System of systems views of safety and vulnerability in dangerous goods transportation by road
Summary of the presentation

• Introduction to DELab
• Dangerous goods transportation: problem description
• SoS research views of the problem
• Conclusions
What is DELAB?

DELAB is a joint laboratory between:

- DIST (Department of Computer Communication and System Sciences) now DIBRIS (Department of Informatics, Bioengineering, Robotics and System Engineering) of the University of Genova

- ENI, the largest Italian oil company
DELAB objectives

• DELAB research focuses on Dangerous Goods Transportation (DGT) and Logistics integrating intelligent systems in order to prevent accidents to people and to the environment.
• Collaboration started in 2002, formalised as joint lab in 2009.
• From a methodological viewpoint, DELab approaches focuses on SoSE.
• DELAB has organised the IEEE SoSE conference in Genova in 2012.
Introduction to DELab
Eni DGT transportation process indicators

- 50Mkm per year;
- About 1200 vehicles in Italy
- 8Mt delivered petrol products per year
- About 4500 service stations
- Travels average length 171 km
- about 2.3 drops per travel

Accidents?
Results
Accidents at Eni in the transport by road of petrol products
DELAB main activities

• Real time monitoring architecture for vehicles carrying dangerous goods

• Support tools for staff and drivers including training, resource management and advanced data mining

• Decision Support Systems (e.g. orders and fleet management)
Dimension of the problem (1/3)

Unit of measures

- **tkm, tonnxkm**
  - eg. the transport of 4 tonn x 3 km represents 12 tonnxkm

- **vkm, veh x km**,
  - Eg. It represents simply the sum of km used in transport

- **BTO, basic transport operation.**
  - Defined by a couple of load / unload of the vehicle
### Dimension of the problem (2/3): DGT in Italy (Eurostat)

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Dimension of the problem (3/3): DGT in Italy (Eurostat)

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DeLAB monitoring

- About 80% of the Eni vehicles working in Italy are monitored in real-time
- representing about 10% of the vehicles travelling on the Italian territory
DGT Information System (1)

• On board architecture
• Transmission system
• Database
• GIS-based Applications
• DSS
DGT Information System (2)

DGT: the problem

Information Flow

Physical Data

Sensors

Database

GIS

DSS
On-board Architecture (1)

A suggested On-board architecture might be based on:

- a collection of analog/digital sensors
- a “road box” containing a functional unit (OBU On Board Unit) interfaced with sensors and equipped with a GPS and a GPRS transmitter/receiver
On-board Architecture (2)
On-board sensors (1)

Accelerometer / Inclinometer

Products temperature and pressure
On-board sensors (2)

Electronic Counter
Air Suspension Pressure
Loading station door
On-board sensors (3)

Odometer

Emergency button

Canbus
Canbus

Controller–area network is a vehicle bus standard designed to allow devices to communicate with each other within a vehicle. CAN is a message based protocol, designed specifically for automotive applications.

The concentrator should be linked to the CANBUS in order to acquire all the relevant data for the information system.
The OBU (1)

Each sensor is linked to the on-board unit (OBU) through different communication channels (e.g. serial cable, coaxial cable, air-pressure cable)
The OBU (2)

In case that vehicles operate in explosive atmosphere the road box must satisfy some specific requisites ruled by ATEX directive (94/9/CE).
The Transmission Module (1)

Messages should be sent by a GPRS module equipped with one or more SIM-cards.

SIM redundancy copes with the frequent issue of the signal loss due to the GSM uncovering of some areas.
The Transmission Module (2)

Italian Department of Infrastructure and Transport is evaluating the realisation of a SIM card dedicated to DGT. It should work with every mobile operator and it should have transmission priority.
DGT Database Infrastructure
Transmissions Database (T-DB)

This database receives all the raw messages from the Web Service and stores data for the diagnostic activities. The high rate of transmission can lead to a danger of table locking. In order to avoid this issue it is suggested that messages are not immediately parsed: an external application could provide this operation at regular intervals (e.g. 1 minute), moving the unpacked data to the main DB server (M-DB).
Main Database (M-DB)

• Receives data from the T-DB via application server
• Provides data to the web applications
• Collects generic data (e.g. Trucks Registry, Users authentication) integrating them with the real time information
• Provides efficient mechanism to historicize daily data
Backup Database (B-DB)

• The M-DB backups all data on a local storage device
• The application server sends the backup files to the B-DB. It can quickly substitute the M-DB in event of failure with minimum data loss
• Periodically the application server provides to delete oldest backup files (e.g. older than one month)
Web GIS Architecture

Web Server

Db Server

Client

Web service

Java script

Web Map Server
Server side GIS

The Web server:

• Receive the client GIS request
• Call the Web map server to obtain the desired information
• If necessary process the received data
• Send the data back to the client
Client side GIS

The client:

• Send a request to the Web server
• Receive from the web server links and methods to call and manage remote JavaScript functions
• Use the information retrieved from the Web server to request data from the Web map server
DGT: the problem
DGT: the problem
DGT: the problem
DATA RILEVAZIONE: 11/02/2013 15:00:19

BASE DI CARICO: LIVORNO - 491 (0 km)

Viaggi Giornalieri:
- Viaggio 1
- Viaggio 2

DETTAGLIO SCOMPARTI:

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Totalizzatore Benzina: 7106114 l
Totalizzatore Gasolio: 13090540 l

Tracing Mezzo

PDF Sheda Tecnica
DGT: the problem
TIP sections
Remote monitoring

This module is equipped with real-time monitoring of transport through the representation of data in tabular form and on geo-referenced maps and interactive maps. This allows the users to constantly monitor the operations and to extract and export data about the position of the trucks and the quality and quantity of the products loaded and unloaded.
The control room ensures a continuous, real-time monitoring of vehicles equipped with remote control, both during the day services both where the service is done at night. The control room is equipped with advanced tools for managing alarms and anomalies detected during transport.
Trip planning

The planning section is used to provide comparison in real time between the service planned and the service implemented. It’s possible to compare the trip programmed with the data coming from the remote monitoring system installed on board.
Service stations

This feature provides the data of the level of products that is in underground tanks at service stations. This makes possible to know the volume of product in the service stations in order to check the correctness of the unloading of the trucks and automatically generate orders for delivery.
Technical management

All the technical controls of the trucks are included in this section and are constantly updated during each inspection. Are available, in addition to the controls, all the expiring dates (audits, insurance, fire extinguishers, certificates, etc...).
Train management

Similar to the truck management in this section all the technical controls of the rail tank wagons are stored and constantly updated during each inspection.
Operational controls

The section of operational control is used in combination with technical checks to verify the proper implementation of the operation of the service, procedures and standards contained in the contracts of transport. The correct application of procedures for loading, transport and unloading of goods is the basis for good customer service and to prevent accidents and injuries.
With the Training module is possible to create training sessions. After each course is concluded, the tests are automatically generated by the system and the results are stored in the data base. The training section is integrated with the service Safety Game, an innovative feature that permits to send to the users a daily question about the courses that he attended.
Document management

All documents relating laws, regulations, standards and business processes are classified and available in this module to ensure a continuous updating and better sharing of information with suppliers.
Accident reporting

The section of accident reporting is designed to collect information on any anomalous event that has caused damage or injury. All event information is placed in the database and classified by type of service and product transported, in order to be analyzed, exported and used for statistical purposes.
The management of information flows, procedures, definition of roles and responsibilities and verification of operation of business processes are managed in a quality system. This module provides all the tools (non-compliance, improvement programs, complaints, corrective actions, etc...) to keep under control the Quality Management System in Secondary Logistics.
Continuous assessment of compliance with laws and procedures is the best way to detect abnormalities in business processes and ensure continuous improvement. This module is available to suppliers in order to make self-assessments and audits concerning: Environment, Fire Prevention, Safety (D. Lgs. 81/08), Relevant accidents (D. Lgs. 334/99), ADR, RID, IMDG, LPG Storage, Fuel Storage, Quality.
Performance index

All modules above make it possible to produce "dynamic" performance indexes, which are generated in real time. In this way there is a continuous monitoring of performance and it’s possible to make comparisons between different suppliers on the basis of objective parameters.
SoS research views of the problem

- DGT risk definition
- DGT near miss accidents
- Automatic knowledge acquisition
- Risk averse routing
- Optimal decentralised control
- Robust decentralised control
DGT risk definition

- Type of risk: LPHC (low probability high consequences), given by:
  - frequency of accident (alea)
  - exposure (enjeux) and related vulnerability
Frequency of accident

- The frequency of accident is very low given by different systems:
  - Vehicle accident frequency ($10^{-6}$, $10^{-8}$, yr, km)
  - Flow of DGT (4%-7% of goods transport)
  - Scenario probability (eg explosion, of the order of $10^{-2}$)
  - Amplification or decrease effects on risk due to weather and traffic conditions, or infrastructure conditions.
Exposure and related vulnerability

- Individual/social
  - Estimation of density of population
    - Research aspect: dynamic estimated by GPS/GPRS cell use, traffic on the road
- Environment: interaction with other risks
- Property: economic value of interest for insurance company
An example: train vs road on Frejus
An example: train vs road on Frejus

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DGT near miss accidents

- Low frequency of accidents
- Enhance knowledge by the collection of near miss accidents
Near Miss Accidents

✓ The objective is to identify the causes leading to truck accidents enhancing the knowledge base through the data collection of near miss accidents.

✓ A system of systems engineering approach was developed where the main system components relate to the driver, the truck and the external environment conditions.
SoSE Architecture

- Wearable sensors system.
- Sensors and Can Bus.
- Central on board unit.
- Additional information from other sources such as road plans, traffic, weather and cartography.
Biometric Data

- Heart Rate (HR)
- Breathing Rate (BR)
- Posture
- Movement
Biometric Information

- Electrocardiogram (ECG)
- Heart Rate Variability (HRV)
- Respiration Trend
- Breathing Rate
- Analysis of Movement Level
Can Bus Data

- Abs / Ebs Status
- Pressure on Pedals
- Wheel based speed
- Speed of every wheel
Can Bus Information

- Precursor to rollover
- Sudden breaking
- Anomalous use of the steering wheel
- Anomalous use of pedals
SoSE Architecture (2)

Biometric Data

Behavioral Data
Experimental Context

The project included two different approaches. Firstly, a context based on simulation, and secondly the experimentation on real trucks.
Experimental Context (2)
Experimental Context (3)
Near miss accident
The future
• Automatic knowledge acquisition

• Objective
  • Is a good driver in theory also good in practise?
  • We have data from training, we have data from driving styles.
  • Applying data mining, data fusion, knowledge discovery technologies, the final objective is to give feedback to training to enhance safety
Risk averse routing

- Objective
  - To define a Vehicle Routing Problem (VRP) defined on risk
- Methodologies
  - Classic VRP
  - Multiobjective VRP (costs, risk etc…)
  - Risk averse min max risk
    - On 1 vehicle it might be quite obvious
    - If we imagine of a security problem or to play against nature, on a fleet of vehicles or even on 1 vehicles, the mixed routing strategy is better… game theory approaches
Risk averse routing

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Network model

- We define a network $G$ which consists of a vertex set $V = \{1, \ldots, n\}$ and a set of undirected arcs $A$ with $|A| = m$.
- $q_{ij}$ Probability of an incident on arc $(i, j)$ given that the arc is used.
- $c_{ij}$ Population inside a circle of given impact radius, centred at any point on the arc $(i, j)$. This is the chosen measure of the consequence of an incident on arc $(i, j)$.
- The expected number of people $X$ affected by a shipment along route $P$ is therefore

$$E[X(P)] = \sum_{(i,j) \in P} q_{ij} c_{ij}$$
Risk averse dispatching

- Problems for LPHC events.
- Consequently, a risk-averse dispatcher may prefer to base choices on pessimistic assumptions about arc incident probabilities.
- One strategy proposed is to choose the route $P$ that minimises the maximum population exposure, namely choose $P$ that minimises...
Single Route Minmax Problem

Examples  (Erkut and Ingolfsson, 2000)

- Arc elimination process: When no route can be found then the threshold needs to be increased to add some arcs.
- A shortest route algorithm taking $c_{ij}$ as arc lengths may be modified, replacing the sum operator by a max operator.
- Since the solution is embedded in a minimal spanning tree, provided the arcs are undirected and provided the minimal spanning tree is unique, Kruskal’s or Prim’s algorithms may be used. The relationship between the solution to the minmax routing problem and minimal spanning trees leads to the observation that the solution may be extremely circuitous as minimal spanning trees tend to be sparse.
Mixed Route Minmax Problem

- The risk-averse dispatcher will in general not wish to use one route but rather a mix of routes.
- Apart from equity of exposure considerations addressed in other work, which are of course important, the OD exposure as defined above can in general be reduced by using a mix of routes.
Route 1

City 1
10000 inh

Route 2

City 2
1000 inh

OD exposure:
-10000 choosing just route 1
-1000 choosing just route 2
-- with a mixed route strategy?

SoS research views of the problem
Mixed route strategy

• Let p represent the probability of choosing route 1, so (1 - p) is the probability of choosing route 2. The exposure on route 1 is 10.000p while on route 2 it is 1.000(1 - p).

• The value of p that minimises OD exposure defines the safest mix of routes.
Min_p C \text{ subject to} \\
10,000 \quad p \leq C \\
1,000 \quad (1 - p) \leq C

The solution, which is found by solving

10,000p = 1,000(1 - p),

is to use route 1 about 9.1% of the time and route 2 about 91% of the time, yielding an **OD exposure of 910**, which is less than the OD exposure when only one route is used.

The expected loss conditional on an incident

\[ C^* = q10,000p^* + (1 - q)1,000(1 - p^*) = q910 + (1 - q)910 = 910 \]
SoS approaches here can result in min max distributed control approaches as shown in the following slides.
Optimal decentralised control

• Theoretical background in brief (Rantzer, 2009) on team theory revised:
  • A convex problem can be decomposed in a set of decoupled problems (Arrow, Hurwics, Usawa; 1958)
  • This has been recently used for team problems (Rantzer 2009) based on LQ formulations, and more specifically on a DGT application (Roncoli, Bersani, Sacile, in press)
Real-time INFOMOBILITY SYSTEM

- Generic Traffic Flow Monitoring
- DG Traffic Flow Monitoring

- Forecasted Risk System
- Daily DG scheduled routing Plan System
- Forecasted Speed System

DG Transport Control System

- Optimal DG Flow
- Optimal DG Speed
Centralised control scheme:

- the unique DM must know the complete state of the system and send the control variables to all nodes and links
- high amount of information that must be exchanged within the network (require high levels of connectivity)
- more sensitive to failures and modelling errors

A DMPC method based on a dual decomposition technique is applied to solve the optimal control problem for a set of subsystems, in which only the control related to the DG flow between two neighbouring SSs are exchanged between each couple of them.

The solutions of individual subproblems, when put together, may not constitute a feasible schedule since coupling constraints have been relaxed by proper multipliers (iteratively adjusted based on the degree of constraint violations between neighbouring nodes). Subproblems are then re-solved based on the new set of multipliers until the convergence is reached (Rantzer, 2009).
\[ V \] the set of vertices, that is a set of \(|V|\) SSs
\[ E \] the set of edges \(ij\), connecting vertices \(i,j \in V\)
\[ P(i) \subset V \] the set of vertices predecessor of vertex \(i\)
\[ S(i) \subset V \] the set of vertices successor of vertex \(i\)
\[ x(t) \in \mathbb{R}^{|V|} \] the system state vector, where element \(x_i(t)\) is the state of subsystem \(i\) at time \(t\)
\[ u(t) \in \mathbb{R}^{|E|} \] the control vector
\[ w_i(t) \in \mathbb{R}^{|S(i)|} \] limited part of the control vector that includes the leaving flows for each vertex \(i\), whose elements \(w_{ij}\) are defined as \(w_{ij} = u_{ij}\) where \(j \in S(i)\)
\[ v_i(t) \in \mathbb{R}^{|P(i)|} \] limited part of the control vector that includes the incoming flows of each vertex \(i\), whose elements \(v_{ij}\) are defined as \(v_{ij} = u_{ji}\) where \(j \in P(i)\)
\[ c_{ij}(t) \] the “price” to create a consensus among the two different subsystems \(i\) and \(j\)
The general minimisation problem to be solved is:

$$H^* = \min_{x(t), u(t)} \sum_{t=1}^{T} H(x(t), u(t))$$

Let $H(x(t), u(t))$ denote a function supposed to be strictly convex, and separable in terms of functions $H_i(x_i(t), u_{ik}(t), u_{hi}(t))$, which are related to each subsystem $i \in V$, depending on the subsystem state $x_i(t)$, and on the shared control:

$$H^* = \min_{x_i(t), u_{ik}(t), u_{hi}(t)} \sum_{t=1}^{T} \sum_{i \in V} \sum_{k \in S(i)} \sum_{h \in P(i)} H_i(x_i(t), u_{ik}(t), u_{hi}(t))$$

where $u_{ik}(t)$ and $u_{hi}(t)$ are the outcoming and incoming control variables for vertex $i$. 
Specifically, for a generic link $ij$, there is one component of vector $w_i(t)$, say $w_{ij}(t)$, and one component of vector $v_j(t)$, say $v_{ji}(t)$, for which an agreement must be found so that $w_{ij}(t) = v_{ji}(t)$.

The problem could be rewritten as:

$$H^* = \max_{c_{ij}(t)} \min_{x_i(t), w_i(t), v_i(t)} \left\{ \sum_{t=1}^{T} \sum_{i \in V} H_i(x_i(t), w_i(t), v_i(t)) \right\}$$

$$+ \sum_{ij \in E} \left[ c_{ij}(t)(w_{ij}(t) - v_{ji}(t)) \right]$$
The optimisation problem can be therefore completely decentralised in $N + (M(T - 1))$ problems, that are:

$$
H^*_i = \min_{x_i(t), w_i(t), v_i(t)} \left\{ H_i(x_i(t), w_i(t), v_i(t)) \right\}
$$

$$
= \min_{x_i(t), w_i(t), v_i(t)} \left\{ \sum_{t=1}^{T} \left( H_i(x_i(t), w_i(t), v_i(t)) + \sum_{j \in S(i)} c_{ij}(t)w_{ij}(t) - \sum_{k \in P(i)} c_{ki}(t)v_{ik}(t) \right) \right\}
$$

$$
K^*_{ij} = \max_{c_{ij}(t)} \left( w_{ij}(t) - v_{ij}(t) \right)
$$

$t = 1, \ldots, T$, and $\forall ij \in E$

This approach provides the possibility of solving the problem performing a fair bargaining in order to optimise its own objective function under a cooperative vision of the problem. The optimal solution can be computed applying an unconstrained optimisation method (i.e. the gradient search method).
Robust decentralised control

- A min max quadratic problem can be transformed into a Linear Matrix Inequality problem
SoS research views of the problem

\[
\inf_{\mu} \sup_{0 \neq x \in \mathbb{R}^n} \frac{J(x, u)}{\|x\|^2}
\]

subject to \( y_i = C_i x \)

\[ u_i = \mu_i(y_i) \]

for \( i = 1, \ldots, N \)

\[
J(x, u) = \begin{pmatrix} x & u \end{pmatrix}^T \begin{pmatrix} Q_{xx} & Q_{xu} \\ Q_{ux} & Q_{uu} \end{pmatrix} \begin{pmatrix} x \\ u \end{pmatrix}
\]

\[
\min_{\gamma, K} \gamma
\]

subject to \( K = \text{diag}(K_1, \ldots, K_N) \)

\[
\begin{pmatrix} Q_{xx} - \gamma I + Q_{xu} K C + C^T K^T Q_{ux} & C^T K^T \\ K C & -Q_{uu} \end{pmatrix} \preceq 0
\]
System of systems network

Variables

\( X_i \) inoperability of subsystem \( i \)
\( U_i \) resilience allocated to subsystem \( i \)
\( W_i \) random attack to system \( i \)

The static relationship governing the SoS is given by:

\[ x = Ax + Bu + w \]
An example

Subsystem 1: A power plant
Subsystem 2: A transportation system (roads, signs, signaling facilities, etc.)
Subsystem 3: A hospital
Subsystem 4: A urban area
The system of systems networks

- Influence: each subsystem state may influence another one
- Control/logistics: allocation/covering of resiliance
- Information: access to state observability
The system of systems influence network

\[
A = \begin{bmatrix}
0 & 0 & 0 & 0 \\
0.2 & 0 & 0 & 0 \\
0.5 & 0.5 & 0 & 0 \\
0.8 & 0.6 & 0.1 & 0 \\
\end{bmatrix};
\]

Values \( a_{ij} \) shows the degree of system \( j \) affecting subsystem \( i \).
The SoS control/logistics network

Centre A u1
Centre B u2
Centre C u3

Values $b_{ij}$ show the resilience of system $j$ affecting subsystem $i$.
The SoS information network

\[
\begin{aligned}
C_1 &= \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \\
C_2 &= \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix} \\
C_3 &= \begin{bmatrix} 0 & 1 & 0 & 0 \\
& 0 & 0 & 1 \\
& & 0 & 1 \end{bmatrix}
\end{aligned}
\]

Values $c_{ij}$ shows the (1) observability of $j$ by centre $i$ or (0) not.
SoS research views of the problem

Three network views

Diagram 1: 1, 2, 3, 4

Diagram 2: A, B, 1, 2, 3, 4, C

Diagram 3: A, B, 1, 2, 3, 4, C
Question

Which is the more robust control law to optimize the following problem?

\[ \inf_{\mu} \sup_{w} J(x, u, w) = \frac{x'Qx + u'Ru}{\|w\|^2} \]

\[ y_i = C_i w \]

\[ u_i = \mu_i (y_i) = K_i y_i = K_i C_i w \quad \text{for } i=1,\ldots,N. \]

where

\[ y_i \]

represent the observability of the system \( i-th \);
In [*] it is demonstrated that for the minimax team problem with a quadratic cost given by (12), the linear policy $u = KCx$ is optimal, and the optimal values for $K$ can be computed by solving the Linear Matrix Inequality (LMI) (14).

$$J(x, u) = \begin{bmatrix} x \\ u \end{bmatrix} \begin{bmatrix} Q_{xx} & Q_{xu} \\ Q_{ux} & Q_{uu} \end{bmatrix} \begin{bmatrix} x \\ u \end{bmatrix}$$

where

$$Q_{uu} > 0$$

subject to:

$$\begin{bmatrix} Q_{xx} - \gamma I + Q_{xu}KC + C^T K^T Q_{ux} & C^T K^T \\ KC & -Q_{uu}^{-1} \end{bmatrix} > 0$$
Let be the following generic team decision problem

\[
\inf_{u} \sup_{w} J(u, w) \tag{15}
\]

s.t.
\[
x(k + 1) = Ax(k) + Bu(k) + w(k) \tag{16}
\]
\[
y_i(k) = C_i x_i(k) \tag{17}
\]
\[
u_i(k) = \mu_i(y_i(k)) \tag{18}
\]
for \(i = 1, \ldots, N\).

The observation of the system \(i\) at the time \(k\) is given by (17).

Let be \(J(u, w)\) defined as follows:

\[
J(u, w) = x'(K)Q_{xx}x(K) + \sum_{k=0}^{K-1} \left\{ x'(k) \begin{bmatrix} Q_{xx} & Q_{xu} \\ Q_{ux} & Q_{uu} \end{bmatrix} x(k) - \gamma \|w(k)\|^2 \right\} \tag{19}
\]

\[
\begin{bmatrix} Q_{xx} & Q_{xu} \\ Q_{ux} & Q_{uu} \end{bmatrix} \in \mathbb{R}^{M \times M} \tag{20}
\]
\[
Q_{xx} \in \mathbb{R}^{M \times M} \tag{21}
\]
\[
Q_{xu} \in \mathbb{R}^{M \times M} \tag{22}
\]
\[
Q_{ux} \in \mathbb{R}^{M \times M} \tag{23}
\]
\[
Q_{uu} \in \mathbb{R}^{M \times M} \tag{24}
\]

and
\[
Q_{xx} \succeq 0 \tag{25}
\]
\[
Q_{uu} > 0 \tag{26}
\]
Now write $x(k)$ as

$$x(k) = A^5 x(k) + \sum_{n=0}^{\tau-1} A^n B u(k - n - 1) + \sum_{n=0}^{\tau-1} A^n w(k - n - 1) =$$

(27)

Replacing (27) in (19), we obtain the following minimization problem

$$J(u, w) = \left[ A^T x(0) + \sum_{n=0}^{\tau-1} A^n B u(k - n - 1) + \sum_{n=0}^{\tau-1} A^n w(k - n - 1) \right]^T Q_{xx}$$

$$\left[ A^T x(0) + \sum_{n=0}^{\tau-1} A^n B u(k - n - 1) + \sum_{n=0}^{\tau-1} A^n w(k - n - 1) \right]$$

$$+ \sum_{\tau=0}^{\tau-1} \left\{ \left[ A^T x(0) + \sum_{n=0}^{\tau-1} A^n B u(k - n - 1) + \sum_{n=0}^{\tau-1} A^n w(k - n - 1) \right]^T \begin{bmatrix} Q_{xx} & Q_{xu} \\ Q_{ux} & Q_{uu} \end{bmatrix} \right\}$$

$$- \gamma \|w(t)\|^2$$

(28)
Dynamic version \((4)\)
Decentralised robust control of a system of inventory systems

Problem definition
The proposed system of Inventory System (SOIS):

- the production of one product.
- this product is manufactured and consumed within local economies.
- production is supposed to be a priori defined by a production planning previously computed according to the demand forecast. Alternatively production depends on external uncontrolled factors and its behaviour can be forecasted.
- in both cases, the balance between local production and local demand results in a stochastic process whose trend can be predicted with a certain degree of reliability.
Decentralised robust control of a system of inventory systems: application to a microgrid network

The SOIS has the following discrete state equation:

\[ x(k + 1) = Ax(k) + Bu(k) + p(k) + w(k) \quad k = 0..K - 1 \]

Each local economy has an optimal reference inventory to be stocked in its local inventory representing the local economy state.
Problem formulation

State equation

\[ x(k+1) = Ax(k) + Bu(k) + p(k) + w(k) \quad k = 0..K - 1 \]
\[ x(0) \text{ given} \]

(1)

where:
- \( x(k) \in R^S \) is the product inventory vector, whose \( m \)-th element is \( x_m(k) \), that is the inventory in the \( m \)-th IS;
- \( u(k) \in R^2 \) is the control vector of product flows exchanged in the SOIS in time interval \( (k,k+1] \); the generic component is \( u_i(k) \).
- \( p(k) \in R^S \) is the vector expressing the product balance in the SOIS as previously planned or however forecasted, whose \( m \)-th element is \( p_m(k) \);
- \( w(k) \in R^S \) is the vector expressing the error in the forecast of \( p(k) \). The generic element \( w_m(k) \) is modelled as a stochastic variable \( N(0,W_m) \);
- \( A \in R^{S,S} \) is a diagonal matrix whose \( m \)-th diagonal coefficient is \( a_m \);
- \( B \in R^{S,2} \) is the incidence matrix of \( G \). Specifically let \( B \) be defined as \( B = [b_1 \ b_2 \ \ldots \ b_2] \), where the value of the generic element of \( b_i \in R^{S} \), that is \( b_{ij} \), is equal to 1 if a link exists between \( i \) and \( j \), with direction towards \( i \) (that is according to the convention \( i > j \)), -1 if a link exists between \( i \) and \( j \), with direction towards \( j \) (that is according to the convention \( i < j \)), 0 if no link exist between \( i \) and \( j \).
Problem formulation

Let the problem be defined as

\[
\inf_{\mu} \sup_{w} J(u, w)
\]

\[
J(u, w) = x'(K)Q_{xx}x(K) + \sum_{k=0}^{K-1} \begin{bmatrix} [x(k)]' \\ [u(k)]' \end{bmatrix} \begin{bmatrix} Q_{xx} & 0 \\ 0 & Q_{uu} \end{bmatrix} \begin{bmatrix} x(k) \\ u(k) \end{bmatrix} - \gamma \|w(k)\|^2
\]

s.t.

\[
x(k + 1) = Ax(k) + Bu(k) + w(k) + p(k) \\
y_i(k) = b_i' x(k) \\
u_i(k) = \mu_i(y_i(k)) \quad i = 1 \ldots Z
\]

how to replace \( p(k) \)?

\( x(k) \) is the variation of the inventory with respect to a reference level \( x_0 \)

\( p(k) \) is a deterministic known sequence of values given by

\[ p(k) = (A - I)z^* + f(k) \]
Adaptation to dangerous goods transport

• The problem can be here seen again as a distributed problem, where regions containing a transport infrastructure wish to cooperate to minimise the maximum risk
  – The inventory is made by dangerous goods
  – Information is decentralised
  – Flow is controlled on the network of infrastructures
Conclusions

- Dangerous goods transport is a promising example of application of SoS technologies
- If you are interested to start a similar adventure, I would be happy to collaborate with you