

From Mathematics to Computer Science, a Logical Algorithm

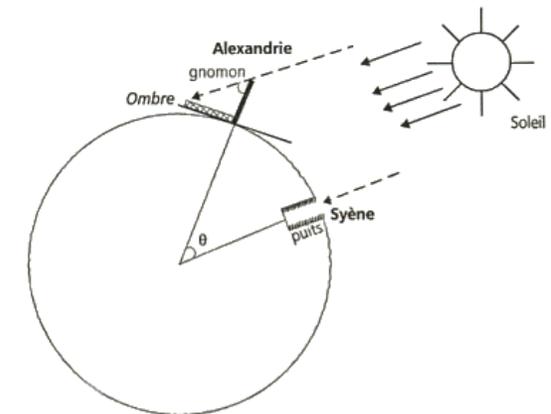
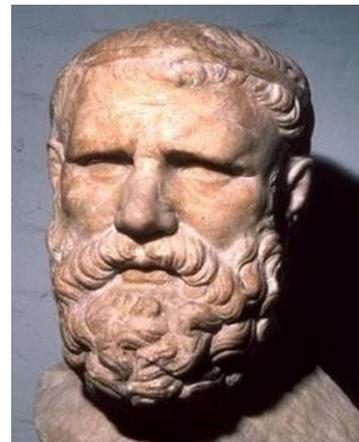
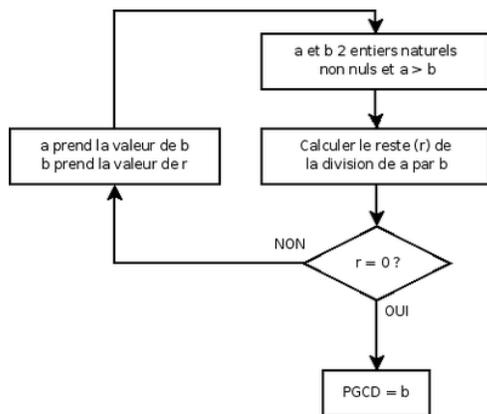
Dritan Nace

In the beginning...

In the beginning there were Babylonian, Egyptian, Chinese mathematics, and especially the Greeks...

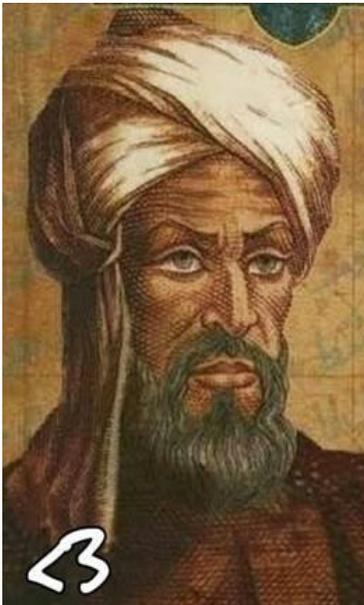
The main object of study was the calculation of various laws of nature, small practical problems...

The most famous work of Greek time is Euclid's "Elements" (in thirteen books), presenting mathematics for the first time in a systematic and formal way, based on axioms and postulates, as well as theorems accompanied by their demonstrations. There is also the first simple algorithm: calculating the Greatest Common Divisor!



Algoritmi

- Muhammad Ibn Mūsā al-Khwarizmī, (in latin *Algoritmi*), born in 780...
- The father of algebra.



Merveilleuse Équation de l'homme d'après Al-Khwarizmi (le père de l'algèbre) :

On interrogea le père de l'algèbre sur l'homme. Ce dernier a répondu humblement:

"Si l'homme est éthique et plein de morale, c'est = 1 ;

S'il est en plus charmant, on lui ajoute un zéro, c'est = 10 ;

S'il est riche, on lui ajoute un autre zéro, c'est = 100 ;

S'il est d'origine noble, on lui ajoute un autre zéro et c'est = 1000 ;

Si la valeur morale (nombre 1) de cette personne disparaît, il ne lui reste que les zéros qui n'ont aucune valeur."

Paroles des sages à travers les âges

An inheritance problem that is modelled as an equation of the second degree $ax^2 - bx - c$:
where the number of children and (b) the number of wives and c the size of the parcel of land as a inheritance.

Entrer a, b, c

$$\Delta \leftarrow b^2 - 4ac$$

Afficher Δ

SI $\Delta < 0$ Alors

Afficher « Pas de racine »

Sinon

$$x_1 \leftarrow \frac{-b - \sqrt{\Delta}}{2a}$$

$$x_2 \leftarrow \frac{-b + \sqrt{\Delta}}{2a}$$

Afficher x_1, x_2

FinSI

Leibniz: logic, what links philosophy with mathematics

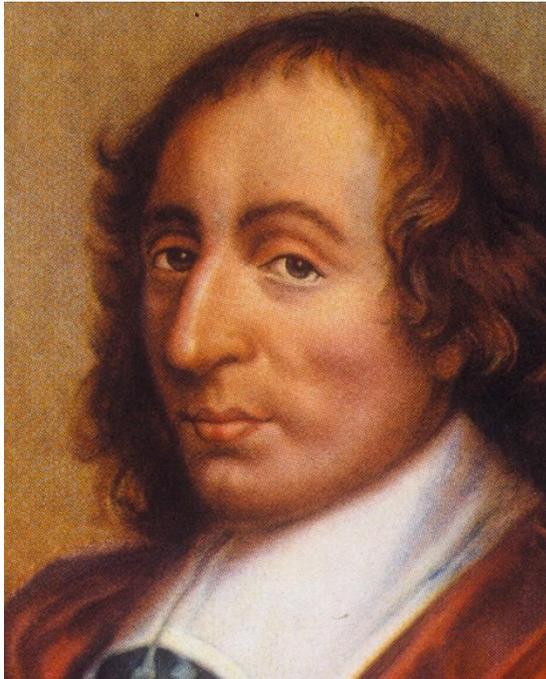
For Leibniz philosophy and mathematics form an inseparable whole whose logic ensures cohesion.

1. The idea that Logic is the Key to Nature. The *calculus ratiocinator* is an algorithm or theoretical computational machine that would unravel the true from the false in any discussion whose terms would be expressed in the universal philosophical language, the **universal characteristic**. To solve a question, you just have to come together and say, "Let's calculate!"
2. Leibniz's calculator machine, a technological exploit for the time.



Pascal and pascaline

- **Blaise Pascal and his machine**



The machines ... programmable (around the 18th century)



Logic on the move: advances and paradoxes

1850 - propositional calculation (Boole)

1880 - calculation of the predicates

1884 - theory of sets

1900 - crisis of the foundations of mathematics? Formalize arithmetic with a deductive system

1904 – Russell's paradox

Soit $A = \{x \mid x \notin x\}$

Si $A \notin A$ alors $A \in A$

The naïve theory of sets is inconsistent.

David Hilbert – le programme universel

Hilbert advocated that arithmetic can be formalized by a deductive system of axioms, deduction rules and that satisfy three properties.

Axioms;

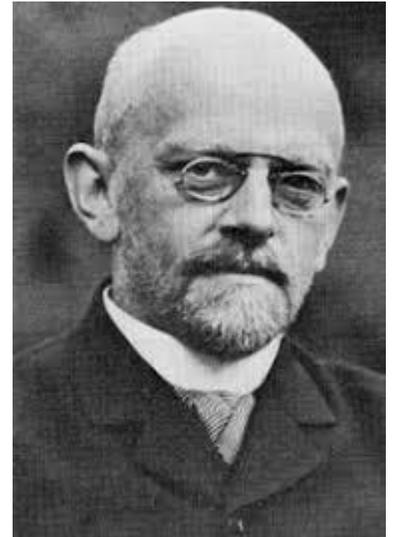
Deduction rules: $P \Rightarrow Q$ then if P is true implies that Q is true;

Properties:

You can't have P and not P (coherence)

We always have either p or non P (completeness)

There is an algorithm that, given P decides whether P or not P (decidability)



The new generation: Kurt Gödel...

Gödel – 1932

First theorem of incompleteness. Any consistent formalization of arithmetic contains a statement P as can be inferred neither P nor $\neg P$



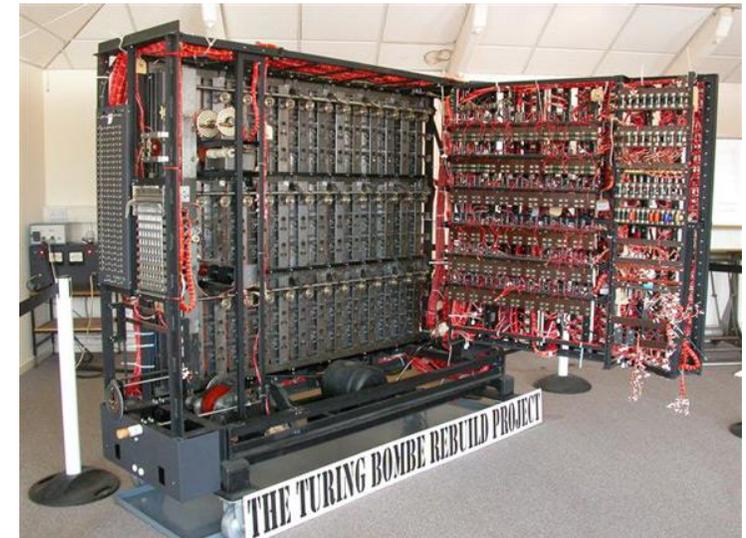
Church et Turing

Church and Turing - 1936

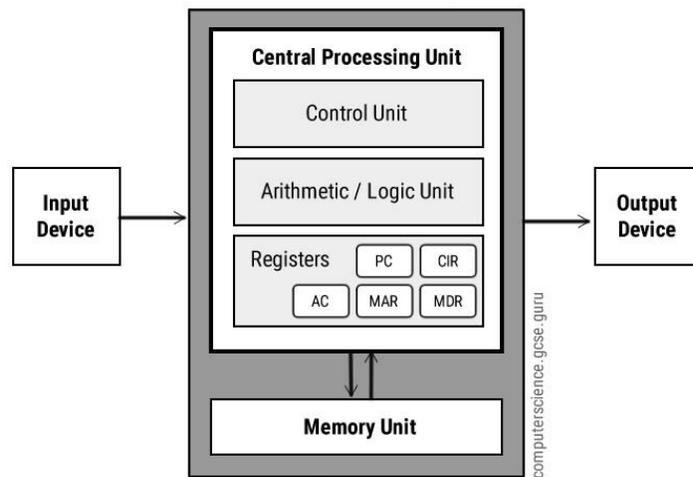
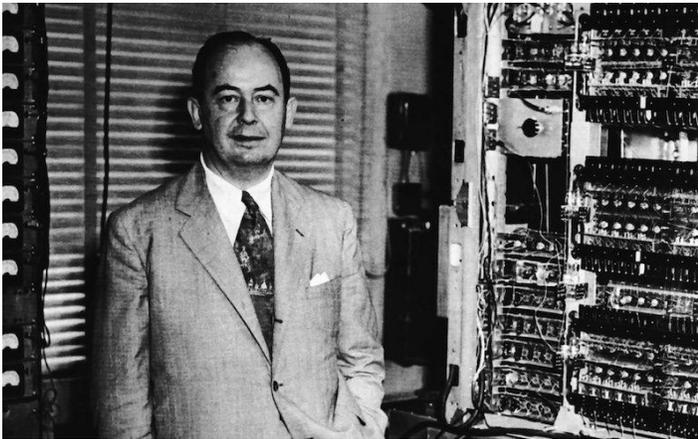
Questions: What functions are decidable?

P is decidable if there is an algorithm that for each x says "YES" or "NO" to the question: "Is P (x) true?"

Alan **Turing** proved in 1936 that a general algorithm running on a **Turing** machine that solves the halting problem for all possible program-input pairs necessarily cannot exist. Hence, the halting problem is **undecidable** for **Turing** machines.



John von Neumann arrived and Watson did it!



Ju faleminderit!

Wir müssen wissen — wir werden wissen!
David Hilbert

