Scheduling Overview

Conjunctive Graph;

Potentials, properties and existence conditions;

Methods for scheduling;

- Potential task method
- Pert method;

Scheduling problem

A scheduling problem consists in determining execution dates for jobs/tasks which use a quantity of limited resources.

- ✓ Production systems;
- ✓ Transport ;
- ✓ Exam timetable;
- ✓ *etc.*

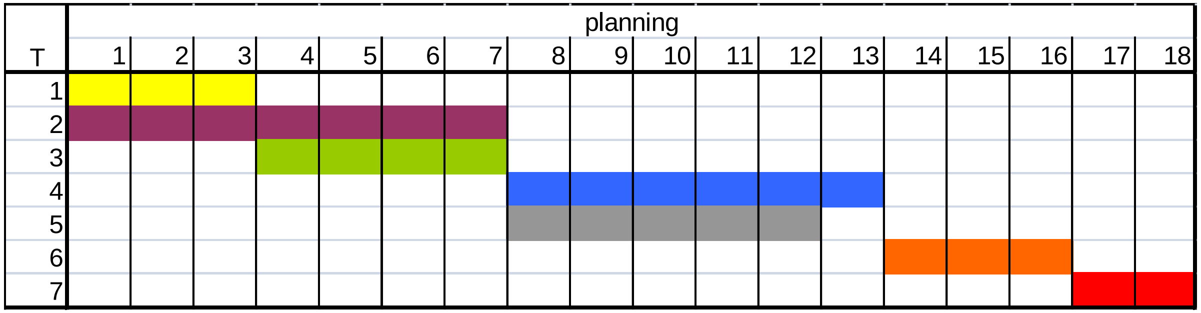
Goal: minimizing the makespan (the time difference between the start and finish of a sequence of jobs/tasks).

Other criteria:

- ✓ Minimizing the number of delayed tasks;
- ✓ Minimizing the sum of delays;
- ✓ Minimizing the sum of weighted execution dates;
- ✓ *etc.*

An example

Tasks	Durations	Constraints
1	3	
2	7	
3	4	1→3
4	6	(1,2)→4
5	5	3→5
6	3	(3,4)→6
7	2	6→7



Scheduling a project

Techniques that allows to manage the scheduling of a project:

- The **potential-task method**, (construction of cruiser FRANCE, 1958)
- The **PERT method (**It was designed by the US navy to allow for coordination of the work of several thousand persons to build missiles with POLARIS nuclear warheads, 9000 sub contractuals, 250 furnishers, the delay were reduced from 7 years to 4 thanks to PERT.)

Method:

Searching for maximal paths in a particular graph

Conjunctive graphs and potentials

- Conjunctive graph :
  - Graph G = (X, U,v) with root 0 and an antiroot n+1 such that there exists a positive path from root to any other vertex, and from any vertex to the antiroot.
- Set of potentials $T= \{t_i / i \in X\}$:
  - A set of potentials in G=(X, U) is an application t from X to R such that :
    1. $t_o = 0$,
    2. $v_{ij} \leq t_j - t_i$, for all arc (i,j) in U.
- Scheduling is a set of potentials defined on the associated conjunctive graph.

Properties

<u>Existence theorem:</u>

– A Necessary and Sufficient Condition of existence of a set of potentials in conjunctive graph $G = (X, U, v)$ is that there are no negative cycles.

Let l(i,j) be the maximal value of a path going from i to j.

Let $R = \{r_i = l(0,i) \ / \ i \in X\}$.

R gives a set of potentials (said at the earliest).

It can be shown that $F = \{ f_i = l(0,n+1) - l(i,n+1) \ / \ i \in X \}$ gives as well a set of potentials (said at the latest).

Properties

Proposition 1 :

For any potentials set T= $\{t_i \,/\, i \in X\}$, we have $r_i \leq t_i$ for all $i \in X$.

In particular: $t_{n+1} \geq t^*$*, where* $t^* = l(0,n+1)$ *gives the optimal makespan.*

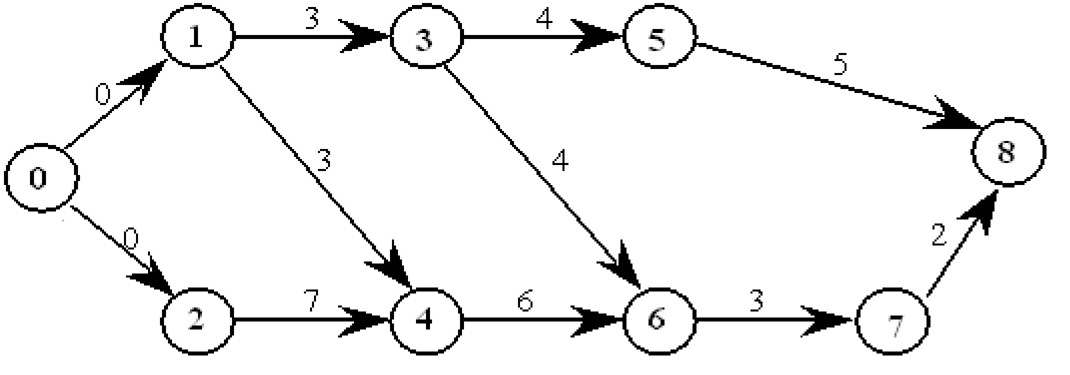
Proposition 2 :

If T is a scheduling with makespan $t^* = l(0,n+1)$, then $t_i \leq f_i$, for all $i \in X$.

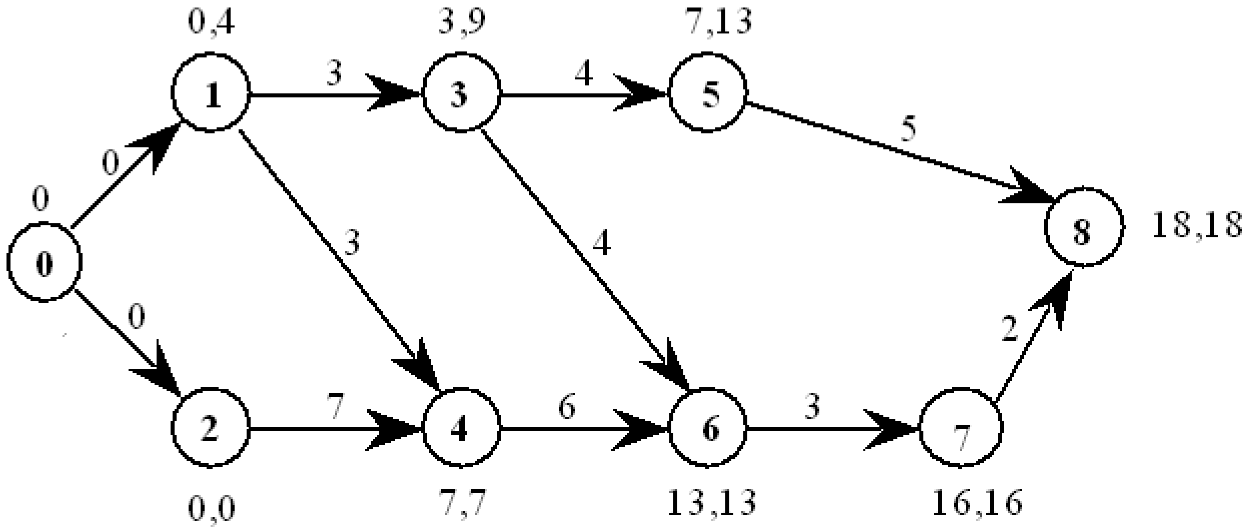
An example (I)

Tasks	Durations	Constraints
1	3	
2	7	
3	4	1→3
4	6	(1,2)→4
5	5	3→5
6	3	(3,4)→6
7	2	6→7

The induced potential graph
(two fictive nodes, 0 and 8)



Task potential method



Earliest time calculation :
$r_0 = 0, \quad r_j = \max (r_i + v_{ij})\ (i \in U\text{-}(j)).$

Latest time calculation :
$f_{n+1} = t^*, f_i = \min (f_j - v_{ij})\ (j \in U\text{+}(i)).$

Pert method (I)

PERT : Project **E**valuation and **R**eview **T**echnique

For any task i :

An event D_i : start of task i

An event F_i : end of task i

An event D : start of scheduling

An event F : end of scheduling

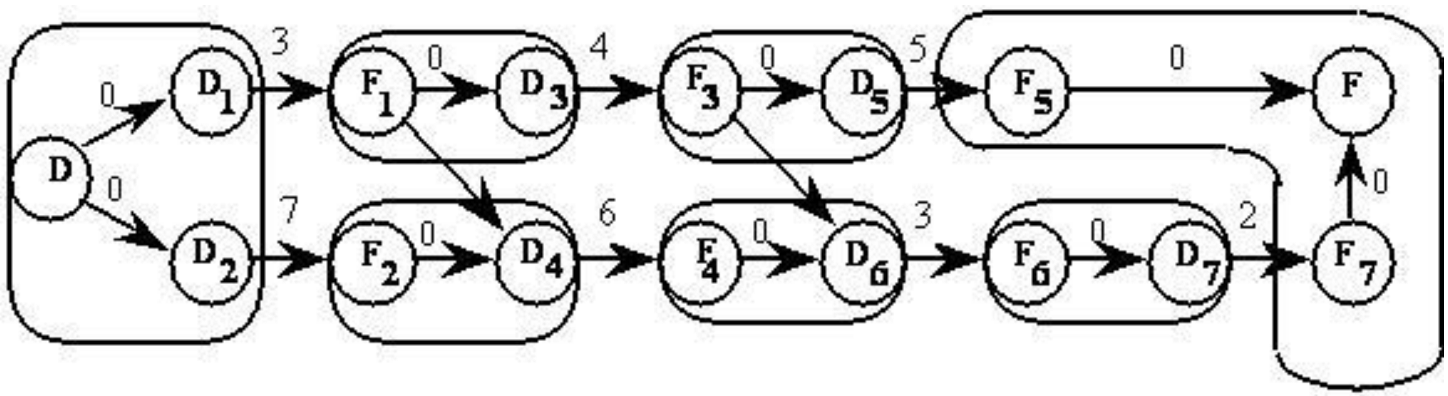
The associated conjunctive graph:

vertices : set of events

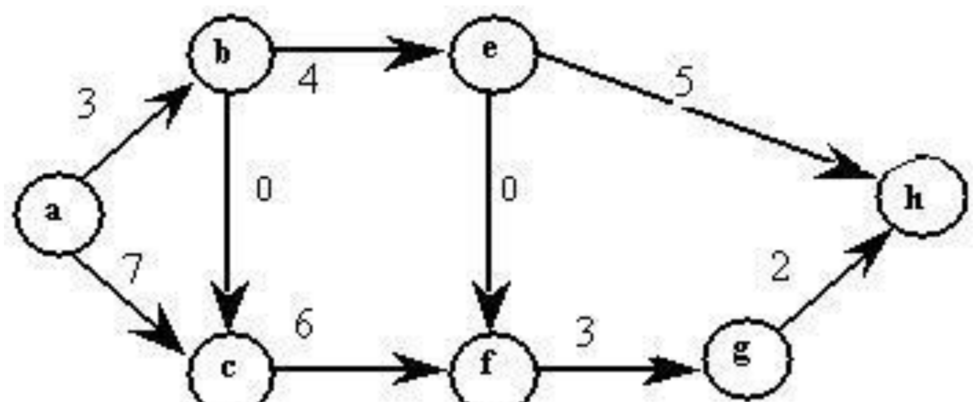
arcs :

- all tasks,
- fictive arcs representing the precedence constraints.

PERT method (II)



Detailed graph



Simplified graph