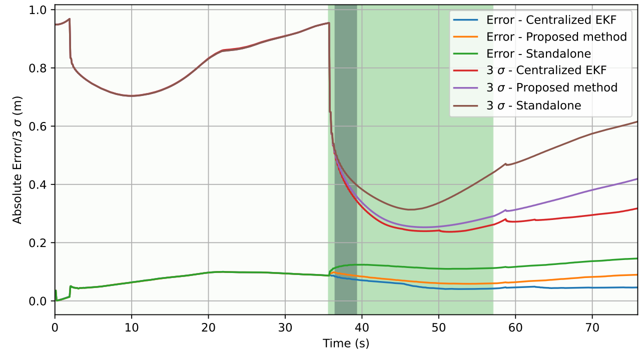



Fig. 3: Errors and uncertainties of robot 2. Collaboration clearly improves the localization accuracy of this robot.

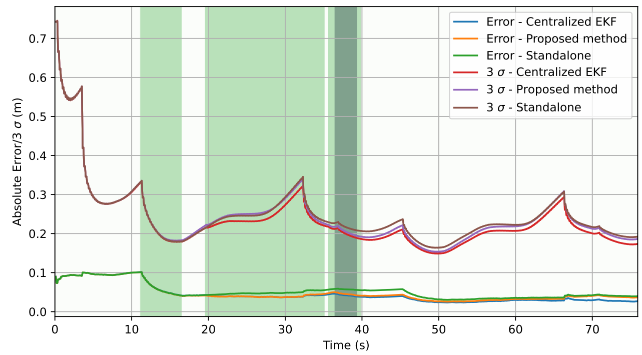



Fig. 4: Errors and uncertainties of robot 3. The gain in accuracy due to collaborative localization can be seen.

### *A. Collaborative robot localization*

Figs. 2, 3 and 4 show the errors and the covariances of the three robots. Table I shows the mean errors from $t = 40$s. It can be seen, that collaboration, in general, improves the results in terms of accuracy and confidence. Likewise, the covariances of the standalone filter are the highest as it has less information compared to the collaborative filters, which induces a lower confidence.

By comparing our approach to the optimal EKF, it can be seen that our estimated covariance is close to the centralized EKF while being higher. This is an important result that verifies that the proposed approach is neither overconfident nor so pessimistic.

Let's look now at the results of each robot. The value of the collaboration is particularly visible on robot 2 (Fig. 3). Indeed, it operates alone by observing landmark 3 a long time, then around $t = 36$s, all the robots start to collaborate by observing landmark 4. It can be seen that the new landmark improves the confidence of the standalone filter. Regarding the collaborative approach, further improvements can be noticed in terms of accuracy and uncertainty. This is due to landmark 4 which is better estimated thanks to the collaboration, as shown in Fig. 6b.

For robot 1 (Fig. 2), it can be seen that the different methods

TABLE I: Mean error norms of the different approaches, and their empiric standard deviations, from $t = 40$s.

| Robot / Method | Mean ± Std. (mm) |
|---|---|
| R1 / Centralized EKF | 39 ± 25 |
| R1 / Collaborative | 48 ± 27 |
| R1 / Standalone | 58 ± 31 |
| R2 / Centralized EKF | 48 ± 21 |
| R2 / Collaborative | 71 ± 24 |
| R2 / Standalone | 122 ± 17 |
| R3 / Centralized EKF | 30 ± 15 |
| R3 / Collaborative | 34 ± 15 |
| R3 / Standalone | 40 ± 17 |
| All / Centralized EKF | 39 |
| All / Collaborative | 51 |
| All / Standalone | 74 |

tend to a similar result because of the observation of the perfect landmark (landmark 1), at the beginning and at the end. Indeed, robot 1 observes landmark 2 at the same time than the perfect landmark 1, the pose of landmark 2 is then strongly corrected by the standalone method (Fig. 5). Robot 1 continues to observe landmark 2 for a long time, so robot accuracy in standalone mode is maintained. However, a difference is visible between $t = 36$s and $t = 63$s when robot 1 does not observe any landmark or can observe only landmark 4 leading to a drift in the standalone mode. This result is confirmed in table I where the mean error from $t = 40$s is smaller for the collaborative method compared to the standalone.

Robot 3 shown in Fig. 4 is perhaps the one where the benefit of collaboration is the least obvious on the covariances. Indeed, this robot observes firstly landmarks 2 and 4 and observes them regularly. The collaboration with the other robots by observing landmark 4 brings a little more confidence. On the other hand, the robot 3 is very often in collaboration with the other two, which explain the gain in terms of accuracy compared to the standalone method as shown in table I.

### *B. Collaborative map update*

After presenting the contribution of the collaboration on robots localization, the collaborative map updates are presented. As each robot has its own map, a consensus on the maps is done using the KLA as explained in section IV-C. In accordance with the theoretical part, the proposed approach leads to a more pessimistic solution compared to the robots' state. Therefore, the uncertainties of the landmarks using the decentralized collaborative method deviate from the optimal case (centralized EKF). For example, Fig. 6 shows the results of landmark 4 as seen by the different robots.

When robots exchange their maps, the consensus may decrease the accuracy of some robots on their map. For example, it is the case of robot 3 that observes landmark 4 at the very beginning (Fig. 6c). When it collaborates with robot 1 at $t = 11$s, the maps are shared and merged using the KLA. At this time sample, the collaborative method leads to a higher uncertainty compared to the standalone approach. Indeed, each time a fusion using the KLA is done, the covariance of the most confident increases and the covariance of the

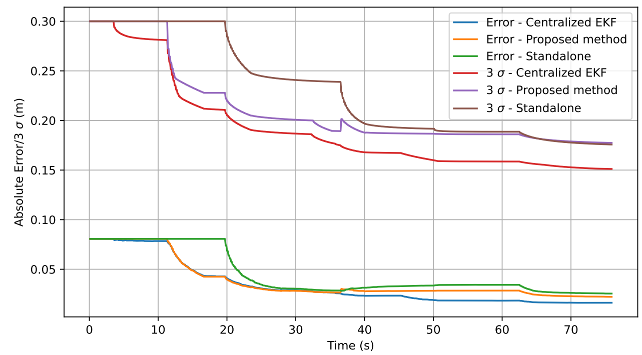



Fig. 5: Errors and uncertainties of landmark 2 as seen by robot 1. Collaboration clearly reduces the mapping error of this landmark.

TABLE II: Mean error norms of the inaccurate landmarks by the tested methods, and their empiric standard deviation, from $t = 40$s.

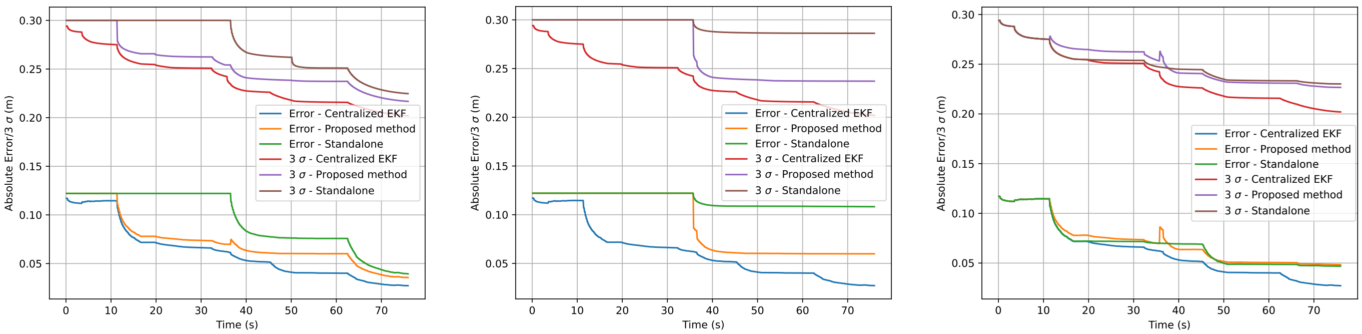
| Robot / Method | Landmark: mean ± std. (mm) | | | |
|---|---|---|---|---|
| | 2 | 3 | 4 | all |
| Centralized EKF | 19 ± 9 | 64 ± 29 | 39 ± 15 | 41 |
| R1 / Collaborative | 27 ± 11 | 86 ± 36 | 53 ± 17 | 55 |
| R1 / Standalone | 31 ± 12 | 97 ± 38 | 66 ± 21 | 65 |
| R2 / Collaborative | 28 ± 10 | 95 ± 34 | 60 ± 12 | 61 |
| R2 / Standalone | 81 ± 0 | 147 ± 15 | 109 ± 3 | 112 |
| R3 / Collaborative | 23 ± 9 | 96 ± 27 | 53 ± 15 | 57 |
| R3 / Standalone | 30 ± 12 | 112 ± 0 | 52 ± 16 | 65 |
| All / Collaborative | 26 | 93 | 56 | 58 |
| All / Standalone | 47 | 119 | 76 | 81 |

less confident decreases, to reach an average. For the most confident, it is always a loss of precision.

In the considered problem, let us recall that only one landmark is accurately located. It is interesting to observe that the poses of the others landmarks tend to converge to the real poses thanks to the collaboration as shown in table II. The collaborative method provides a better overall estimation of the landmarks.

## VI. Conclusion and Future Work

In this paper, a new decentralized approach for collaborative localization has been proposed using an inaccurate prior map. The collaboration is done when robots observe the same landmarks. The method is based on the Schmidt-Kalman Filter that allows to estimate jointly robots and landmarks poses with limited communication. The method has been evaluated in simulation and compared with the centralized and the standalone approaches. It has been shown that the decentralized collaborative approach produces better results than the non-collaborative version in terms of accuracy and maintains a consistent and non-pessimistic behavior quite similar to the centralized version with only a slight loss of accuracy. In addition, the contribution of the collaboration for updating the map has been analyzed in detail in the scenario under consideration. For the map as well, the estimates made by the




(a) Seen by robot 1. (b) Seen by robot 2. (c) Seen by robot 3.

Fig. 6: Errors and uncertainties of landmark 4. In the map of each robot, this landmark is better located thanks to collaboration.

robots remain consistent and gain in accuracy as the robots move, as soon as a landmark is well located.

In future work, we will focus on improving maps merging. The final step will be the evaluation of the approach on real experimental data.

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