Application de la théorie des fonctions de croyance aux études de la fiabilité

F. Aguirre, M. Sallak, W. Schön
faguirre@utc.fr

22 Novembre, 2011

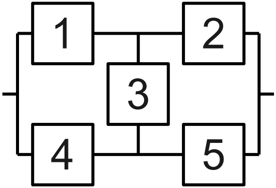
HEUDIASYC UMR CNRS 6599
Université de Technologie de Compiègne

Outline

- Introduction
  - Classical reliability analysis
  - How to treat epistemic uncertainty
- Belief function theory
- Reliability analysis using belief functions
- Reliability benchmark
- Conclusion

Introduction

Reliability of non repairable systems SIT50.



Reliability The ability of a system or component to perform its required functions under stated conditions for a specified period of time.

Introduction

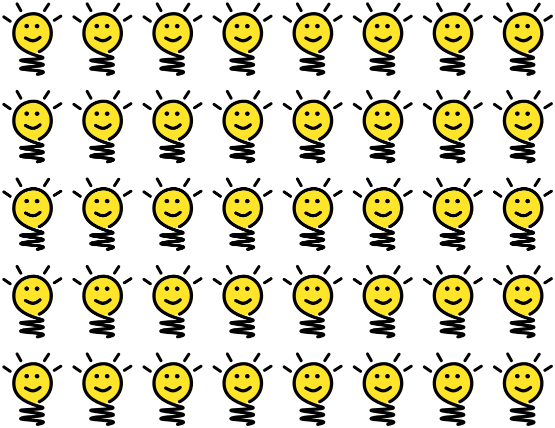
How to predict the failure ?

- ↪ Is a future and uncertain event.
- ↪ We want to understand, model and predict it in order to take a decision
- ↪ **If** enough background model **Then** Probability theory
- ↪ **Else** chance, propension, possibility, probability, degree of belief or degree of logical support ? ? ?

What should we do ?

Introduction

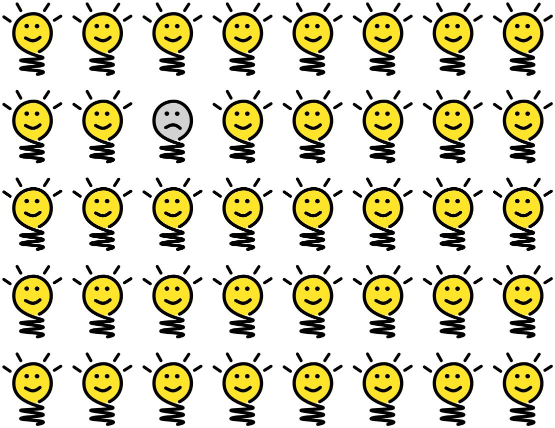
What is a good background knowledge ?



Life (h)	

Introduction

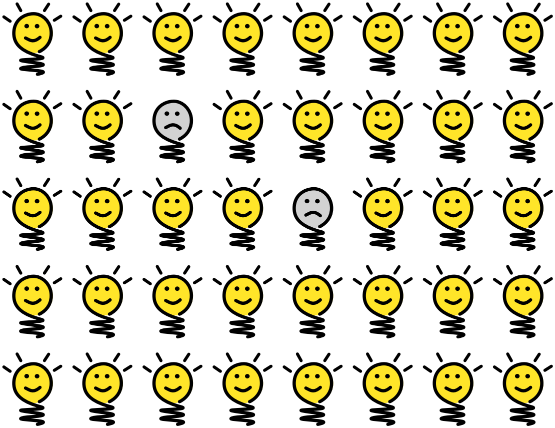
What is a good background knowledge ?



Life (h)	
4925	

Introduction

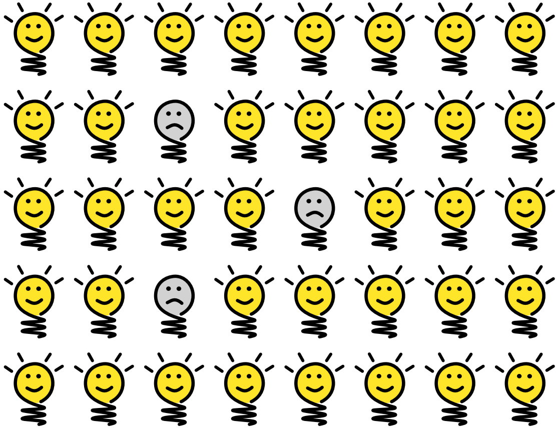
What is a good background knowledge ?



Life (h)	
4925	
5165	

Introduction

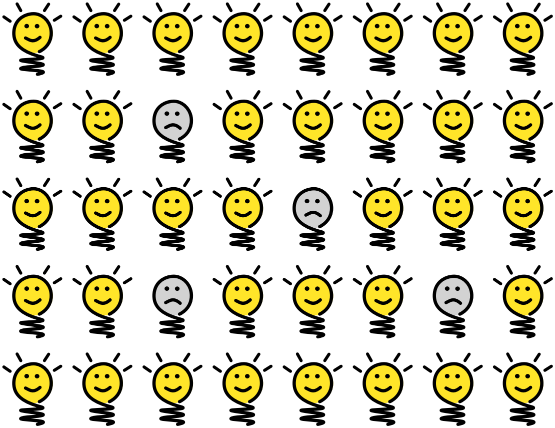
What is a good background knowledge ?



Life (h)	
4925	
5165	
5300	

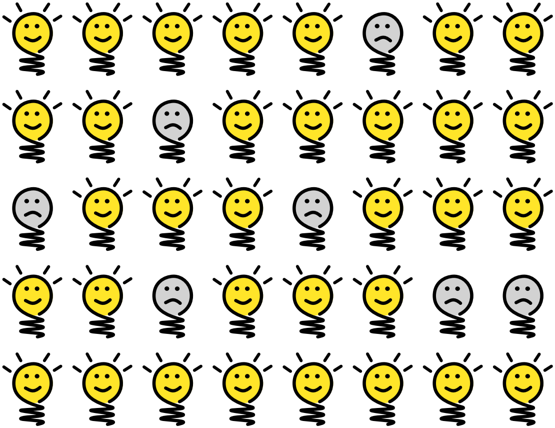
Introduction

What is a good background knowledge ?



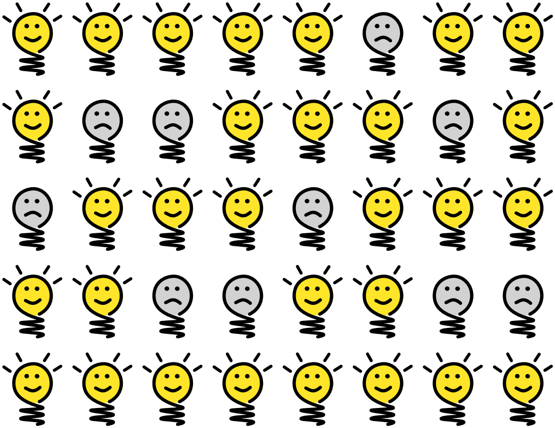
Life (h)	
4925	
5165	
5300	
5482	

Introduction
What is a good background knowledge ?



Life (h)	
4925	
5165	
5300	
5482	
5593	
5597	
5700	

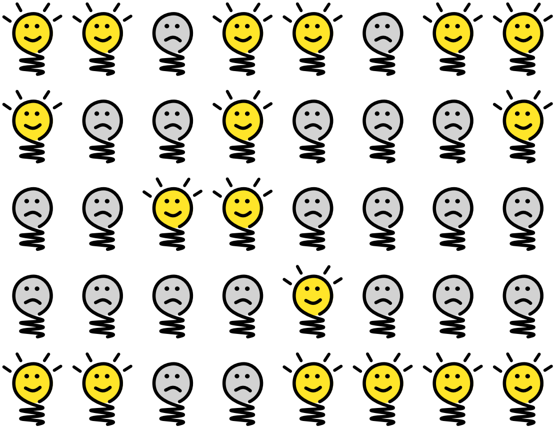
Introduction
What is a good background knowledge ?



Life (h)	
4925	
5165	
5300	
5482	
5593	
5597	
5700	
5852	
5862	
5880	

Introduction

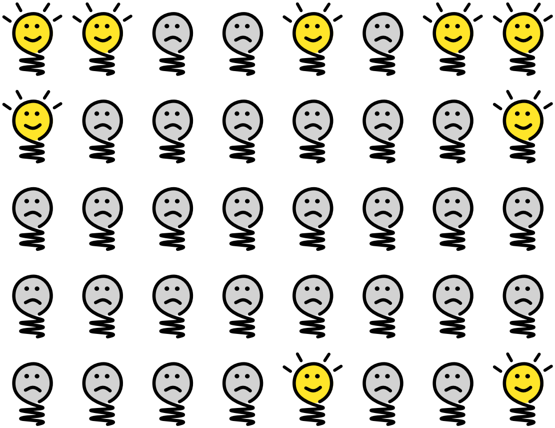
What is a good background knowledge ?



Life (h)	
4925	6360
5165	6372
5300	
5482	
5593	
5597	
5700	
5852	
5862	
5880	
5942	
5970	
5986	
6025	
6070	
6100	
6182	
6204	
6282	
6350	

Introduction

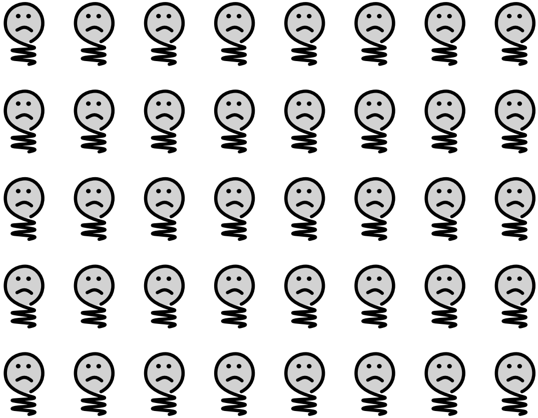
What is a good background knowledge ?



Life (h)	
4925	6360
5165	6372
5300	6405
5482	6450
5593	6482
5597	6512
5700	6572
5852	6652
5862	6710
5880	6720
5942	6735
5970	
5986	
6025	
6070	
6100	
6182	
6204	
6282	
6350	

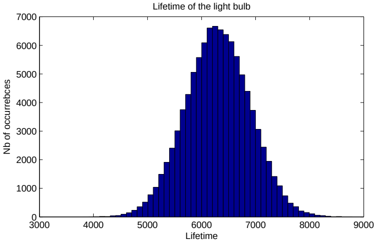
Introduction

What is a good background knowledge ?



Life (h)	
4925	6360
5165	6372
5300	6405
5482	6450
5593	6482
5597	6512
5700	6572
5852	6652
5862	6710
5880	6720
5942	6735
5970	6800
5986	6822
6025	6940
6070	7014
6100	7050
6182	7090
6204	7180
6282	7325
6350	7402

Introduction
Failure rate estimations

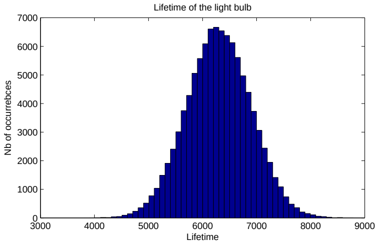


MTTF Mean time to failure $\approx 6302h$

λ Failure rate $\approx \frac{1}{MTTF} \approx 1.587E-4$

$$R(t) = e^{-\lambda t}$$

Introduction
Failure rate estimations



MTTF Mean time to failure $\approx 6302h$

λ Failure rate $\approx \frac{1}{MTTF} \approx 1.587E-4$

$$R(t) = e^{-\lambda t}$$

Aleatory uncertainty

Introduction

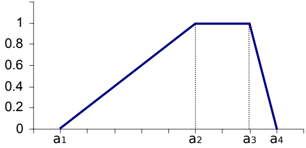
And if there isn't a good background knowledge ? SIT53

- Fuzzy sets theory.
- Imprecise probabilities.
- Monte Carlo simulations.
- Dempster-Shafer theory.

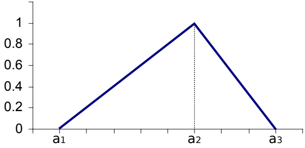
Aleatory vs. Epistemic

Introduction
Fuzzy set approach

- Reliability is represented by fuzzy numbers



TFN(a1,a2,a3,a4)



TFN(a1,a2,a3,a4)

$$fOR = \left(1-\prod_{i=1}^{n}(1-a_{i1}), 1-\prod_{i=1}^{n}(1-a_{i2}), 1-\prod_{i=1}^{n}(1-a_{i3}),\right)$$

$$fAND = \left(\prod_{i=1}^{n}a_{i1}, \prod_{i=1}^{n}a_{i2}, \prod_{i=1}^{n}a_{i3},\right)$$

Introduction
Imprecise probabilities

- Reliability is represented by an upper and lower probability

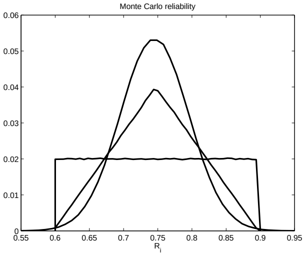
$$\underline{P}(S_{path}) = \max\left\{0, \max_{1\leq i\leq N_T}\left(\sum_{j=1}^{n(T_i)} \underline{P}(W_{T_i(j)}) - (n(T_i)-1)\right)\right\}$$

$$\overline{P}(S_{cut}) = \min\left\{1, \min_{1\leq i\leq N_C}\sum_{j=1}^{n(C_i)} \overline{P}(W_{C_i(j)})\right\}$$

$$R_s \in [\underline{P}(S_{path}), \overline{P}(S_{cut})]$$

Introduction
Monte Carlo simulations

- Reliability is represented by a probability distribution.



Outline

- Introduction
- Belief function theory
  - Fundamental concepts
  - Combination of information
- Reliability analysis using belief functions
- Reliability benchmark
- Conclusion

Belief functions theory [1]

- It is well adapted to model Ignorance $\rightarrow m^{\Omega}(\Omega)$
- There is a clear distinction from what you can and can't support (Given your background knowledge)
- Yields the same results as probability theory when epistemic uncertainty is eliminated. Consistency with current theories

1. Adapted from several presentations from Thierry Denoeux

Belief functions theory
Fundamental concepts

- Let X be a variable taking values in a finite domain Ω (frame of discernment).

Mass function

Is a mapping from $2^{\Omega} \rightarrow [0,1]$ such that :

$$\sum_{A \subseteq \Omega} m^{\Omega}(A) = 1$$

- Every subset A of Ω such that $m^{\Omega}(A) > 0$ is called a focal set

Belief functions theory
Belief and plausibility functions

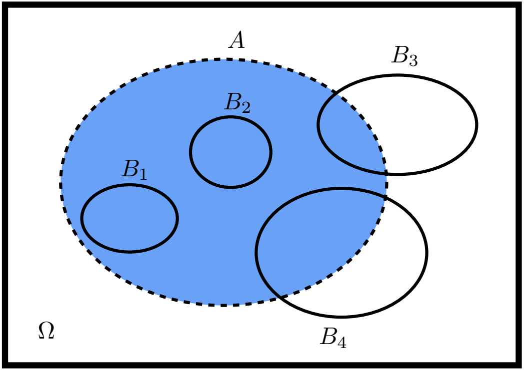
Belief function
Degree to which the evidence supports *A*.

Plausibility function
Maximal degree of support that could be assigned to *A* if there were more available evidence.

$$Bel(A) = \sum_{\emptyset \neq B \subseteq A} m^{\Omega}(B) \quad \& \quad Pl(A) = \sum_{B \cap A \neq \emptyset} m^{\Omega}(B)$$

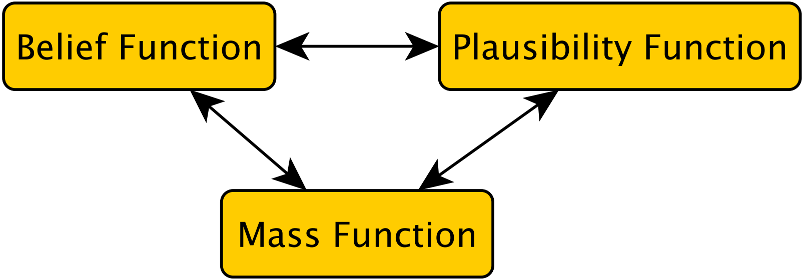
$$\forall A, B \subseteq \Omega \tag{1}$$

Belief functions theory
Belief and plausibility functions



$$Bel(A) = m(B_1) + m(B_2)$$
$$Pl(A) = m(B_1) + m(B_2) + m(B_3) + m(B_4)$$

Belief functions theory
Equivalence between different functions



Belief functions theory
Combination of information

- The two pieces of information are reliable and come from distinct pieces of evidence.

Conjunctive combination rule

$$m^{\Omega}_{i\cap j}(H) = \sum_{A\cap B=H,} m^{\Omega}_{i}(A)m^{\Omega}_{j}(B), \quad \forall A,B \subseteq \Omega$$

- If there is a conflict between both pieces of evidence :
  - Open world assumption : Answer is elsewhere
  - One of the sources is wrong (or both, why not..)

Belief functions theory
Combination of information

- If there is a conflict and you want to normalize.

Dempster's combination rule

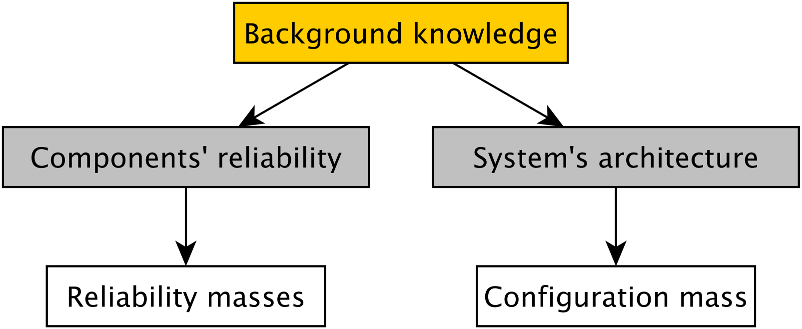
$$m^{\Omega}_{i\oplus j}(H) = \frac{m^{\Omega}_{i\cap j}(H)}{1-k}, \quad \forall A,B \subseteq \Omega$$

With :$k = m^{\Omega}_{i\cap j}(\emptyset)$ Conflict factor

Outline

- Introduction
- Belief function theory
- Reliability analysis using belief functions
  - Initial model
  - Generalized reliability expressions using the TBM
  - Aggregation of Experts' Opinions
  - Failure dependencies
- Reliability benchmark
- Conclusion

Reliability assessments using the TBM



Mass construction
Components' reliability masses

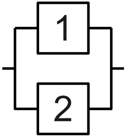
- Experts opinions $\rightarrow m^{\Omega_i}(\{W_i, F_i\})$

$$
\begin{aligned}
m^{\Omega_i}(\{F_i\}) &= f_i \\
m^{\Omega_i}(\{W_i\}) &= w_i \\
m^{\Omega_i}(\{W_i, F_i\}) &= 1 - w_i - f_i
\end{aligned}
$$

- Failure rate $\rightarrow [\underline{\lambda}, \overline{\lambda}]$

$$
\begin{aligned}
m^{\Omega_i}(\{F_i\}) &= 1 - e^{-t\underline{\lambda}} \\
m^{\Omega_i}(\{W_i\}) &= e^{-t\overline{\lambda}} \\
m^{\Omega_i}(\{W_i, F_i\}) &= e^{-t\underline{\lambda}} - e^{-t\overline{\lambda}}
\end{aligned}
$$

Mass construction
System's configuration

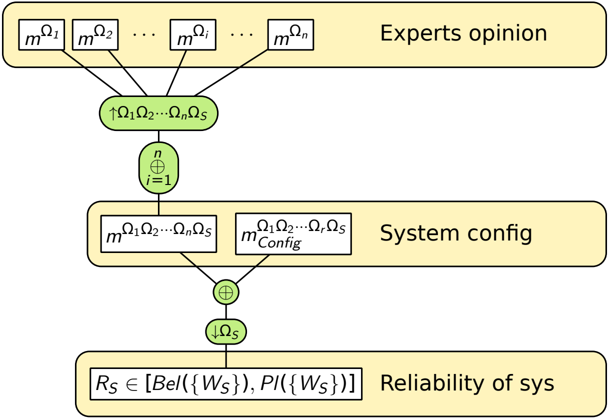


x_1	x_2	x_S
0	0	0
0	1	1
1	0	1
1	1	1

TABLE: Truth table

$$m_{parallel}^{\Omega_1 \times \Omega_2 \times \Omega_S}(\{(F_1, F_2, F_S), (F_1, W_2, W_S), (W_1, F_2, W_S)(W_1, W_2, W_S)\}) = 1$$

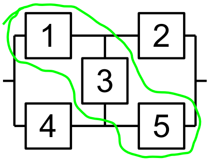
Reliability of a system using the TBM



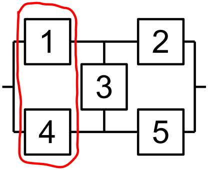
$$m^{\Omega_S} = \left(\oplus_{i=1}^{n} m^{\Omega_i \uparrow \Omega_1\Omega_2\cdots\Omega_n\Omega_S} \oplus m_{Config}^{\Omega_1\Omega_2\cdots\Omega_n\Omega_S}\right)^{\downarrow\Omega_S}$$

Generalized reliability expressions

Minimal cuts and paths



Minimal path



Minimal cut

Minimal paths :

$A_1 = \{1, 2\}, \quad A_2 = \{4, 5\}, \quad A_3 = \{1, 3, 5\}, \quad A_4 = \{2, 3, 4\},$

$R_s \rightarrow P(A_1 \cup A_2 \cup A_3 \cup A_4) \rightarrow$ Poincare's theorem

Generalized reliability expressions using the TBM

Minimal cuts

$$Bel(W_S) = \prod_{i=1}^{N_C}\left(1 - \prod_{j=1}^{n(C_i)}\left(1 - m\{W_{C_i(j)}\}\right)\right)$$

$$Pl(W_S) = \prod_{i=1}^{N_C}\left(1 - \prod_{j=1}^{n(C_i)} m\{F_{C_i(j)}\}\right)$$

Minimal paths

$$Bel(W_S) = 1 - \prod_{i=1}^{N_T}\left(1 - \prod_{j=1}^{n(T_i)} m\{W_{T_i(j)}\}\right)$$

$$Pl(W_S) = 1 - \prod_{i=1}^{N_T}\left(1 - \prod_{j=1}^{n(T_i)}\left(1 - m\{F_{T_i(j)}\}\right)\right)$$

Generalized reliability expressions using the TBM

- When minimal paths and/or minimal cuts are not disjoint :

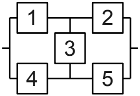
$$\begin{aligned} \forall n \geq 1 : & \quad A_1, A_2, \ldots A_n \subseteq \Omega \\ Bel(A_1 \cup A_2 \cup \ldots A_n) & = \sum_i Bel(A_i) - \sum_{i>j} Bel(A_i \cap A_j) \\ & \cdots - (-1)^n Bel(A_1 \cap A_2 \cap \ldots A_n) \end{aligned}$$

$$\begin{aligned} Bel(W_s) = & \quad Bel(A_1 \cup A_2 \cup \ldots A_n) \\ Pl(W_s) = & \quad 1 - Bel(C_1 \cup C_2 \cup \ldots C_n) \end{aligned}$$

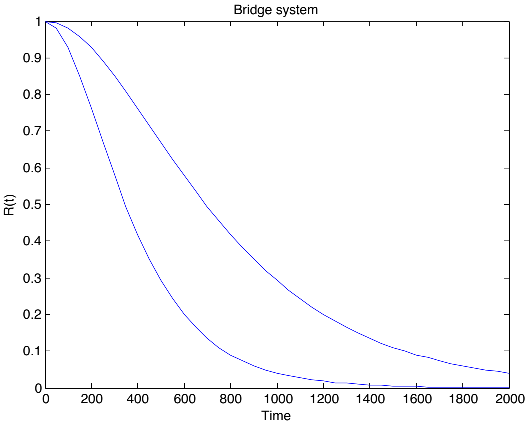
$$\Downarrow$$

$$R_s \in [Bel(W_s), Pl(W_s)]$$

Generalized reliability expressions using the TBM



$\lambda = [0.001, 0.002]$



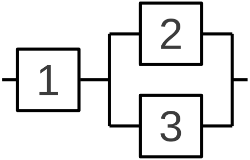
Aggregation of experts opinions

- Experts opinion are not independent (Based on common background)
- Idempotent combination rules $\rightarrow m_1 \oplus m_1 = m_1$

Combination rules	Idempotent
Conjunctive	No
Disjunctive	No
Dempster	No
Averaging	Yes
Cautious	Yes
Yager	No

Dependencies between components

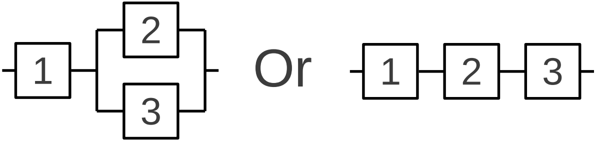
First proposition



$$m_{Config}^{\Omega_1\times\Omega_2\times\Omega_3\times\Omega_S} = 1-\delta$$

$$m_{Depen}^{\Omega_1\times\Omega_2\times\Omega_3\times\Omega_S} = \delta$$

Dependencies between components
Second proposition



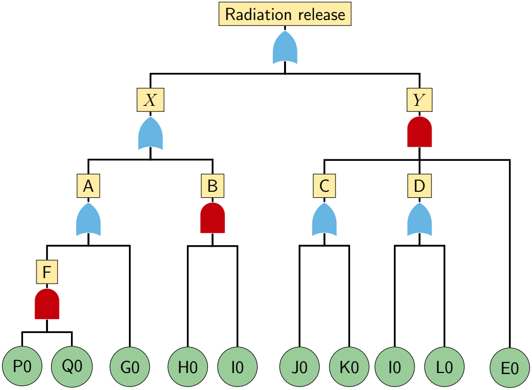
x_1	x_2	x_3	x_S
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	{0,1}
1	1	0	{0,1}
1	1	1	1

TABLE: Truth table

Outline

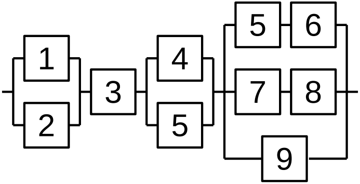
- Introduction
- Belief function theory
- Reliability analysis using belief functions
- Reliability benchmark
  - Case study
  - Belief functions vs. Imprecise probabilities
- Conclusion

Nuclear power plant



Fault tree of a nuclear power plant

Nuclear power plant



Reliability block diagram of a nuclear power plant

$$C = [1,2],[3],[4,5],[5,7,9],[5,8,9],[6,7,9],[6,8,9]$$

Nuclear power plant

	E0		G0		P0		J0		L0	
Expert	L	U	L	U	L	U	L	U	L	U
1	0.008	0.012	0.032	0.052	0.075	0.085	0.042	0.058	0.04	0.16
2	0.026	0.034	0.044	0.056	0.046	0.054	0.055	0.065	0.16	0.34
3	0.020	0.030	0.034	0.046	0.065	0.075	0.038	0.052	0.16	0.24

	Q0		H0		I0		K0	
Expert	L	U	L	U	L	U	L	U
1	0.072	0.093	0.012	0.032	0.07	0.15	0.12	0.19
2	0.055	0.075	0.020	0.046	0.18	0.23	0.16	0.26
3	0.045	0.060	0.034	0.048	0.23	0.29	0.28	0.35

TABLE: Experts opinion

Nuclear power plant

Comb rule →	Dempster		Cautious		Average		Fuzzy		IP	
Event	L	U	L	U	L	U	L	U	L	U
E0	0.0000	0.0000	0.0129	0.0169	0.0180	0.0253	0.0163	0.0237	0.026	0.012
G0	0.0001	0.0001	0.0436	0.0554	0.0367	0.0513	0.0380	0.5200	0.044	0.046
P0	0.0004	0.0004	0.0592	0.0671	0.0620	0.0713	0.0703	0.0797	0.075	0.054
J0	0.0002	0.0002	0.0550	0.0650	0.0450	0.0583	0.0450	0.0590	0.055	0.052
L0	0.0164	0.0190	0.1600	0.2400	0.1200	0.2467	0.1120	0.2380	0.160	0.016
Q0	0.0005	0.0005	0.0511	0.0660	0.0573	0.0760	0.0590	0.0760	0.072	0.060
H0	0.0001	0.0001	0.0340	0.0480	0.0220	0.0420	0.0220	0.0410	0.034	0.032
I0	0.0164	0.0168	0.1895	0.2389	0.1600	0.2233	0.1520	0.2180	0.230	0.150
K0	0.0306	0.0315	0.2414	0.3017	0.1867	0.2667	0.1950	0.2750	0.280	0.190

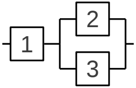
TABLE: Combination of experts opinion

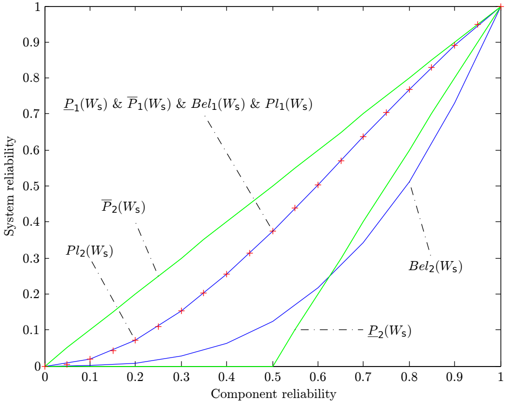
Nuclear power plant

Theory	Likelihood of top event	
Belief Functions	$Bel(F_s)$ 0.04607	$Pl(F_s)$ 0.06897
Imprecise probabilities	P_{low} 0.03800	P_{up} 0.19270
Monte-Carlo simulations	$P_{1\%}$ 0.04912	$P_{99\%}$ 0.06503
Fuzzy sets theory	a_1 0.04600	a_4 0.06900

TABLE: Results obtained using the different theories

TBM vs. Imprecise probabilities





Outline

- Introduction
- Belief function theory
- Reliability analysis using belief functions
- Reliability benchmark
- Conclusion

Conclusion

- Uncertainties about : the model, the components reliabilities and the dependencies between failures can be modeled.
- In a similar way, multi-state systems can also be studied
- The obtained results are consistent with the classical theories
- Reliability of the systems doesn't converge to $[0,1]$ as $n \rightarrow \infty$ systems.
- Graphical methods - Fault tree analysis
- Toolbox Matlab.

Thank you for you attention ! !

faguirre@utc.fr