On robust FSO network dimensioning

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***Abstract*—The presented work is motivated by Free Space Optics (FSO) communications. FSO is a well established wireless optical transmission technology considered as an alternative to radio communications for example in metropolitan wireless mesh networks. An FSO link is established by means of a laser beam between the transmitter and the receiver placed in the line of sight. A major disadvantage of FSO links (with respect to fiber links) is their sensitivity to weather conditions such as fog, rain and snow, causing substantial loss of the optical transmission power over the channel due mostly to absorption and scattering. Although the FSO technology allows for a fast and low-cost deployment of broadband networks, its operation will be affected by vulnerability of the link transmissions to weather conditions that can be manifested by substantial losses in links' transmission capacity with respect to the nominal capacity. Therefore, proper dimensioning of an FSO network should take such losses into account so that the level of carried traffic is satisfactory under all observed weather conditions. In the paper we introduce and study an optimization problem for such enhanced dimensioning using the robust optimization approach.**

I. Introduction

The considerations of this paper are devoted to dimensioning communication networks resilient with respect to multiple partial link failures. The main application area we have in mind are networks that apply Free Space Optics (FSO) – a well established broadband wireless optical transmission technology where the communication links are provided by means of a laser beam sent from the transmitter to the receiver placed in the line of sight. FSO links are considered as an alternative to radio links for example in metropolitan Wireless Mesh Networks (WMN).

FSO networks exhibit several important advantages: transmission range of several kilometers, high transmission bandwidth, secure communication, quick and easy deployment, lower cost as compared with the fiber optical technology, immunity to electromagnetic interference, license-free long-range operation (contrary to radio communications). A disadvantage is vulnerability of the FSO links to weather conditions such as fog, rain, and snow (and pollution, for that matter), causing substantial loss of the optical transmission power over the channel due mostly to absorption and scattering. This makes the problem of network dimensioning important, and, as a matter of fact, difficult.

Typically, a given weather condition affects a subset of the network links and each affected link looses a portion (fraction) of its nominal capacity (i.e., of the bit-rate realized when weather is fine). Such a loss is called the link failure (degradation) ratio that can assume some value between 0 (no capacity loss) and 1 (total capacity loss). As we will see in Section II, typical values of the link failure ratio are $0.25, 0.50$ and 1. As a consequence, each particular weather condition that may occur defines a failure state referred to as the multiple partial link failure state.

The above discussion motivates the network optimization problem studied in this paper: how to dimension the network links at the lowest cost and at the same time assure the traffic demand satisfaction at an acceptable level for all observed/predicted weather conditions. As we will demonstrate, the problem can be approached by the methods of robust optimization [1].

The impact of weather conditions on the wireless networks transmission capacity has been studied to some extent, but the majority of works have considered failure modeling for a single region failure (some investigations for the case of weather perturbations occurring simultaneously in different regions can be found in [2]). In particular, paper [3] discusses three measures of the WMN survivability for a regional failure scenario assuming circular failure areas with random location of failure epicenters. As observed in [3], from the network topology viewpoint, networks with nodes covering a square area in a regular way have a better performance in terms of the total traffic surviving a regional failure. It is also shown that in the case of heavy rain storms, using information on forecasted attenuation of links based on radar measurements allows for periodic updates of the network topology in advance, and thus dealing efficiently with these phenomena. In [4], a mixed-integer programming (MIP) model for network reconfiguration in case of unfavorable weather conditions is presented. The model looks for alternative routing paths ensuring rerouting some of the disturbed traffic while reducing the interference between adjacent transmission links. Other studies, such as [5], have considered the relevance and impact of specific weather factors on the FSO links capacity. In Section II, we will present a short study on such an impact in terms of simultaneously affected FSO links.

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As already mentioned, the paper introduces a (new) optimization model for robust dimensioning of FSO networks in order to achieve resilience with respect to multiple partial link failures. Similar problems have been widely investigated in the literature but for a given list of failure states, see for instance [6] and the list of literature therein. More recently, robust optimization approaches to survivable network dimensioning have been developed in [7]–[9] (under traffic demand uncertainty), and in [10] (under signal propagation uncertainty in wireless networks). Yet, to the best of our knowledge, no models have been developed for uncertainty of available capacity on transmission links, i.e., for the case relevant to networks employing FSO. Also, as majority of works on network restoration has been done for wired networks, multiple partial failures are virtually not considered despite common appearance of this phenomenon in modern wireless networks with adjustable modulation and coding schemes (MCS). Multiple partial failures were addressed in [6] for the so called Global Rerouting mechanism, and recently in [11] and [12] (paper [12] along with [13] are the starting point for this work). An optimization approach, based on disjoint path routing, to FSO networks with multiple partial failures is presented in [14].

The contribution of this work consists in studying a network dimensioning problem taking into account resilience with respect to multiple partial link failures. In contrast to the previous investigations, we deal with the problem involving a very large number of failure states, described by the so called uncertainty set (an instance of the budgeted uncertainty set formally described in [1]) for which we propose an optimization model together with a cut generation solution algorithm, and use it for a numerical study on robust dimensioning of a specific FSO network.

The paper is organized as follows. In Section II we discuss link failure modeling based on a set of meteorological records for a given time period. In Section III, we present a mathematical formulation of the robust network dimensioning problem central to our investigations. Section IV develops the solution approach while Section V presents numerical results of its application. Finally, in Section VI, we discuss possible enhancements of the introduced approach, followed by an outline of the future work in Section VII.

II. Link Failure Model

As already mentioned, the optimization approach developed in this paper is motivated by the necessity of dealing with multiple partial failures of FSO links caused by weather conditions. Therefore we start with a short study of the impact of meteorological phenomena on the transmission capacity of the FSO links in order to build a representative set of failure states to be used as the reference uncertainty set in the robust optimization approach presented in the next sections. The particular goal of the study is to show how such a set can be deduced from a given (historic) weather conditions record.

Just like radio technology, FSO suffers from a major disadvantage – sensitivity of the links transmission capacity to weather conditions. To take this feature into account in optimization modeling, we need to estimate *link failure ratio* defined as the fraction of the nominal capacity of the link lost in a given degraded weather condition. Note that the nominal capacity (i.e., the maximum data bit-rate) is realized on the link during good weather when the most effective (in terms of the data bit-rate) modulation and coding scheme (MCS) can be applied to the transmitted optical signal.

In case of bad weather conditions, the data transmission quality may be severely affected if no special action is undertaken. A solution is to adjust the current signal modulation scheme applied at the transmitter of the affected link to secure correct data transmission. In our considerations we assume four operation modes for each link. The first (basic) mode realizes the nominal link capacity (failure ratio 0), which is typically of the order of 1 Gbps [15], and is applied in normal (good) weather conditions using the 16-QAM (quadrature-amplitude modulation, [16]) modulation and coding scheme. The second mode corresponds to weather conditions worse than normal when the scheme is switched to 4-QAM – this assures approximately 75% of the nominal link capacity (failure ratio 0.25). When the weather conditions are even more degraded, the third mode is applied by switching to the QPSK (quadrature phase-shift keying, see [17]) scheme, losing approximately 50% of the nominal link capacity (failure ratio 0.5). The last mode corresponds to severely bad weather conditions that make correct data transmission impossible (failure ratio 1).

Having distinguished the four operation modes, we can determine what is the proper operation mode for each FSO link in a given weather state (note that in general different links see different weather condition in a given weather state). To do this we apply the formulae given in [18] and deduce the vector of the link failure ratios for each of the considered weather states. The set of all such vectors constitutes the *reference failure set* denoted by $\mathcal{S}$. We also consider virtual failure sets, called K-sets, parameterized by an integer value K, where K is less than or equal to the number of all links in the network. For a given K, the K-set contains all states corresponding to all combinations of K, or less, simultaneously affected links. In our optimization approach such K-sets are used as an approximation of the reference failure set. The approach consists in using K-sets (for a set of selected values of parameter K) as an input (instead of the true reference failure set) to the robust network dimensioning problem formulated in Section III, and then testing the link capacities solving the problem (called the robust solution) on the true reference failure state. Clearly, the larger the K value, the more failure states the K-set contains, resulting in general in more robust (and at the same time likely more costly) solutions.

For the numerical study presented in Section V we analysed the weather states observed over a one-year period (from August 16, 2014, to August 15, 2015) in network $polska$ described in [19]. The network is composed of the 12 largest metropolitan areas in Poland connected by 18 links. For the sake of simplicity we considered only the weather conditions observed in the metropolitan areas, and assumed that the link connecting tow given areas is affected by the worse of the weather conditions in the two areas. The observed conditions were translated into the multiple partial link failure states which constitute the reference failure set $\mathcal{S}$ corresponding to all hourly periods in the considered one-year time horizon, which gives $365 \times 24 = 8760$ states in total. In 60.75% of the states there are no affected links at all (these are the nominal states). The majority of the link failure states are characterized only by the 16-QAM to 4-QAM modulation and coding scheme change that reduce the nominal capacity of the affected links by 25% (link failure ratio 0.25) – there are 39.18% of such states in $\mathcal{S}$. In the remaining 0.07% of the failure states in $\mathcal{S}$ the modulation and coding scheme of some affected links is changed from 16-QAM to QSPK which reduces the nominal capacity by 50% (link failure ratio 0.5). Finally, there is a very small number of states with a few links affected in 100% (link failure ratio 1 – total loss of communication).

To give a rough idea of the value of applying the K-sets instead of the reference failure set $\mathcal{S}$ in robust optimization let us consider $polska$ with the above described set $\mathcal{S}$ of the failure states (derived from historical data). The presented calculations follow an ad-hoc heuristic procedure based on several simplifying assumptions.

Consider the robust network optimized (dimensioned) for a given K-set (for some K between 1 and 18) where for each state of the K-set the same 0.25 failure ratio is assigned to all its affected links (corresponding to the 16-QAM to 4-QAM modulation scheme adjustment). This means that the network can carry the assumed nominal traffic demand for all K (or less) simultaneous partial link failures with failure ratio 0.25. We assume (this is the first simplification) that in the nominal state (no links affected) the optimization assigns the same traffic carried (equal to 1 traffic unit) to all 18 links. For such an optimized network we now wish to (roughly) estimate the percentage of the traffic carried in a given state s in $\mathcal{S}$ which is not in the K-set. For this we assume (the second simplification) that the traffic carried in the nominal state is equal to the sum of the traffic carried on the links, i.e., to 18, and this value is at the same time equal to the total traffic demand. Certainly, for the states in $\mathcal{S}$ that are also in the K-set this percentage is equal to 100%. Yet, for the states in $\mathcal{S}$ but not in the K-set, this percentage is in general less than 100%.

For each state $s \in \mathcal{S}$ outside the assumed K-set the total lost traffic (i.e., the total traffic carried in the nominal state minus the total traffic carried in state s) is computed in the following way. Let K' be the number of affected links in state s. Suppose (w.l.o.g.) that the first K' consecutive links are affected in s and that their failure ratios (denoted by $\beta(k)$, $k = 1, 2, \ldots, 18$, where $\beta(k) \in \{0, 0.25, 0.5, 1\}$) are arranged in the non-increasing order, i.e., $\beta(1) \geq \beta(2) \geq \ldots \geq \beta(18)$. Note that $\beta(K') > 0$ and $\beta(k) = 0$ for $k = K'+1, K'+2, \ldots, 18$. Our formula for computing the total volume of the traffic lost in state s (denoted by $L(s)$) is as follows:

$$L(s) = \sum_{k=1}^{K'} (\beta(k) - 0.25), \text{ if } K' \leq K$$

$$L(s) = \sum_{k=1}^{K} (\beta(k) - 0.25) + \sum_{k=K+1}^{K'} \beta(k), \text{ if } K' > K.$$

Thus, the estimated percentage of traffic carried in state s is equal to

$$P(s) := \frac{18 - L(s)}{18} 100\%.$$

The rational behind the above formulae follows from the third (simplifying) assumption. In the network dimensioned in the robust way against the failure states in the K-set, the portion of the nominal traffic that cannot be directly carried on an affected link is, by optimization assumption, restored and realized on the paths using other links. In our setting, the nominal traffic carried on any link is equal to 1. Hence, if we assume that the portion of the traffic in question is proportional to the link capacity loss which is 25% (recall that in the states of the K-set the affected link failure ratio is always 0.25), the volume of the restored traffic is equal to 0.25 units. In effect, the network is capable of restoring 0.25 units of the nominal traffic on the affected links provided not more than K links fail simultaneously. Thus, coming back to the considered state $s \in \mathcal{S}$ outside the K-set, when $K' \leq K$, the 0.25 units of the nominal traffic carried on the first K' links can be restored while the remaining portion, i.e., $\beta(k) - 0.25$, cannot. Thus, $\beta(k) - 0.25$ units of traffic is lost on each affected link. Observe that when $\beta(k) = 0.25$ no traffic on link k is lost, and, to be on the safe side in the estimation, we assume that potential capability of restoring 0.25 units of traffic on links $k = K'+1, K'+2, \ldots, K$ is not exploited. Similarly, when $K' > K$, no traffic on the links $k = K+1, K+2, \ldots, K'$ can be restored so that for those links the traffic loss is just $\beta(k)$.

As an example, consider the state with $K' = 3$ and the following vector of the link failure ratios:

$$(0.25, 0.25, 0.25, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0).$$

There are 15 unaffected links and 3 affected links. Thus, for all $K \geq 3$ the state is covered (meaning that the traffic carried on the affected links can be restored). For $K = 2$, however, there will remain one affected link with the capacity loss not taken into account in dimensioning. Hence, in this state (case $K' > K$) the percentage of traffic carried is equal to $\frac{18-0.25}{18}100\% = 98.61\%$.

Now consider the state also with $K' = 3$ but with a different link failure ratio vector:

$$(0.5, 0.25, 0.25, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0).$$

For $K = 2$ ($K' > K$) the percentage in question is equal to $\frac{18-(0.25+0.25)}{18}100\% = 97.22\%$. For $K \geq 3$ ($K' \leq K$) , though, it is equal to $\frac{18-0.25}{18}100\% = 98.61\%$.

Table I: Estimated traffic carried averaged over all states.

K	0	1	2	3	4	5
[%]	89.94	91.23	92.50	93.78	94.05	96.33
K	6	7	8	9	10	11
[%]	97.61	98.88	99.48	99.89	99.99	99.99

The above described calculations were accomplished for all states of the failure reference set $\mathcal{S}$ described above. The resulting average carried traffic percentage for $K = 0, 1, \ldots, 11$ (for all $K \geq 12$ the result is virtually the same as for $K = 11$) are reported in Table I. Certainly, the calculation gives only a rough upper bound on the amount of the expected carried traffic for the failure states outside the K-set, and a more accurate calculation method would be useful when dealing with realistic networks.

Yet, the results presented in Table I give some evidence that considering K-sets for network dimensioning instead of all possible states can be effective in terms of the capability of traffic handling in the states outside the assumed K-set. This is important as the list of all possible states is in general not known or difficult to retrieve from available historical data.

III. Problem formulation

The considered network is modeled using a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ composed of the set of nodes $\mathcal{V}$ and the set of links $\mathcal{E}$. In the sequel the number of nodes $|\mathcal{V}|$ and the number of links $|\mathcal{E}|$ will be denoted by V and E, respectively. Each link $e \in \mathcal{E}$ represents an arc, i.e., an ordered pair (v, w) of nodes for some $v, w \in \mathcal{V}, v \neq w$. If $e = (v, w)$ then $a(e) := v$ and $b(e) := w$ denote the origin and destination of arc $e \in \mathcal{E}$, respectively. Furthermore, we define $\delta^+(v) := \{e \in \mathcal{E} : a(e) = v\}$ (the set of links outgoing from node v) and $\delta^-(v) := \{e \in \mathcal{E} : b(e) = v\}$ (the set of links incoming to node v). To each link $e \in \mathcal{E}$ there corresponds a non-negative unit capacity cost $\xi(e)$ (a parameter), and capacity denoted by y_e (when link capacity is an optimization variable) or $c(e)$ (when capacity is fixed). The total cost of the network is thus given by $\sum_{e \in \mathcal{E}} \xi(e) y_e$ or $\sum_{e \in \mathcal{E}} \xi(e) c(e)$ (where $y := (y_e,\ e \in \mathcal{E})$, $c := (c(e),\ e \in \mathcal{E})$).

Traffic demands are represented by the set $\mathcal{D}$ (the number of demands $|\mathcal{D}|$ will be denoted by D). Each demand $d \in \mathcal{D}$ is represented by an ordered pair of nodes $(o(d), t(d))$ (its origin and termination) and the volume $h(d)$ (a parameter) of the traffic that has to be realized from $o(d)$ to $t(d)$. The demand volumes and link capacities are expressed in the same units.

The set of network links is subject to multiple partial failures where each failing link e loses a portion $\beta(e)$ (where $0 < \beta(e) \leq 1$ is a given parameter called failure ratio) of its nominal capacity $c(e)$. Consequently, each failure state $s \in \mathcal{S}$, where $\mathcal{S}$ denotes the set of failure states (the number of states $|\mathcal{S}|$ will be denoted by S), is identified with a binary vector $s = (s(e),\ e \in \mathcal{E})$ of failure coefficients where $s(e) = 1$ when link e is available only partially in state s, and $s(e) = 0$ when link e is fully available in state s. This means that in state $s \in \mathcal{S}$ the available capacity of link $e \in \mathcal{E}$ is equal to $(1 - \beta(e)s(e))c(e)$.

Finally, we assume that in any failure state $s \in \mathcal{S}$ each demand $d \in \mathcal{D}$ can be routed in a bifurcated way along all possible paths from $o(d)$ to $t(d)$, and the resulting traffic flows in particular states are independent of each other. This means that the network applies the so called *Global Rerouting* or *Unrestricted Reconfiguration* mechanism [6], [20].

The problem considered in the paper is formulated in the node-link notation (using the link-flows variables x and capacity variables y) as follows:

Problem P($\mathcal{S}$) (main):

$$C(\mathcal{S}) = \min \sum_{e \in \mathcal{E}} \xi(e) y_e \tag{1a}$$

$$\sum_{e \in \delta^+(v)} x^s_{ed} - \sum_{e \in \delta^-(v)} x^s_{ed} = \begin{cases} h(d) & \text{if } v = o(d) \\ -h(d) & \text{if } v = t(d) \\ 0 & \text{otherwise,} \end{cases}$$
$$d \in \mathcal{D}, v \in \mathcal{V}, s \in \mathcal{S} \tag{1b}$$

$$\sum_{d \in \mathcal{D}} x^s_{ed} \leq (1 - \beta(e)s(e)) y_e, \quad e \in \mathcal{E},\ s \in \mathcal{S} \tag{1c}$$

$$x, y \geq 0 \text{ and continuous.} \tag{1d}$$

Above, x^s_{ed} is the flow on link e dedicated to carry the traffic of demand d in state s. The objective of P($\mathcal{S}$), i.e., minimizing the cost of links, is specified in (1a). Constraint (1b) represents the flow conservation equation for each demand d at each node v in each state s, assuring the realization of $h(d)$ for all d in each state s. Finally, the capacity constraint (1c) ensures that the capacity of link e is not exceeded in any state s.

In the sequel we will consider a subfamily of the above described failure states $\mathcal{S}$, namely the family of K-sets (already discussed in Section II). A K-set is denoted with $\mathcal{S}(K)$ and defined, for any integer $K = 1, 2, \ldots, E$, as $\mathcal{S}(K) := \{s \in \{0,1\}^E : \sum_{e \in \mathcal{E}} s(e) \leq K\}$. From the viewpoint of problem (1), every set S considered for P($\mathcal{S}$) is dominated by the K-set with $K = \max\{\sum_{e \in \mathcal{E}} s(e) : s \in \mathcal{S}\}$ in the sense that each feasible solution of P($\mathcal{S}(K)$) is feasible for P($\mathcal{S}$). Thus the optimal solution of P($\mathcal{S}(K)$) will be resilient with respect to all states in $\mathcal{S}$. For convenience, the optimization problem (1) corresponding to a K-set $\mathcal{S}(K)$ will be from now on denoted by P(K) instead of P($\mathcal{S}(K)$).

Note that the number $S(K) := \sum_{k=0}^{K} \binom{E}{k}$ of the states in $\mathcal{S}(K)$ grows exponentially with the number of links E, provided K increases linearly with E, for example when $K = \lfloor \frac{E}{2} \rfloor$. Thus, although problem P(K) is linear, it is in general non-compact as the number of variables and constraints is proportional to $S(K)$ and hence exponential with E. Therefore, the direct approach to P(K) by means of an LP solver is not applicable for large networks. For this reason, we have developed a more efficient approach to P(K). The approach, similar to Benders' decomposition [21] (see also [6]), is presented in the next section.

IV. Solution algorithm

Let $\mathcal{Y}$ denote the set of all capacity vectors feasible for (1) and suppose we wish to test whether a given capacity vector $c = (c(e),\ e \in \mathcal{C})$ is in $\mathcal{Y}$. Because the demand routing in a particular failure state is independent of the demand routing in the remaining states, we can perform the test in question by checking the feasibility separately for each state by means of the following linear program:

Problem F(c, s) (feasibility of c in state s):

$$O(c,s) = \min \sum_{e \in \mathcal{E}} z_e \tag{2a}$$

$$\sum_{e \in \delta^+(v)} x_{ed} - \sum_{e \in \delta^-(v)} x_{ed} = \begin{cases} h(d) & \text{if } v = o(d) \\ -h(d) & \text{if } v = t(d) \\ 0 & \text{otherwise,} \end{cases} \quad d \in \mathcal{D}, v \in \mathcal{V} \tag{2b}$$

$$\sum_{d \in \mathcal{D}} x_{ed} \le (1 - \beta(e)s(e))c(e) + z_e, \quad e \in \mathcal{E} \tag{2c}$$

$$x, z \ge 0 \text{ and continuous,} \tag{2d}$$

where $O(c,s)$ expresses the minimum of the sum of links' overloads. The test is valid since c is feasible for s if, and only if, $O(c,s) = 0$. If the result of the test is negative (i.e., $O(c,s) > 0$) we need to find an inequality that separates c from $\mathcal{Y}$. This can be done by considering the dual to (2) (whose variables $\lambda := (\lambda_d^v,\ v \in \mathcal{V}, d \in \mathcal{D})$ and $\pi := (\pi_e,\ e \in \mathcal{E})$ correspond to the primal constraints (2b) and (2c), respectively):

Problem D(c, s) (dual to F(c, s)):

$$W(c,s) = \max\{\sum_{d \in \mathcal{D}} \lambda_d^{t(d)} h(d) - \sum_{e \in \mathcal{E}} \pi_e (1 - \beta(e)s(e))c(e)\} \tag{3a}$$

$$\pi_e \le 1,\ e \in \mathcal{E};\ \ \lambda_d^{o(d)} = 0,\ d \in \mathcal{D} \tag{3b}$$

$$\lambda_d^{b(e)} - \lambda_d^{a(e)} \le \pi_e,\ e \in \mathcal{E}, d \in \mathcal{D}; \tag{3c}$$

$$\pi \ge 0 \text{ and continuous, } \lambda \text{ continuous.} \tag{3d}$$

Let λ^*, π^* be an optimal solution of problem D(c, s). Since $W(c,s) = O(c,s)$ (where $O(c,s)$ is defined by (2a)), the inequality that separates c from $\mathcal{Y}$ (provided $W(c,s) > 0$) is as follows:

$$\textstyle\sum_{e \in \mathcal{E}} \pi_e^* (1 - \beta(e)s(e)) y_e \ge \sum_{d \in \mathcal{D}} \lambda_d^{t(d)^*} h(d). \tag{4}$$

Since $\sum_{e \in \mathcal{E}} \pi_e^* (1 - \beta(e)s(e))c(e) = \sum_{d \in \mathcal{D}} \lambda_d^{t(d)^*} h(d) - W(c,s)$ and $W(c,s) > 0$, $y = c$ does not fulfil (4).

Note that to make sure that $c \notin \mathcal{Y}$ we in general need to perform test (3) for all states s in $\mathcal{S}$. This, however, is virtually impossible for a general form of the set of failure states $\mathcal{S}$ because of the exponential number of states. Yet, for a K-set $\mathcal{S}(K)$ the test can be improved by finding the maximum $W(c) := \max_{s \in \mathcal{S}(K)} W(c,s)$. The adjusted test is as follows:

Problem G(c) (maximum violation):

$$W(c) = \max\{\sum_{d \in \mathcal{D}} \lambda_d^{t(d)} h(d) - \sum_{e \in \mathcal{E}} \pi_e c(e) + \sum_{e \in \mathcal{E}} \beta(e)c(e)\pi_e u_e\} \tag{5a}$$

$$\pi_e \le 1,\ e \in \mathcal{E};\ \ \lambda_d^{o(d)} = 0,\ d \in \mathcal{D} \tag{5b}$$

$$\lambda_d^{b(e)} - \lambda_d^{a(e)} \le \pi_e,\ e \in \mathcal{E}, d \in \mathcal{D} \tag{5c}$$

$$\sum_{e \in \mathcal{E}} u_e \le K;\ \ u_e \in \{0,1\},\ e \in \mathcal{E} \tag{5d}$$

$$\pi \ge 0 \text{ and continuous, } \lambda \text{ continuous.} \tag{5e}$$

Above, any vector of binary variables u fulfilling constraints (5d) defines a state $s \in \mathcal{S}(K)$ simply by putting $s(e) := u_e,\ e \in \mathcal{E}$. Moreover, for any fixed feasible u the value of the objective function maximized over λ, π is equal to $W(c,s)$ for the so defined s. Since in (5) we are maximizing also over u, we will finally obtain $W(c) = \max_{s \in \mathcal{S}(K)} W(c,s)$, as required.

Observe that formulation (5) involves multiplications of variables $u_e \times \pi_e$, $e \in \mathcal{E}$ with u_e binary and π_e upper bounded by 1. To get rid of these bi-linearities we introduce additional (continuous) variables U_e, $e \in \mathcal{E}$, that will be equal to $u_e \times \pi_e$ in the optimal solution. This is done in the next formulation:

Problem GMIP(c) (MIP version of G(c)):

$$W(c) = \max\{\sum_{d \in \mathcal{D}} \lambda_d^{t(d)} h(d) - \sum_{e \in \mathcal{E}} \pi_e c(e) + \sum_{e \in \mathcal{E}} \beta(e)c(e)U_e\} \tag{6a}$$

$$\pi_e \le 1,\ e \in \mathcal{E};\ \ \lambda_d^{o(d)} = 0,\ d \in \mathcal{D} \tag{6b}$$

$$\lambda_d^{b(e)} - \lambda_d^{a(e)} \le \pi_e,\ e \in \mathcal{E}, d \in \mathcal{D} \tag{6c}$$

$$\sum_{e \in \mathcal{E}} u_e \le K,\ \ e \in \mathcal{E} \tag{6d}$$

$$U_e \le \pi_e,\ U_e \le u_e,\ \ e \in \mathcal{E} \tag{6e}$$

$$\pi \ge 0 \text{ and continuous, } \lambda, U \text{ continuous, } u_e \in \{0,1\}. \tag{6f}$$

Now let λ^*, π^*, U^* (and u^*) be an optimal solution of (6). If $W(c) > 0$ then the capacity vector c is infeasible for the main problem P(K) formulated in (1), and the following (Benders) inequality (written in variables y)

$$\textstyle\sum_{e \in \mathcal{E}} \pi_e^* (1 - \beta(e)U_e^*) y_e - \sum_{d \in \mathcal{D}} \lambda_d^{t(d)^*} h(d) \ge 0 \tag{7}$$

separates c from the set of capacity vectors feasible for P(K). Note that c breaks the above inequality by $W(c) = \max_{s \in \mathcal{S}(K)} O(c,s)$, i.e., by the sum of the link overloads (minimized over the demand routing) in the "worst state" $s \in \mathcal{S}(K)$.

The iterative algorithm for solving (1) is given below. In each iteration the master problem involving only the capacity variables y is solved and then its optimal solution y^* is tested for feasibility with respect to P(K). If the test is positive, the algorithm is stopped and y^* is optimal for (1). If not, a new inequality deduced from the feasibility test is added to the master problem and the algorithm is reiterated. (Below, the notation $y \in \Omega$ means that y fulfills all inequalities in the set of inequalities Ω.)

Algorithm for P(K).

Step 0: $\Omega := \{y \geq 0\}$.
Step 1: Solve the master problem:

$$c := \text{ argmin}_{y \in \Omega} \; \sum_{e \in \mathcal{E}} \xi(e) y_e.$$

Step 2: Solve the feasibility test (6). If $W(c) \leq 0$ then stop (c is the capacity vector optimal for problem (1)).
Step 3: Otherwise, add inequality (7) to Ω and go to Step 1.

V. Computational Results

We tested the proposed algorithm on a real-life network instance called $polska$ considered in Section II. The network connects $V = 12$ nodes (metropolitan areas) and is composed of $E = 18$ links. The unit of link capacity is 1 Mbps and the cost of the capacity unit on each link is equal to 1, i.e., $\xi(e) = 1, e \in \mathcal{E}$. There are $D = 66$ traffic demands with the volumes expressed in Mbps – these volumes can be found in [19]. The reference set of failure states $\mathcal{S}$ examined in the study is that described in Section II. Typically, the links in any $s \in \mathcal{S}$ are affected with failure ratio $\beta(e) = 0.25$. In some states, however, a few links can be affected with $\beta(e) = 0.5$, and, in rare cases, with $\beta(e) = 1$ (the state with no affected links is not considered). In the reported numerical experiments we first solve the instance of problem P(K) (1) for each value of $K = 1, 2, \ldots, 18$ assuming the failure ratios 0.25 (corresponding to changing the modulation and coding scheme from 16-QAM to 4-QAM). After that, for each K, we calculate how much traffic is not carried in the states in $\mathcal{S} \setminus \mathcal{S}(K)$, i.e., in the states outside the set $\mathcal{S}(K)$ assumed for robust optimization.

All the experiments were performed on a 2.70 GHz computer with 8 GB of RAM. The code was written in C++ and the optimization problems were solved by IBM ILOG CPLEX 12.5 (using Concert Technology) running with the default setting.

Tables II and III report the computational results for different K, shown in the consecutive rows. In Table II, c^* is the minimum cost for P(K), "*no. of cuts*" is the number of Benders's cuts (7) generated to reach the optimal robust solution, and "*sol. time*" (in seconds) is the computation time.

Table III reports, for each K, the results of testing the resilience of the optimal solutions described in Table II with respect to all states in the reference failure set $\mathcal{S}$. Certainly, the optimization goal assures that all the states in the K-set $\mathcal{S}(K)$ are covered, meaning that all the traffic demands $h(d)$, $d \in \mathcal{D}$, can be satisfied when the optimal link capacities $y^*_e, e \in \mathcal{E}$, are assumed. Yet, for the states $s \in \mathcal{S} \setminus \mathcal{S}(K)$ this is not guaranteed and hence for each such state s we have minimized the lost traffic using a linear programming formulation similar to (2). In the formulation, link capacities are set to $c(e) := (1 - \beta(e)s(e))y^*_e, e \in \mathcal{E}$, and the routing is optimized in order to minimize the total traffic loss $\sum_{d \in \mathcal{D}} z_d$, where $0 \leq z_d \leq h(d)$, and $h(d) - z_d$ is the actual traffic carried for demand d. In Table III column "*% of avg. carried traffic*" is the average percentage of traffic carried (with respect to the traffic offered) over all of the 8760 states in $\mathcal{S}$ (including the nominal states, see Section II), and "*% of failure states not covered*" is the percentage of the states in $\mathcal{S}$ for which some part of the offered traffic is not realized.

The first important observation is that, already for the network dimensioned for $K = 1$, it is possible to guarantee robustness for almost 74% of the states (only about 26% of the states in $\mathcal{S}$ experience carried traffic degradation). For increasing K, the percentage of the states with no traffic degradation continues to increase until $K = 9$, the case for which our computations reveal the second interesting fact: the network dimensioned for $K = 9$ is already robust against all the states in $\mathcal{S}(18)$, i.e., for the states with all $E = 18$ links or less affected with a failure ratio equal to 0.25 – this effect is clearly seen in Tables II and III. This means

Table II: Results of the robust optimization.

K	c^*	no. of cuts	sol. time
0	21192	0	5
1	23127	56	87
2	24890	78	158
3	26557	70	192
4	27640	92	246
5	27940	127	225
6	28008	155	297
7	28100	139	318
8	28180	115	343
9	28256	136	381
10	28256	128	296
11	28256	116	285
12	28256	119	259
13	28256	109	192
14	28256	118	203
15	28256	112	180
16	28256	98	93
17	28256	115	89
18	28256	127	58

Table III: Post-processing results.

K	% of avg. carried traffic	% of failure states not covered
0	90.44	39.25
1	90.67	26.07
2	91.29	25.12
3	93.38	18.63
4	94.19	17.57
5	95.41	13.99
6	97.04	7.43
7	97.92	4.92
8	99.04	1.82
9	99.56	0.02
10	99.56	0.02
11	99.56	0.02
12	99.56	0.02
13	99.56	0.02
14	99.56	0.02
15	99.56	0.02
16	99.56	0.02
17	99.56	0.02
18	99.56	0.02

that taking into account all $\binom{18}{9}$ states with exactly 9 links affected with the failure ratio 0.25 makes the network robust to all multiple partial link failure states with $\beta(e) \equiv 0.25$. This "saturation effect", for which an intermediate factor of robustness may already offer full protection against all deviations in the input data can be observed also in other (telecommunications) applications of robust optimization (see for example [7] and its references). A similar discussion can be found in [22] in the context of demand uncertainty.

Still, this is not sufficient to cover all the states in $\mathcal{S}$ as shown by the results presented in Table III. This is due to the existence of two "nasty" states that include several links affected by 0.50 failure ratio and even links whose entire capacity is lost, for which the traffic matrix cannot be realized even for large values of K. As a consequence, protection for these two peculiar states, corresponding to about the 0.02% of all states, can never be granted, even imposing protection against all links failing with failure ratio 0.25. Note also that the additional cost for $K \geq 9$ is around 25%, as compared with the (nominal) case $K = 0$. Besides, it is worth observing that the results of the ad-hoc estimation method presented in Table I presented in Section II fit quite well to the analogous (exact) results in Table III.

Concerning the time needed to obtain optimal solutions through the proposed cutting-place algorithm, we note that solving problem P(K) for various K for the considered network instance takes a reasonable amount of time. However, we expect that for large network instances the separation algorithm, in its current form, could take high amount of time and thus the inclusion of additional valid inequalities (like those proposed in Section VI) could become crucial to speed up the algorithm.

We also observe that the number of generated cuts and the separation time are higher for intermediate values of K than for values of K close to 0 or 18 which require considerably less time. This computational behaviour is due to the fact that when the value of K becomes close to its lower or upper bound there are less relevant combinations of failing links to be considered in the separation procedure, as also observed in [7] and the references therein.

VI. Discussion

The node-link formulation (1) can be reformulated for the case of undirected graphs. This is achieved in the standard way by substituting each undirected edge $\{v, w\} \in \mathcal{E}$ by two oppositely directed arcs (v, w) and (w, v) (leading to a bi-directed network graph) and defining the load of edge $\{v, w\}$ as the sum of the loads of arcs (v, w) and (w, v). If also the demands are undirected, then each demand $d \in \mathcal{D}$, connecting nodes v and w, say, should be made directed through selecting one of v, w for the originating node $o(d)$ and the other for the terminating node $t(d)$.

The above derivations are valid for the link-path formulation of P($\mathcal{S}$) (for a given list of allowable routing paths) as well. Although the link-path formulation leads to the dual polytope different from (3b)-(3d), the dual function (3a) used for testing feasibility of the link capacity vector c remains essentially the same. The link-path formulation is valid for both directed and undirected networks, but may require path generation for defining the proper lists of allowable paths.

Observe that capacity vector y is feasible for (1) if, and only if, it is feasible for each state in the following set (admitting fractional failure coefficients $s(e)$, $e \in \mathcal{E}$) called the uncertainty polytope [23]:

$$\{s = (s(e), e \in \mathcal{E}) : \textstyle\sum_{e \in \mathcal{E}} s(e) \leq K, 0 \leq s(e) \leq 1,\, e \in \mathcal{E}\}.$$

This follows from the fact that the version of G(c) with λ and π fixed and the binary variables u in (5d) relaxed, i.e., with $0 \leq u_e \leq 1$ instead of $u_e \in \{0, 1\}$, $e \in \mathcal{E}$, is an LP problem with the totally unimodular matrix of coefficients (recall that K is a positive integer).

The problem studied in this paper may be seen as a special case of the two-stage problem studied in [23] and hence is most likely $\mathcal{NP}$-hard (see also [24], [25]). Moreover, the Benders cutting plane approach may need a large number of Benders inequalities to converge to optimum. This may pose an efficiency issue since test (6) (called also the separation problem) performed in Step 2 of our algorithm contains binary variables. One way to deal with this issue is to try to speed up the convergence of the algorithm by using additional valid inequalities on top of the Benders cuts in the master problem solved in Step 1 of the algorithm.

A valid inequality can be obtained by considering $K \times B$ instead of the last sum in (5a), where $B := \frac{\sum_{e \in \mathcal{E}} \beta(e)c(e)\pi_e}{E}$ is the average value of the terms $\beta(e)c(e)\pi_e$, $e \in \mathcal{E}$. The appropriate test is as follows:

$$\max \sum_{d \in \mathcal{D}} \lambda_d^{t(d)} h(d) - \sum_{e \in \mathcal{E}} \pi_e c(e) + \frac{K}{E} \sum_{e \in \mathcal{E}} \beta(e)c(e)\pi_e \quad \text{(8a)}$$

$$\pi_e \leq 1, \;\; e \in \mathcal{E} \quad \text{(8b)}$$

$$\lambda_d^{b(e)} - \lambda_d^{a(e)} \leq \pi_e, \; e \in \mathcal{E}, d \in \mathcal{D}; \;\; \lambda_d^{o(d)} = 0, \; d \in \mathcal{D} \quad \text{(8c)}$$

$$\pi \geq 0 \text{ and continuous, } \lambda \text{ continuous.} \quad \text{(8d)}$$

When the resulting maximum is positive, the following valid inequality

$$\sum_{d \in \mathcal{D}} \lambda_d^{t(d)^*} h(d) - \sum_{e \in \mathcal{E}} \pi_e^* y_e + \frac{K}{E} \sum_{e \in \mathcal{E}} \pi_e^* \beta(e) y_e \leq 0 \qquad (9)$$

is obtained. Note that (8) is a linear programming problem while formulation (6) requires binary variables,

Similar valid inequalities can be obtained for subsets $\mathcal{E}'$ of links with $k := |\mathcal{E}'| \leq K$. In the corresponding feasibility test the objective is as follows:

$$\max \sum_{d \in \mathcal{D}} \lambda_d^{t(d)} h(d) - \sum_{e \in \mathcal{E}} \pi_e c(e) + \frac{K-k}{E-k} \sum_{e \in \mathcal{E} \setminus \mathcal{E}'} \beta(e)c(e)\pi_e + \sum_{e \in \mathcal{E}'} \beta(e)c(e)\pi_e.$$

Another useful class of valid inequalities are the cutset inequalities, first introduced in the robust context in [9]. Given a partition $\mathcal{V} = \mathcal{V}_1 \cup \mathcal{V}_2$ of the nodes, the cutset inequality associated with the partition states that the amount of capacity installed on the links going from $\mathcal{V}_1$ to $\mathcal{V}_2$ should not be less than the sum of the demands going from $\mathcal{V}_1$ to $\mathcal{V}_2$. Defining $\mathcal{E}(\mathcal{V}_1, \mathcal{V}_2) = \{e \in \mathcal{E} : a(e) \in \mathcal{V}_1 \wedge b(e) \in \mathcal{V}_2\}$ and $\mathcal{D}(\mathcal{V}_1, \mathcal{V}_2) = \{d \in \mathcal{D} : o(d) \in \mathcal{V}_1 \wedge t(d) \in \mathcal{V}_2\}$, the inequality is formally defined as

$$\min_{s \in \mathcal{S}} \sum_{e \in \mathcal{E}(\mathcal{V}_1, \mathcal{V}_2)} (1 - \beta(e)s(e))c(e) \geq \sum_{d \in \mathcal{D}(\mathcal{V}_1, \mathcal{V}_2)} h(d) \quad (11)$$

While the exact separation of (11) requires solving mixed-integer programs, this can also be done heuristically. For instance, in [26] the nodes are randomly partitioned into two subsets and then a local search is performed, picking up one node and moving it to the other subset until there is no more improvement in the violation. If no violated inequality has been found, another initial partition is considered up to a maximum of 5 iterations.

VII. Future Work

The model presented and tested in this paper takes into account one level of failure ratios $\beta(e)$. In practice, however, links may be affected with different levels of failure ratios depending on the severity of weather conditions and the corresponding modulation and coding scheme. As mentioned in Section II, these ratios can for example be equal to 0.25, 0.5 and 1. A more general model taking multiple levels of failure ratios into account will be the subject of a future, more comprehensive study.

In that study, we will also enhance the optimization procedure described in Section IV by using valid inequalities discussed in Section VI to speed up the algorithm convergence, and provide a more complete numerical study of the method.

Also, improvement of the ad-hoc heuristic (for rough estimation of the traffic loss in the states outside the given K-set) presented in Section II would be beneficial for speeding up the approach presented in the paper.

Finally, modularity of link capacity should be added to our optimization model since this is an important characteristic of FSO networks where links are composed of parallel light beam systems. Formally, this feature does not influence the form of (1) and of the master problem but transforms each of them from a linear to a mixed-integer formulation.

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References

[1] D. Bertsimas and M. Sim. The Price of Robustness. *Operations Research*, 52(1):35–53, 2004.

[2] A. Sen, S. Murthy, and S. Banerjee. A new paradigm for the design of fault-tolerant networks. In *Proc. of the 15th International Conference on High-Performance Switching and Routing (HPSR'09)*, pages 1–7, 2009.

[3] J. Rak. Measures of region failure survivability for wireless mesh networks. *Wireless Networks*, 21:673–684, 2015.

[4] J. Rak. A new approach to design of weather disruption-tolerant wireless mesh networks. *Telecommunication Systems*, 61:311–323, 2016.

[5] F. Gjata. Measuring the impact of weather factors to FSO transmission. *Intership Report, Information and Systems Technologies, Université de Technologie de Compiègne*, 2016.

[6] M. Pióro and D. Medhi. *Routing, Flow, and Capacity Design in Communication and Computer Networks*. Morgan Kaufmann, 2016.

[7] T. Bauschert, C. Büsing, F. D'Andreagiovanni, A.M.C.A. Koster, M. Kutschka, and U. Steglich. Network planning under demand uncertainty with robust optimization. *IEEE Commununications Magazine*, 52:178–185, 2014.

[8] F. D'Andreagiovanni, J. Krolikowski, and J. Pulaj. A fast hybrid primal heuristic for multiband robust capacitated network design with multiple time periods. *Appl. Soft Comput.*, 26:497–507, 2015.

[9] A.M.C.A. Koster, M. Kutschka, and C. Raack. Robust Network Design: Formulations, Valid Inequalities, and Computations. *Networks*, 61(2):128–149, 2013.

[10] F. D'Andreagiovanni. Revisiting wireless network jamming by SIR-based considerations and multiband robust optimization. *Optim. Lett.*, 9:1495–1510, 2015.

[11] G. Classen, D. Coudert, A.M.C.A. Koster, and N. Nepomuceno. A chance-constrained model and cutting planes for fixed broadband wireless networks. In *Proc. INOC 2011*, 2011.

[12] M. Pióro, Y. Fouquet, D. Nace, and M. Poss. Optimizing flow thinning protection in multicommodity networks with variable link capacity. *Operations Research*, 64(2):273–289, 2016.

[13] Y. Fouquet, D. Nace, M. Pióro, and M. Poss. An optimization framework for traffic restoration in optical wireless networks with partial link failures. *Optical Switching and Networking*, 23, Part 2:108–117, 2017.

[14] Y. Li, N. Pappas, V. Angelakis, M. Pióro, and D. Yuan. Optimization of free space optical wireless network for cellular backhauling. *IEEE JSAC*, 33(9):1841–1854, 2015.

[15] FSona optical wireless. http://www.fsona.com/product.php.

[16] A. Svensson. An introduction to adaptive qam modulation schemes for known and predicted channels. *Proceedings of the IEEE*, 95(12):2322–2336, Dec 2007.

[17] J.S. Malhotra, M. Kumar, and A.K. Sharma. Performance Comparison of PS-QPSK and PM-QPSK Modulation Schemes in High Capacity Long Haul DWDM Optical Communication Link. *International Journal of Engineering Sciences*, 2(5):154–159, 2013.

[18] Priyanka J, Bhuperdra S, and Rashmi C. Survey on performance of free space optical communication links under various field parameters. *IOSR Journal of Electrical and Electronics Engineering (IOSR-JEEE)*, 9(2):71–75, 2014.

[19] S. Orlowski, M. Pióro, A. Tomaszewski, and R. Wessäly. SNDlib 1.0 – Survivable Netwrok Design Library. *Networks*, 55(3):276–285, 2010.

[20] S. Orlowski and M. Pióro. Complexity of column generation in network design with path-based survivability mechanism. *Networks*, 59(1):132–147, 2012.

[21] J.F. Benders. Partitioning procedures for solving mixed variable programming problems. *Numerische Mathematik*, 4:238–252, 1962.

[22] M. Poss and C. Raack. Affine recourse for the robust network design problem: Between static and dynamic routing. *Networks*, 61(2):180–198, 2013.

[23] J. Ayoub and M. Poss. Decomposition for adjustable robust linear optimization subject to uncertainty polytope. *Computational Management Science*, 2:219–239, 2013.

[24] A. Gupta, J. Kleinberg, A. Kumar, R. Rastogi, and B. Yener. Provisioning a virtual private network: A network design problem for multicommodity flow. In *Proc. 33rd annual ACM Symp Theory Comput (STOC 2001)*, 2001.

[25] C. Chekuri, G. Oriolo, M.G. Scutellá, and F.B. Shepherd. Hardness of robust network design. *Networks*, 50:50–54, 2007.

[26] S. Mattia and M. Poss. A comparison of different routing schemes for the robust network loading problem: polyhedral results and computation. Submitted, available online on HAL.