

# Network Planning under Demand Uncertainty with Robust Optimization

Thomas Bauschert, Chemnitz University of Technology

Christina Büsing, RWTH Aachen University

Fabio D'Andreagiovanni, Zuse Institute Berlin

Arie M. C. A. Koster, RWTH Aachen University

Manuel Kutschka, RWTH Aachen University

Uwe Steglich, Chemnitz University of Technology

## ABSTRACT

The planning of a communication network inevitably depends on the quality of both the planning tool and the demand forecast used. In this article, we show by example how the emerging area of robust optimization can advance the network planning by a more accurate mathematical description of the demand uncertainty. After a general introduction of the concept and its application to a basic network design problem, we present two applications: multi-layer and mixed-line-rate network design. We conclude with a discussion of extensions of the robustness concept to increase the accuracy of handling uncertainties.

## INTRODUCTION

Mathematical tools play a vital role in the design and operation of communication networks. The concept of (directed) graphs and elementary algorithms for computing a shortest path or a spanning tree are core components of communication networking. Many new innovations in technology and network management are first rendered precisely by a mathematical model of the optimization problem (e.g., an integer linear program) that needs to be solved. The network planner is then assisted by mathematical software tools in solving such models. In addition, the challenges to plan more and more complicated communication networks have been one of the main driving forces of new solution methods in the mathematical optimization community.

This interaction between theory and practice has been spurred on by technical progress allowing for large-scale collection of historical traffic data. Until recently, the most successful applications (in telecommunications and beyond) of mathematical optimization involved a deterministic estimation of all relevant parameters like traffic demand values between core router locations. However, at a time when traffic can be

logged in very small time intervals, network planning based on a single traffic matrix seems outdated. The emerging branch of robust optimization addresses this issue by taking into account the uncertainty of the input parameters beyond estimations.

In this article, we provide an introduction to robust optimization and its application to different communication network settings recently studied by the authors in the context of a three-year research program supported by the German government, and in collaboration with Nokia Siemens Networks and DFN-Verein (the operator of the German national research and education network).

## ROBUST OPTIMIZATION

For simplicity, let us consider a single link in a communication network and two traffic flows that can use this link. Historical data for both flows can be represented by an  $(x, y)$ -point in two-dimensional space. Figure 1a shows the traffic values during 15 points in history. The average traffic values are 477 Mb/s ( $x$ -axis) and 637 Mb/s ( $y$ -axis), displayed by the red point. Taking those values and a link capacity of 1024 Mb/s, one observes that on average 92 percent of the traffic can be routed across the link, or alternatively, 100 percent of the first traffic flow and almost 86 percent of the second traffic flow. These solutions correspond to solutions of the following linear program (LP):

$$\begin{aligned} \max & x_1 + x_2 \\ \text{s.t.} & 477x_1 + 637x_2 \leq 1024 \\ & 0 \leq x_1, x_2 \leq 1 \end{aligned}$$

where  $x_1, x_2$  define the fraction of traffic routed. Feasible solutions are  $(x_1, x_2) = (0.92, 0.92)$  and  $(x_1, x_2) = (1.00, 0.86)$  (the latter solution is optimal for the above LP). However, if we consider the historical data, only 8 out of 15 traffic flows do not exceed the capacity in the first solution,

This work was supported by the German Federal Ministry of Education and Research (BMBF grants 05M10PAA, 05M10OCA, and 05M10ZAA), project ROBUKOM — Robust Communication Networks, <http://www.robukom.de>.

whereas in the second solution 10 out of 15 flows can be routed. Thus, the probability that the network link is overloaded is 46 percent in the first case and 33 percent in the second case.

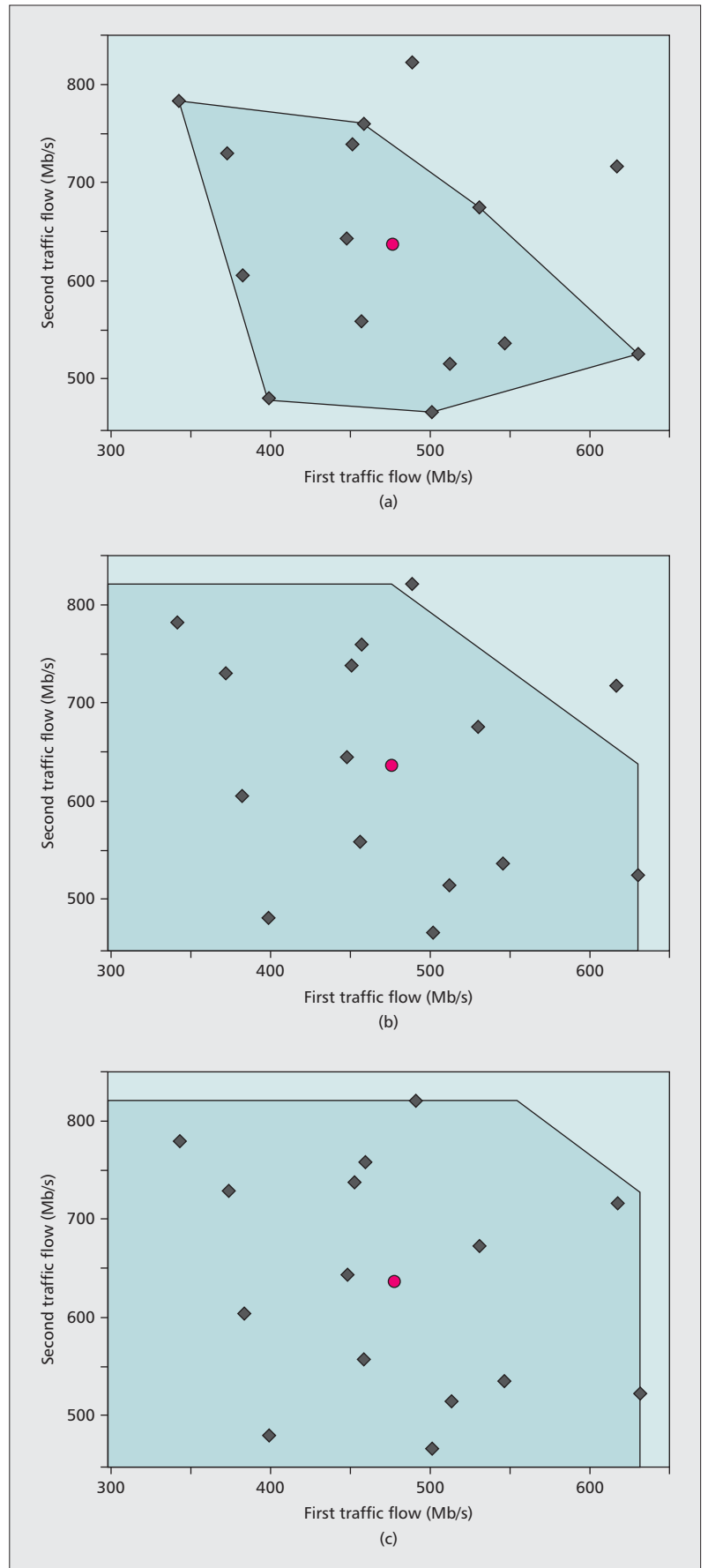
If we would like to have a solution such that in less than 15 percent of the historical cases the link is overloaded, we have to solve a robust linear program. Clearly, the two coefficients are uncertain, and taking the average traffic volume does not suffice. Robust optimization offers an adequate way to incorporate uncertainties into our model: The uncertain coefficients are considered as random variables drawn from an *uncertainty set*. This uncertainty set describes all possible interactions between the uncertain coefficients and might look like the polyhedra in Fig. 1a–c. In fact, the polyhedron in Fig. 1a is the convex hull of 13 out of 15 historical data points.

The task of robust optimization is to find a solution that is feasible for all considered realizations of the uncertain coefficients (from the uncertainty set) and maximizes the objective among these solutions. In the case of our example, the solution  $(x_1, x_2) = (1.00, 0.73)$  satisfies the constraint regardless of the values drawn from the uncertainty set in Fig. 1a and maximizes the sum among all robust feasible solutions (w.r.t. this uncertainty set). Accordingly, its usage would lead to a probability of overloading in about 13 percent of historical cases.

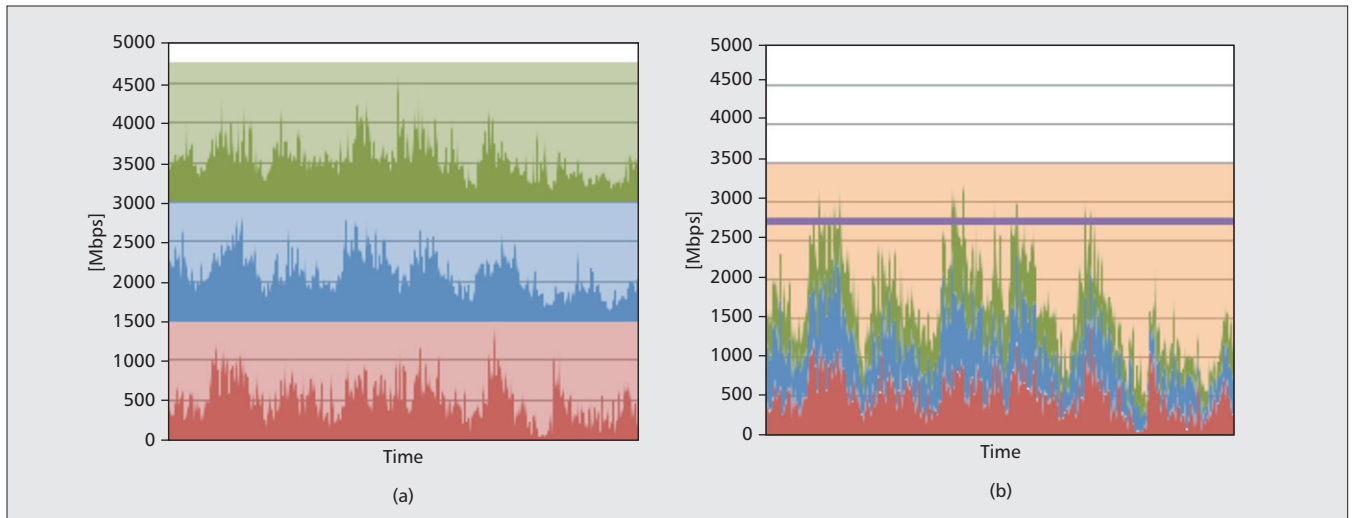
A major challenge in robust optimization is the construction of a reasonable uncertainty set (in our example, the set depicted in Fig. 1a). The book by Ben-Tal *et al.* [1] provides a thorough discussion on this topic. Bertsimas and Sim [2] developed a generic uncertainty set that can be adjusted by a parameter  $\Gamma \geq 0$ . For each uncertain coefficient  $a_i$  we define a nominal value  $\bar{a}_i$  and a maximum deviation  $\hat{a}_i \geq 0$ . The  $\Gamma$ -robust uncertainty set is now defined as values  $a_i \in [\bar{a}_i - \hat{a}_i, \bar{a}_i + \hat{a}_i]$  such that the sum of the relative excesses  $|a_i - \{\bar{a}_i\}_i / \{\hat{a}_i\}_i|$  of the nominal values is at most  $\Gamma$  (for integer values of  $\Gamma$  this corresponds to the simultaneous deviation of at most  $\Gamma$  values toward their maximum value). In Figs. 1b and 1c, the nominal values are set to the average traffic volumes, and the deviations are set to the difference between the maximum traffic volumes and the averages. The uncertainty set in Fig. 1b corresponds to  $\Gamma = 1$ , in Fig. 1c to  $\Gamma = 1.5$  (if more coefficients can deviate, typically integer values are taken for  $\Gamma$ ). As the graphics show, 13 out of 15 historical data points are included in the uncertainty set induced by  $\Gamma = 1$ , whereas all of them are part of the set induced by  $\Gamma = 1.5$ . Here, the advantage of robust optimization comes into play: robust feasible solutions can be found without setting the input parameters to their most conservative estimation (i.e., the maximum values). By varying the  $\Gamma$ -value, the protection level against traffic fluctuations can be adapted to the needs of the planner. By comparing the network cost and robustness level, network planners can exploit this trade-off for decision support.

Two major advantages of the  $\Gamma$ -robustness concept of Bertsimas and Sim [2] are:

- As long as the uncertain coefficients are independently and symmetrically distributed, the probability that the constraint is violated by an



**Figure 1.** Possible uncertainty sets for two traffic flows covering: a) convex hull of 13 out of 15 historical data points; b)  $\Gamma$ -robustness with averages as nominal values; c)  $\Gamma$ -robustness with 67 percent quantiles as nominal values.



**Figure 2.** Traffic fluctuations for three node-pairs in the US Abilene Internet2 network in time intervals of 5 minutes during one week (publicly available via [6]). In (a) the light colors represent the spare capacity for every single node pair, if the (artificial) decision is taken to reserve capacity by the maximum traffic volume. In (b) the spare capacity is computed by the maximum sum of the traffic volumes. The purple line represents the sum of the 90% quantiles of the traffic volumes.

optimal solution can be bounded by a function depending on the number of uncertain coefficients and the parameter  $\Gamma$ ; that is, given a value, a value  $\Gamma$  can be chosen such that the probability of constraint satisfaction of the actual values is at least  $1 - \epsilon$  (see [2] for details).

- The mathematical description of robust feasible solutions can be reformulated so that the size of the linear program is increased moderately, yielding a *compact* model, that is, a model whose size (number of variables and constraints) is polynomial in the network size (number of nodes and/or links). Accordingly, the complexity increase of solving the linear program is bounded. This property is explained by example in the next section for the network design problem under demand uncertainty.

Although the first point cannot be expected to hold for real-life applications, the concept is still valuable as discussed below. Regarding the second advantage, it should be noted that most network design problems without uncertainty are already NP-hard and thus unlikely to be solved in polynomial time.

In the following sections, we give several examples from network design where a robust approach was successfully applied.

## NETWORK DESIGN UNDER DEMAND UNCERTAINTY

First, let us change our point of view from traffic engineering to network design. The core of traffic engineering in a (backbone) communication network is the following technology-independent question: how do we route the traffic flows from sources to destinations across the links such that the capacities of those links are not exceeded? In the network design problem, this question is accompanied by the decision on the capacity granularities at the links with the aim of finding a solution with minimum capacity installation cost. A vast amount of literature exists exploiting math-

ematical optimization techniques. We refer readers to [3] for a detailed introduction to the topic.

Although there have been several works addressing the uncertainty in traffic volumes since as early as the 1990s (e.g., [4, 5]), until recently, the practice of network planning was based on a single traffic matrix consisting of the forecasted traffic demands between every pair of network nodes. To avoid congestion in the designed network due to dynamic traffic fluctuations (which frequently happens in modern communication networks), as shown in Figure 2, traffic estimates for every node pair have to be very conservative. However, traffic peaks do not occur simultaneously for all traffic flows using the same link, so an unnecessarily high amount of resources are installed using such an approach. Statistical multiplexing balances out such effects, but for the planning it remains unclear how the demand matrix should be defined.

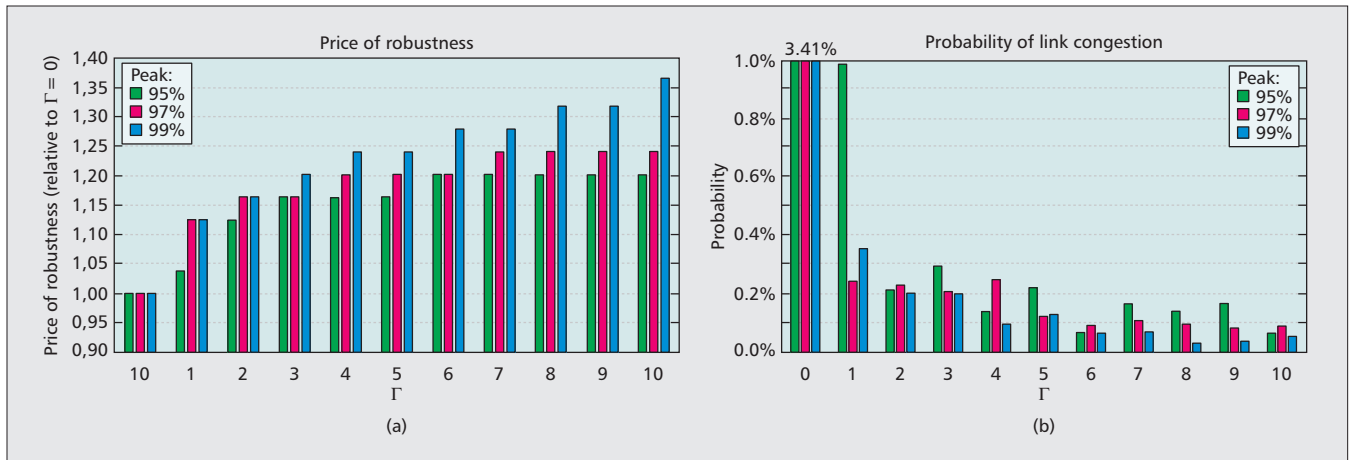
The  $\Gamma$ -robustness concept provides a valuable alternative in this case, in some sense modeling the effect of statistical multiplexing. Instead of a single traffic forecast for every pair  $(s, t)$  of source and target nodes, a nominal demand  $\bar{d}^{st}$  and a deviation  $\hat{d}^{st}$  are defined for every node pair. Let  $C$  be the installable capacity batch size,  $f_{ij}^{st}$  be the decision variable determining the fraction of the traffic flow between  $s$  and  $t$  via the link between nodes  $i$  and  $j$  (single path routing can be modeled by binary flow variables), and let  $x_{ij}$  be the integer decision variable representing the number of capacity granularities to be installed.

Now, the capacity constraint for the link between nodes  $i$  and  $j$  is given by

$$\sum_{(s,t)} \bar{d}^{st} f_{ij}^{st} + \text{DEV}(f, \Gamma) \leq C x_{ij},$$

where  $\text{DEV}(f, \Gamma)$  is the total capacity that has to be reserved to cope with the realized traffic values above the nominal values if the  $\Gamma$ -robust uncertainty set is used.<sup>1</sup> Now, for a moment, let

<sup>1</sup> Of course,  $\Gamma$  can be chosen individually for every link, but in this exposition we simplify the description by considering a single  $\Gamma$  value for all links.



**Figure 3.** Robust network design on the basis of historical data for the pan-European research backbone network GÉANT with observed means as nominal values and subject to different quantiles (95, 97, 99 percent) for the peak demand values (nominal + deviation): a) price of robustness; b) resulting probability of link congestion. Data source: Koster and Kutschka [8].

the flow variables be fixed. Then  $\text{DEV}(f, \Gamma)$  can be computed by the following linear program:

$$\begin{aligned} \text{DEV}(f, \Gamma) = \max & \sum_{(s,t)} \hat{d}^{st} f_{ij}^{st} y^{st} \\ \text{s.t.} & \sum_{(s,t)} y^{st} \leq \Gamma \\ & 0 \leq y^{st} \leq 1 \end{aligned}$$

where  $y^{st} = 1$  if and only if the demand of a pair is increased to its maximum value (on link  $ij$ ). Note that since  $f$  is fixed, the product in the objective is linear in the variables. By linear programming duality, the term  $\text{DEV}(f, \Gamma)$  can be replaced by a linear function (on new variables) and additional linear constraints. These constraints link the new variables with the flow variables  $f_{ij}^{st}$ , yielding a new integer linear formulation for the robust network design problem. This formulation is slightly more complicated than the network design problem for a single traffic matrix. We refer to [7, references therein] for further details.

The minimum network design cost as a function of the parameter  $\Gamma$  is known as the “price of robustness” [2]. It describes the additional cost of increasing the protection (and thus reducing the violation probability) by increasing  $\Gamma$ . Figure 3a shows the price of robustness for a computation based on historical data of a 22-node network. For example, with peak values corresponding to the 95 percent quantile (to be explained below), the cost does not increase beyond and is about 20 percent higher than the cost of a traditional design based on the means only (i.e.,  $\Gamma = 0$ ).

What remains are two highly correlated input decisions. First of all, the nominal and deviation values of demands have to be set. By the absence of known probability distributions, historical data serves as the base for these input values. In Fig. 3a the nominal value for every node pair is chosen as the average historical traffic volume. Results for three different peak values (= nominal + deviation) are shown: the 95, 97, and 99 percent quantiles of the observed volumes.

The second decision is the choice of  $\Gamma$ . With input based on historical data and/or forecasts, no design will be 100 percent robust by definition. However, the freedom of choosing provides the network designer with a novel decision support tool. Multiple designs can be evaluated according to their practical robustness. This can be achieved by simulations or, as in [8], by computing a probability (based on historical data) that an arbitrary link is congested. Figure 3b shows this probability for different values of  $\Gamma$  and peak quantiles. For example, for  $\Gamma = 6$  and peak demands corresponding to the 95 percent quantile, the congestion probability is 0.07 percent.

Based on these and further experiments with historical data [7–9], a good choice seems to be the (arithmetic) mean as nominal values and the observed 95 percent quantile as the peak value. By varying the  $\Gamma$  value and evaluating the resulting designs, the decision maker can make a better informed decision on the capacities to be installed.

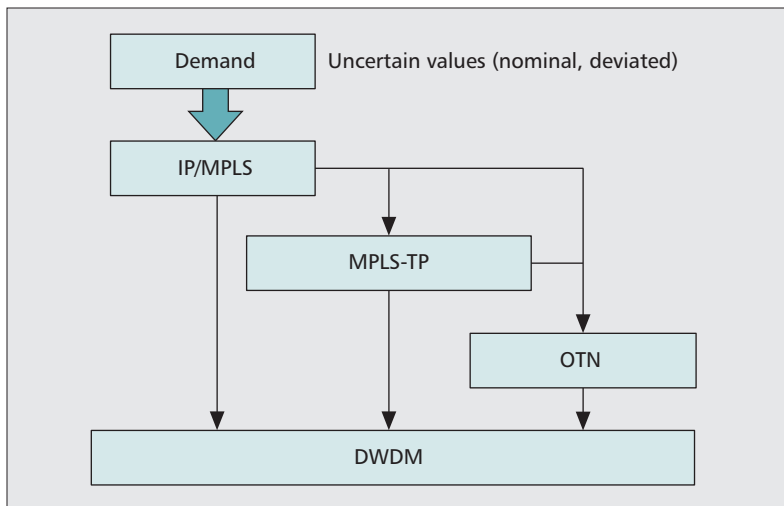
Finally, let us point out that the results discussed above are obtained by solving the integer linear programs to optimality. For small and medium-size networks (up to 25 nodes) such results can typically be achieved with out-of-the-box optimization software on a standard PC within minutes or a few hours. However, as of today, for larger instances (30 and more nodes), optimal results cannot be achieved by advanced tailor-made mathematical techniques. These results are independent of the degree of demand uncertainty.

## ROBUST MULTI-LAYER NETWORK DESIGN

### GENERAL MULTI-LAYER PROBLEM

The design problem outlined in the previous section describes a single-layer network problem. However, many communication networks nowadays consist of two or more technological layers, such as the IP layer, the multiprotocol label switching (MPLS) or MPLS transport profile (MPLS-TP) layer, the optical transport network





**Figure 4.** Feasible multi-layer interconnections and resulting layer configurations.

(OTN) layer, and the dense wavelength-division multiplex (DWDM) layer. Additionally, there is a logical demand layer, which induces traffic demand for arbitrary end-to-end connections. A wide range of technologically feasible layer configurations and possibilities for transporting the traffic demand through the layers exist (Fig. 4). Common layer configurations are, for instance, IP-over-DWDM or IP-over-MPLS-over-OTN-over-DWDM. A multi-layer network optimization formulation has to incorporate all technological and logical layers that should be part of the potential solution space.

Considering all constraints of a multi-layer network design problem in a generic mathematical formulation is a very challenging task. A layer model that is too abstract might neglect important technological constraints. On the other hand, a fine-grained formulation of the layers might lead to huge computational complexity of the multi-layer model.

A comprehensive multi-layer modeling should integrate:

- Layer model (e.g., multi-layer structure and feasible layer interconnections)
- Technological restrictions (e.g., capacity granularities of interfaces and sub-interfaces, number of interface card slots, multiplexing capabilities)
- Cost model, considering, say, capital expenditures (CAPEX), operational expenditures (OPEX), and energy consumption
- Traffic demand model (with or without demand uncertainty)
- Model of resilience mechanisms (e.g., 1+1, 1:1 protection, rerouting)

#### INCORPORATION OF DEMAND UNCERTAINTY IN MULTI-LAYER PLANNING

Like in the preceding case, multi-layer network designs should have the ability to cope with uncertain traffic demand. Traffic demand fluctuations can occur in a temporal and spatial manner. The temporal effects can be classified into short-, mid-, and long-term fluctuations. In particular, the mid- and long-term effects such as

the daytime usage behavior as depicted in Fig. 2 are relevant for robust network design. On the other hand spatial traffic demand fluctuations are either caused by day of time traffic shifts (in large networks spanning over multiple time zones) or by effects outside the network like Border Gateway Protocol (BGP) route flaps or dynamic server selection policies of Content Delivery Networks (CDN).

Concepts like the previously described  $\Gamma$ -robustness can be applied in multi-layer network design similar to the single-layer case. However, the complexity, model size and computation time are substantially increased by introducing  $\Gamma$ -robustness in multi-layer network optimization as shown in Steglich *et al.* [10]. Uncertainty in traffic affects the capacity dimensioning of all subjacent technological layers. In the lower layers traffic demand uncertainty is smoothed by multiplexing traffic from higher layers.

#### LAYER (TECHNOLOGY) SELECTION AND OPTIMIZATION

Further potential challenges in multi-layer network design are the determination of the layers (technologies) that should be used given a set of potential networking technologies and the determination of the optimum connectivity (topology graph) within each layer.

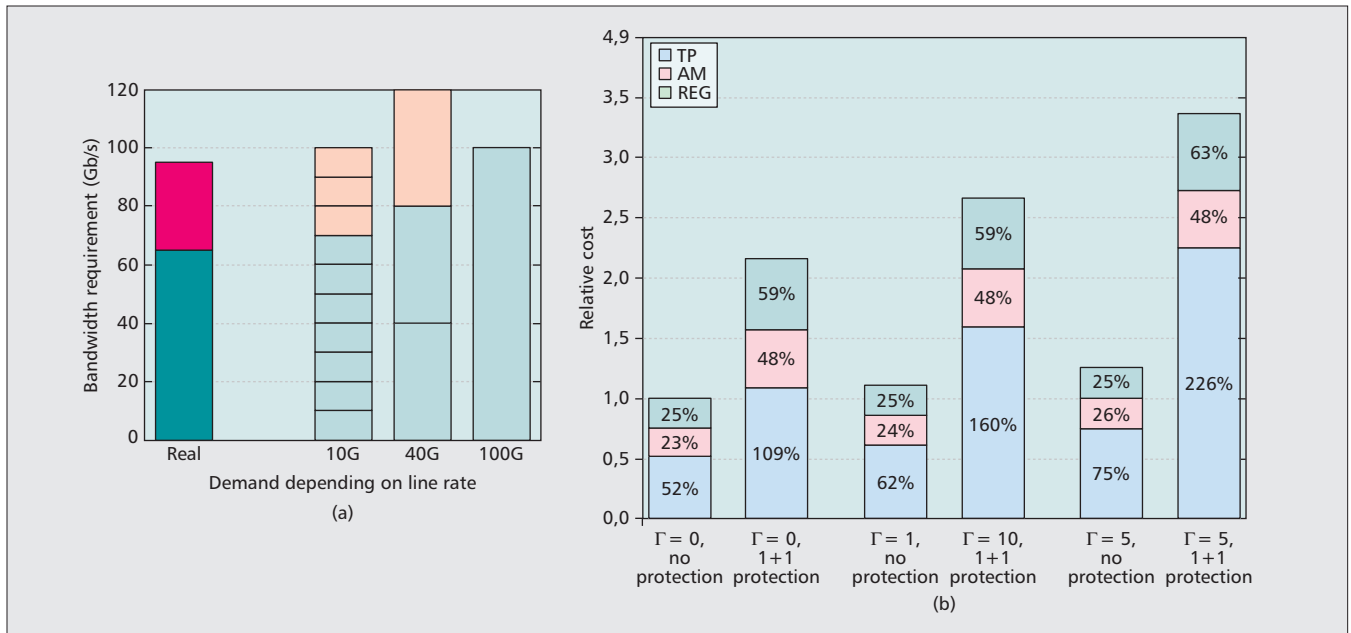
Regarding the first challenge, *layer-skipping* is an option to reduce the network CAPEX. Although interfaces for connecting higher layers to lower ones (e.g., IP to DWDM IFs) are more expensive [11], the overall CAPEX might be cheaper than establishing an intermediate layer with further interfaces. The result of the optimization should reveal which particular layers are used and which layers are omitted. For this, layer configurations with possible layer sequences have to be included into the multi-layer optimization model.

To cope with the second challenge, flexible path sets (per layer) are included in the multi-layer network optimization. These path sets contain three types of paths: *opaque* paths (calculated by a *k*-shortest path algorithm), *transparent* paths (with no intermediate nodes), and *specific* paths where some of the intermediate nodes of opaque paths might be omitted. As a result of the multi-layer optimization the cheapest (in terms of the optimization objective) paths are selected, thus leading to *shortcuts* in some layers. The inclusion of such path sets (allowing the determination of shortcuts) influences the size of the multi-layer network optimization model significantly.

The well-known *IP router offloading problem* [12] can be considered as a combination of the layer skipping and shortcut determination.

#### RESULTS FROM THE ROBUKOM PROJECT

In the ROBUKOM project, a multi-layer network design model with traffic demand uncertainty has been developed. This model applies  $\Gamma$ -robustness to model traffic uncertainty. Moreover, aspects like layer-skipping, shortcuts and router-offloading are included. First computational results with off-the-shelf solvers are provided for small-, mid- and large-scale networks



**Figure 5.** Mixed-line-rate optical network design: a) bandwidth requirement depending on the operated line rate for an example with nominal demand value of 65 Gb/s and a deviation of 30 Gb/s; b) cost of multi-line-rate optical networks normalized to  $\Gamma = 0$  and no protection. The costs are broken down with respect to transponder (TP) cost, amplifier (AM) cost, and regenerator (REG) cost. All cost figures are derived from Rambach et al. [11].

in Steglich et al. [10]. The introduction of  $\Gamma$ -robustness increases the CAPEX costs. For a 5-node network without layer skipping, securing at most 10 demand shifts ( $\Gamma = 10$ ) is 23.0 percent more expensive compared to a non-robust network design. With layer skipping it is 25.6 percent more expensive to consider traffic uncertainty. The uncertainty parameter  $\Gamma$  shows an even higher influence for the GÉANT network: CAPEX is raised here by 117.2 percent ( $\Gamma = 0$  vs.  $\Gamma = 10$ ).

In our future work, we intend to apply other techniques (e.g., meta-heuristics) in order to reduce the computation times and memory requirements when dealing with large-scale multi-layer network design. Note that these drawbacks exist independent of the demand uncertainty.

## MIXED-LINE-RATE OPTICAL NETWORKS

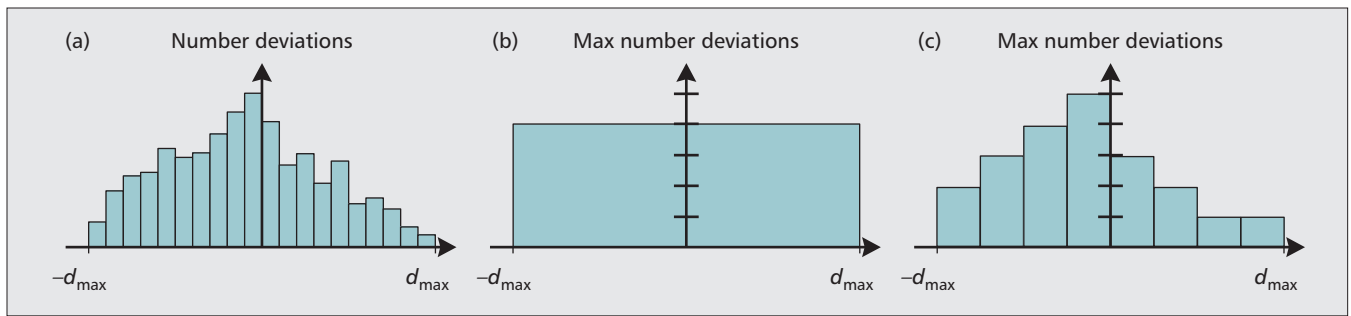
In an optical network, lightpaths are used for transporting traffic flows. Mixed-line-rate optical networks allow for a more resource-efficient handling of small and large traffic volumes by the simultaneous configuration of lightpaths with different bit rates (e.g., 10 Gb/s, 40 Gb/s, and 100 Gb/s).

Given a potential network topology and commodities with (uncertain) demand values, a cost-minimal hardware configuration (line rate used for each demand, installed transponders, amplifiers, and regenerators) and optimum routing have to be determined. Additional survivability requirements may exist.

In Duhovniko et al. [13] a mixed integer linear programming formulation for the design of mixed-line-rate networks with uncertain demands

is given. In addition to the modeling of  $\Gamma$ -robustness, its main feature consists of the computation of the nominal and peak demand values. In contrast to single-line-rate planning, the nominal and deviation values depend on the line rate of the lightpath used for a particular demand. If small demands are routed on a lightpath with a high bit rate, on one hand, additional lightpaths for absorbing traffic peaks are not needed, but on the other hand, the resources are not used efficiently. If lightpaths with a low bit rate are used instead, traffic peaks might exceed the capacity reserved by the lightpaths for the nominal demand, and additional spare lightpaths have to be reserved to handle these peaks. Hence, depending on the line rate used, different nominal and deviation values have to be used. Figure 5a shows an example with a nominal demand (in 1 Gb/s) of 65 Gb/s and a deviation of 30 Gb/s. If a line rate of 10 Gb/s is chosen, 7 lightpaths have to be reserved for the nominal demand, and an additional 3 for peak values. In case 40 Gb/s is chosen, 2 lightpaths are needed for the nominal demand and another one for the peak. However, if 100 Gb/s is chosen, a single lightpath provides enough capacity for the nominal as well as the peak demand, and thus no further deviation value is needed in this case.

Figure 5b shows exemplarily the cost of a robust mixed-line-rate optical network with GÉANT data, with and without 1+1 protection for different values of  $\Gamma$ . The costs are normalized to the case without protection and without robustness ( $\Gamma = 0$ ). Not surprisingly, the costs are more than doubled if 1+1 protection is implemented, but the price of robustness for unprotected cases is rather low. For robust designs with 1+1 protection in particular, the transponder cost increases significantly with increasing  $\Gamma$ , which can be explained by the need to use more and more



**Figure 6.** Visual comparison of a single and a multiband uncertainty set defined over the same overall deviation range: is the histogram of deviations built on the historical data; b) and c): possible single and multiband representations of the histogram.

high-bit-rate transponders since the number of lightpaths per fiber is limited.

This case study also illustrates how robust optimization can be used for the analysis of cost developments in future networks. A higher  $\Gamma$  value corresponds to a larger number of demands deviating to their peak value, which can be interpreted as either a faster than expected increase of traffic volumes or a less precise forecast for a more distant point in time.

## MULTIBAND ROBUST NETWORK DESIGN

In the previous sections, we showed the benefit of adopting robust optimization to tackle traffic uncertainty in the design of communication networks. A robust optimization approach guarantees moderate dimensioning of the installed capacity and the established routing paths that are able to accommodate (bad) traffic variations with respect to expected traffic values. Our primary robustness tool has been the model proposed in [2], essentially based on adopting a single deviation band for each uncertain parameter and an upper bound on the number of data that can simultaneously deviate from their nominal value.

The Bertsimas-Sim model has also attracted considerable attention outside the mathematical programming community and has been applied in many different contexts (see Bertsimas *et al.* [14] for an overview). Key factors of this success are undoubtedly its elegant simplicity and its straightforward application: its use just requires fixing the upper bound  $\Gamma$  on the number of deviating data. Different solutions according to this parameter can then be compared by the corresponding price of robustness, allowing evaluation of the trade-off between stability and price by the decision maker.

However, the adoption of a single deviation band to represent the uncertain value of a coefficient may greatly limit the possibility of modeling uncertainty: in many real-world problems, the deviations distribute asymmetrically within often non-symmetric intervals. In such cases, adopting a single deviation band and thus focusing just on the extreme deviations neglects the inner uncertainty behavior and may lead to over conservative robust solutions that overestimate the impact of variances. It is thus desirable to increase the resolution of the model.

Increasing the resolution of the Bertsimas-Sim model can be done by a simple operation: partitioning the single deviation band into multiple bands, each with its own upper bound on the number of data falling into that band. Moreover, to further increase the power of modeling uncertainty, we can also introduce a lower bound on the number of deviations falling in each band: this simple trick allows also explicitly taking into account good deviations that in a Bertsimas-Sim approach are neglected, but in reality are actually present with the effect of reducing the impact of bad deviations. We call an uncertainty set based on multiple deviation bands a *multiband set* with *multiband robustness* the resulting robust optimization model.

Multiband robustness looks particularly attractive in real-world applications, where it is common to have historical data that shows the past behavior of the uncertainty. These data can be used to define histograms representing the (discrete) distribution of the uncertainty in the past and form a basis on which to build multiband sets, which are now strongly data-driven. We refer to Fig. 6 for a visual representation of the differences between a single and a multiband representation of the uncertainty.

Within the project ROBUKOM, we have started to investigate the theoretical properties of multiband robustness. Here, we summarize the main theoretical results we have obtained, and refer the reader to Büsing and D'Andreagiovanni [15, 16] for a complete and detailed overview of them. Given an uncertain mixed-integer linear program (MILP) and assuming that we represent uncertainty by a multiband set:

- The robust counterpart of an MILP is equivalent to a *compact* MILP, whose size grows linearly with the number of deviation bands of the multiband set and quadratically by the number of variables and constraints.
- Verifying if a solution of MILP is robust w.r.t. the multiband set can be done efficiently by solving a min-cost flow problem (note that this result can be used as a basis on which to define an efficient cutting-plane solution algorithm; see [15] for details).
- If the uncertain MILP includes only binary variables and the uncertainty just affects the objective function, a robust optimal solution can be obtained by solving a polynomial number of original MILPs with modified objective coefficients.

The application of multiband robustness to

network design with demand uncertainty implies that the overall range of deviation  $[\hat{d}^{st} - \hat{d}^{st}, \hat{d}^{st} + \hat{d}^{st}]$  of each demand associated with a source-target pair  $(s, t)$  is partitioned into a number  $K > 1$  of non-overlapping sub-bands. Instead of a single  $\Gamma$  value to bind the number of deviations, each of these bands is then associated with a lower and an upper bound on the number of deviations that may fall in it (these should be derived from the historical data).

We carried out preliminary experiments on the adoption of multiband robustness in network design, referring to the well-known U.S. Abilene Internet2 network instances. The number of deviation bands was fixed to 7, and the extremes of the bands were defined according to the 50th, 70th, 75th, 80th, 85th, 90th, and 95th quantile demand values, derived from historical data. The used bounds of each band took into account the probability of realization of the demands in each band. In comparison to a single-band approach using a comparable and optimistic  $\Gamma$  parameter, the multi-band approach granted a percentage reduction in the price of robustness between 1 and 5 percent, while maintaining the same computational performance (no significant increase in solution time). This is due to the refined representation of the uncertainty, which makes the robust solutions less conservative. These preliminary results have encouraged ongoing investigations of better tuning of the parameters of the multiband set (number, bounds and width of the bands).

## CONCLUSIONS

Robust optimization is an emerging mathematical optimization technique to deal with uncertain input parameters. In recent years, the methodology has also been applied to communication networks in various settings. Its potential has been shown clearly by those case studies and deserves further integration into network planning tools in practice. Moreover, driven by the availability of historical data, the methodology is developed further as well to allow the usage of more accurate models.

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## BIOGRAPHIES

THOMAS BAUSCHERT (thomas.bauschert@etit.tu-chemnitz.de) received his Dipl.-Ing. and Dr.-Ing. degrees from the Technical University of Munich (TUM) in 1990 and 1997, respectively. From 1997 to 2007 he was with Siemens and Nokia Siemens Networks in Munich, responsible for network planning, strategic product management, and research projects in the fields of high-speed fixed networks and mobile networks. In 2007, he was appointed full professor and head of the Chair for Communication Networks at Chemnitz University of Technology.

CHRISTINA BÜSING (buesing@or.rwth-aachen.de) is a senior researcher in the Department of Operations Research at RWTH Aachen University. She received her Ph.D. in 2011 from the Technical University of Berlin and currently works on robust optimization and its extension to recoverable robustness. In 2013, she was appointed as a member of the Junges Kolleg of the North Rhine-Westphalian Academy of Sciences, Humanities and the Arts.

FABIO D'ANDREAGIOVANNI (d.andreagiovanni@zib.de) has been a senior researcher in the Department of Optimization at Zuse-Institut Berlin since 2010. He received his Ph.D. in operations research from Sapienza Università di Roma (2010) and was a research scholar in the Department of Industrial Engineering and Operations Research at Columbia University, New York (2008–2009). His research on network design has received several awards, such as the Accenture M.Sc. Prize 2006, the INFORMS Telecom Doctoral Dissertation Award 2010, and the ESF-JSPS Excellence Award for Mathematics for Innovations 2012.

ARIE M. C. A. KOSTER (koster@math2.rwth-aachen.de) has been, since April 2009, a professor of mathematics at RWTH Aachen University. He holds an M.Sc. degree in technical mathematics (1995) from Delft University of Technology and a Ph.D. degree in mathematics of operations research (1999) from Maastricht University. From 1999 to 2007, he was a senior researcher at Zuse Institute Berlin, and from 2007 to 2009 an assistant professor at the Centre for Discrete Mathematics and Its Applications (DIMAP) of the University of Warwick. His research currently focuses on robust optimization and its applications. During 2010–2013, he coordinated the BMBF-project ROBUKOM — Robust Communication Networks.

MANUEL KUTSCHKA (kutschka@math2.rwth-aachen.de) was a Ph.D. student in the Discrete Optimization group at RWTH Aachen University until June 2013. He received his Diploma degree in business mathematics in 2007 from the Technical University of Berlin. His Ph.D. thesis is about robust optimization, in particular the recoverable robust knapsack and robust network design problems.

UWE STEGLICH (uwe.steglich@etit.tu-chemnitz.de) was a Ph.D. student at the Chair for Communications Networks at Chemnitz University of Technology until September 2013. He received his Diploma degree in information and communications technology in 2010 from Chemnitz University of Technology. His Ph.D. thesis is about robust multi-layer network design considering traffic demand fluctuations.

*Robust Optimization is an emerging mathematical optimization technique to deal with uncertain input parameters. In recent years, the methodology has also been applied to communication networks in various settings. Its potential has been clearly shown by those case studies.*