Applied Soft Computing 61 (2017) 1074–1087

Contents lists available at ScienceDirect

Applied Soft Computing

journal homepage: www.elsevier.com/locate/asoc

Integrating LP-guided variable fixing with MIP heuristics in the robust design of hybrid wired-wireless FTTx access networks☆

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ARTICLE INFO

Article history:
Received 15 July 2016
Received in revised form 21 June 2017
Accepted 10 July 2017
Available online 27 July 2017

Keywords:
Telecommunications access networks
FTTx
Connected facility location
Mixed integer linear programming
Robust optimization
MIP heuristics

ABSTRACT

This study investigates how to model and solve the problem of optimally designing FTTx telecommunications access networks integrating wired and wireless technologies, while taking into account the uncertainty of wireless signal propagation. We propose an original robust optimization model for the related robust 3-architecture Connected Facility Location problem, which includes additional variables and constraints to model wireless signal coverage represented through signal-to-interference ratios. Since the resulting robust problem can prove very challenging even for a modern state-of-the art optimization solver, we propose to solve it by an original primal heuristic that combines a probabilistic variable fixing procedure, guided by peculiar Linear Programming relaxations, with a Mixed Integer Programming heuristic, based on an exact very large neighborhood search. A numerical study carried out on a set of realistic instances show that our heuristic can find solutions of much higher quality than a state-of-the-art solver.

1. Introduction

Since the nineties of the last millennium, telecommunications have played a major role in people's everyday life and the volume of traffic exchanged over worldwide telecommunications networks has astonishingly increased. Major telecommunications companies expect that such growth will powerfully continue: very recent studies forecast that the annual global internet traffic will surpass the *zettabyte* (=10^{21} bytes) by the end of 2016 and that the increase will continue at a compound annual growth rate of 22% from 2015 to 2020 [17]. To support this growth, the need for a new generation of high-capacity *telecommunication access networks*, namely the external part of a network which connects users to service providers, has arisen.

In the last decade, optical fiber connections have become a critical component of high-capacity access networks. Indeed, fibers can provide higher capacity and better transmission rates than old copper-based connections. However, deploying a full fiber-based access network, where a dedicated fiber connection is granted to each single user, entails extremely high costs and thus, in the last years, the trend has been to provide broadband internet access through different types of non-fully fiber-based deployments. These several deployments, usually called *architectures*, are denoted as a whole by the acronym *FTTx* (Fiber-To-The-x), where the *x* is specified on the basis of where the optical fiber granting access to the user is terminated. Major examples of architectures are:

- *Fiber-To-The-Home (FTTH)*, where the fiber is brought directly to the final user;
- *Fiber-To-The-Cabinet (FFTC)*, where the fiber is brought to a street cabinet close to the user and then the user is connected to the fiber termination point through a copper-based connection;

☆ This paper is an extended, improved version of the paper "An (MI)LP-Based Primal Heuristic for 3-Architecture Connected Facility Location in Urban Access Network Design" presented at EvoComNet2016 and published in: Applications of Evolutionary Computation, 19th European Conference, EvoApplications 2016, Porto, Portugal, March 30–April 1, 2016, Proceedings, Part I, LNCS 9597, pp. 283–289 Springer, 2016 (see [27]).

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http://dx.doi.org/10.1016/j.asoc.2017.07.018
1568-4946/

- *Fiber-To-The-Building* (FTTB), where the fiber is brought to the building of the user and then the user is connected to the fiber termination point through a copper-based connection.

We refer the reader to [39] for an exhaustive introduction to FTTx network design and to [49,56,62] for a thorough discussion about realistic aspects of FTTH network design. Until a few years ago, a pure FTTH network appeared to be the best technological solution to provide very fast connections to all users and thus the problem of designing FTTH networks has attracted a lot of attention both in terms of models and solution approaches (e.g., [2,5,44,55,63]). Detailed optimization models used in studies evaluating the deployment of FTTH networks in Turkish and French cities are presented respectively in [60] and [4] (the latter paper also introduces several families of valid inequalities for the mathematical formulation of the considered network design problem). The question of the impact of traffic uncertainty on the design has been addressed in [42]. In [47], a Benders Decomposition algorithm has been developed for the problem of designing passive optical networks, which presents strong similarities with the FTTH network design problem. Finally, it is also interesting to note that the problem of designing FTTH networks has relevant connections with the design of last generation wireless networks (e.g., [16]) and smart grids (e.g., [59]).

Nowadays, an access network fully based on an FTTH architecture is not attractive because it entails extremely high deployment costs and not all users are willing to pay higher fees for faster connections. Furthermore, using a single architecture is limiting. Consequently, deployments that mix *two architectures*, like FTTH and FTTC/FTTB, have received higher attention (e.g., [45,57]). The integration of wired and wireless connections is even more promising: connectivity to the users is not just granted by wired connections, but it is also provided through wireless links, leading to *3-architecture* networks including also the so-called *Fiber-To-The-Air* (FTTA) architecture [37,39]. This integration aims to get the best of both wired and wireless worlds: the high capacity offered by optical fiber networks and the mobility and ubiquity offered by wireless networks (see [48,37] for a discussion). Furthermore, the integration can grant a determinant cost advantage, since deploying wireless transmitters eliminates the time-consuming and costly trenching effort associated with lying optical fibers.

This paper presents a robust optimization model for the design of 3-architecture telecommunications access networks integrating wireless and wired connections. A distinctive feature of our model with respect to models proposed in literature on the topic (see [45] and [39] for an overview) is to include the *signal-to-interference ratios*, namely those mathematical expressions that should be used for assessing wireless signal coverage [1,58]. The inclusion of such expressions is critical in any wireless network design problem considering wireless signal coverage, since the exclusion may lead to wrong design decisions – see [19,21] and [43] for a detailed discussion on this issue. The resulting optimization problem is very difficult to solve even for a state-of-the-art commercial MIP solver like IBM ILOG CPLEX [41].

In this work, our main original contributions are in particular:

1. we propose the first optimization model for the problem of optimally designing a 3-architecture access network, explicitly modelling the *signal-to-interference formulas* that express wireless signal coverage. Specifically, we trace back the design problem to a 3-architecture variant of the *Connected Facility Location Problem* that includes additional variables and constraints for modelling the service coverage of the wireless architecture;
2. in order to strengthen the mathematical formulation of the problem, we propose to include two families of valid inequalities that model conflicts between variables representing the activation of wireless transmitters and the assignment of users to the transmitters;
3. since the coefficients expressing the attenuation of wireless signals in a real environment are naturally subject to uncertainty, we propose a Robust Optimization model based on Γ-Robustness to tackle such uncertainty and obtain design solutions that are protected against deviations in the wireless input data;
4. we propose a primal heuristic for solving the (robust) design problem. The heuristic is based on the combination of a probabilistic variable fixing procedure, guided by suitable *Linear Programming* (LP) relaxations of the problem, with an exact *Mixed Integer Programming* (MIP) heuristic, which provides for executing a *very large neighborhood search* formulated as a *Mixed Integer Linear Program* (MILP) and solved exactly by a state-of-the-art MIP solver;

The new algorithm developed in this study is successfully tested on a set of realistic network instances, showing that our heuristic is able to produce solutions of much higher quality than those returned by a state-of-the-art MILP commercial solver.

The remainder of this paper is organized as follows. Section 2 reviews the 2-architecture Connected Facility Location Problem. Section 3 presents the new formulation for the 3-architecture network design. Section 4 describes the robust formulation for the 3-architecture network design, taking into account signal propagation uncertainty. Section 5 describes the new heuristic algorithm developed in this study. Section 6 discusses computational results. Finally, Section 7 summarizes the main findings of this study and outlines directions of future research.

2. 2-Architecture connected facility location

As first step towards the definition of our new optimization model considering 3 architectures, we consider a generalization of the classical *Connected Facility Location Problem* (ConFL) that includes 2 architectures. The ConFL problem arises in many applications related to the design and management of (telecommunications) networks. For an exhaustive introduction to fundamentals concepts of the theory of network flows on graphs and of the ConFL that we will use throughout the paper, we refer the reader to the books by Ahuja et al. [3] and by Bertsekas [8], and to the paper by Gollowitzer and Ljubic [38]. An essential description of the ConFL is the following: given a set of facilities that can be opened to serve a set of users, the aim is to decide (1) which facilities are opened, (2) how to assign users to open facilities, (3) how to connect open facilities through a Steiner tree, in order to minimize the total cost associated with opening and connecting facilities and with assigning facilities to users. The ConFL is known to be an NP-Hard problem [40]. A hop-constrained version of ConFL that is related to the design of single-architecture access network has been studied in Ljubic and Gollowitzer [46].

The canonical version of the ConFL takes into account *one single* architecture and, in the case of telecommunication access networks, can be associated with the design of networks using one single *technology*. This single technology usually corresponds to a given type of wired connections (e.g., either optical fibers or copper cables). However, since using a single technology may be limitative (see the Introduction), there is an increasing interest in deploying networks that integrate two architectures, mixing optical fiber and copper connection technologies. Considering these 2-architecture networks leads to the definition of a generalization of the ConFL that has been first modelled in [45] and that we denote by 2-ConFL. In the remainder of this section, we define an optimization model for the 2-ConFL, which constitutes the basis for defining our new model considering a further 3-architecture generalization of the ConFL that also includes wireless technology.

The 2-ConFL associated with access network design can be essentially described as follows: (i) we are given a set of potential facilities and a set of potential users; (ii) each facility can install one among two available technologies to provide a telecommunication service to the users; (iii) each open facility must be connected to a central office and each served user must be assigned to exactly one open facility; (iv) the objective is to ensure service coverage to a minimum number of users through each type of technology, while minimizing the total cost of deployment of the network.

In order to formally model the 2-ConFL as a mathematical optimization problem, we rely on a representation of the network by means of a directed graph $G(V, A)$ where:

1. the set of nodes V is the (disjoint) union of:
   (a) a set of users U – each user $u \in U$ is associated with a weight $w_u \geq 0$ representing its importance;
   (b) a set of facilities F – each facility $f \in F$ is associated with an opening cost $c_f^t \geq 0$ that depends upon the technology $t \in T$ used by f;
   (c) a set of central offices O – each office $o \in O$ is associated with an opening cost $c_o \geq 0$;
   (d) a set of Steiner nodes S.
   We call *core nodes* the subset of nodes $V^C = F \cup O \cup S$ (i.e., the subset of nodes that does not include the user nodes). Furthermore, we denote by F_u^t the subset of facilities using technology t that may serve user u and by U_f^t the subset of users that may be served by facility f when using technology t;
2. the set of arcs A is the (disjoint) union of:
   (a) a set of *core arcs* $A^C = \{(i, j) : i, j \in V^C\}$ that represent connections only between core nodes. Each arc (i, j) is associated with a cost of realization $c_{ij} \geq 0$;
   (b) a set of *assignment arcs* $A^{ASS} = \{(f, u) \in A : u \in U, f \in F_u\}$ representing connection of facilities to users. Each arc (f, u) is associated with a cost of realization c_{fu}^t that depends upon the used technology.

An exemplary visualization of the elements introduced above is given in Fig. 1.

We call *core network* that part of the network that represents the potential topology of the optical fiber deployment and that has the *core nodes* as set of nodes and the *core arcs* as set of arcs. Additionally, we call *core graph* the subgraph $G^C(V^C, A^C)$ of $G(V, A)$ corresponding to the core network.

In order to consider the cost of opening central offices, we rely on a common modelling trick that consists in adding an artificial root node r to $G(V, A)$. Such root node is then connected to every central office $o \in O$ through an arc (r, o) associated with a cost c_{ro} that equals the cost c_o of opening o. As a consequence, we also add a set of (artificial) arcs $A^R = \{(r, o) : o \in O\}$ to $G(V, A)$. In the remainder of the paper, we will denote by $A^{R\text{-}C}$ the union of the root and the core arcs (i.e, $A^{R\text{-}C} = A^R \cup A^C$).

The deployment of an access network entails a total cost that is equal to the sum of the cost of opening central offices and facilities, the cost of establishing connections in the core graph and the cost of connecting open facilities to served users.

Concerning the coverage of users, the deployment of an access network typically requires to provide a minimum weighted coverage for each architecture (e.g., [45]). If we denote by $W = \sum_{u \in U} w_u$ the sum of all user weights, we can express the coverage requirements for each architecture corresponding to a technology $t \in T$ by defining thresholds $W_t \in [0, W]$ for each $t \in T$. If we conventionally assume that $t = 1$ is a more costly and performing technology than $t = 2$ (for example, $t = 1$ could be an optical fiber-based technology whereas $t = 1$ could be copper-based), similarly to [45], we can assume that $W_1 \leq W_2$. In other words, the coverage requirement of the upper-class technology $t = 1$ is not higher than that of the lower-class technology $t = 2$. This assumption can be based on the realistic consideration that just a (minor) subset of users is willing to face higher prices for obtaining a telecommunication service of (much) higher quality.

Using all the elements and the notation introduced above, we can eventually present a *mixed integer linear program* modeling the 2-ConFL. First, we need to introduce the following families of decision variables:

1. facility opening variables $z_f^t \in \{0, 1\}\ \forall f \in F, t \in T$ such that:

$$z_f^t = \begin{cases} 1 & \text{if facility } f \text{ is open and uses technology } t \\ 0 & \text{otherwise} \end{cases}$$

2. arc installation variables $x_{ij} \in \{0, 1\}\ \forall (i, j) \in A^{R\text{-}C}$ such that:

$$x_{ij} = \begin{cases} 1 & \text{if the root arc or core arc } (i, j) \text{ is installed} \\ 0 & \text{otherwise} \end{cases}$$

3. assignment arc variables $y_{fu}^t \in \{0, 1\}\ \forall u \in U,\ t \in T, f \in F_u^t$ such that:

$$y_{fu}^t = \begin{cases} 1 & \text{if facility } f \text{ is connected to user } u \text{ by technology } t \\ 0 & \text{otherwise} \end{cases}$$

4. user variables $v_u^t \in \{0, 1\}, \forall u \in U, t \in T$ such that:

$$v_u^t = \begin{cases} 1 & \text{if user } u \text{ is served by technology } t \\ 0 & \text{otherwise} \end{cases}$$

5. flow variables $\phi_{ij}^f, \forall (i, j) \in A^{R\text{-}C}, f \in F$ representing the amount of flow sent on a root or core arc (i,j) for facility f.

The Mixed Integer Linear Program for 2-ConFL (2-ConFL-MILP) is then stated as:

$$\min \sum_{(i,j) \in A^{R\text{-}C}} c_{ij} x_{ij} + \sum_{f \in F} \sum_{t \in T} c_f^t z_f^t + \sum_{u \in U} \sum_{t \in T} \sum_{f \in F_u^t} c_{fu}^t y_{fu}^t \qquad (2-\text{ConFL-MILP})$$

$$\sum_{t \in T} z_f^t \leq 1 \qquad f \in F \tag{1}$$

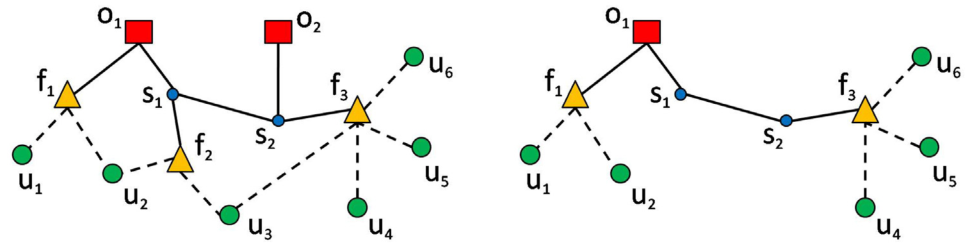
$$\sum_{f \in F_u^t} y_{fu}^t = v_u^t \qquad u \in U, t \in T \tag{2}$$

$$y_{fu}^t \leq z_f^t \qquad u \in U, f \in F, t \in T \tag{3}$$

$$\sum_{u \in U} \sum_{\tau=1}^{t} w_u v_u^t \geq W_t \qquad t \in T \tag{4}$$

$$\sum_{(j,i) \in A^{R\text{-}C}} \phi_{ji}^f - \sum_{(i,j) \in A^{R\text{-}C}} \phi_{ij}^f = \begin{cases} -\sum_{t \in T} z_f^t & \text{if } i = r \\ 0 & \text{if } i \neq r, f \\ +\sum_{t \in T} z_f^t & \text{if } i = f \end{cases} \qquad i \in V^C \cup \{r\}, f \in F \tag{5}$$

$$0 \leq \phi_{ij}^f \leq x_{ij} \qquad (i, j) \in A^{R\text{-}C}, f \in F \tag{6}$$



(a) Potential topology (b) Deployed network

Fig. 1. Example of graphs modelling a very simple FTTx network. The potential topology includes 2 central offices $O = \{o_1, o_2\}$, 2 steiner nodes $S = \{s_1, s_2\}$, 3 facilities $F = \{f_1, f_2, f_3\}$ and 6 users $U = \{u_1, u_2, u_3, u_4, u_5, u_6\}$. Core arcs are represented through continue connecting lines, whereas assignment arcs are represented through dotted lines. In the deployed network, only the facilities f_1, f_3 are activated and are connected to the central office o_1, using both the steiner nodes; also, only 5 among 6 users are served by connecting them to the activated facilities (u_1 and u_2 are served by f_1, whereas u_4, u_5, u_6 are served by f_3 - user u_3 is not served and its corresponding node is deleted from the graph, since it is not connected to any facility).

$$
\begin{array}{lll}
x_{ij} \in \{0, 1\} & & (i, j) \in A^{\text{R-C}} \\
z_f^t \in \{0, 1\} & & f \in F, t \in T \\
y_{fu}^t \in \{0, 1\} & & u \in U, t \in T, f \in F_u^t \\
v_u^t \in \{0, 1\} & & u \in U, t \in T
\end{array}
$$

The objective of the problem is modelled through a function that pursues the minimization of the total cost, expressed by means of a summation that includes the costs of activating root and core arcs, central offices, opened facilities and activated assignment arcs. The constraints (1) express that each facility must be opened using a single technology. The constraints (2) express that if a user u is served through technology t, then exactly one of the assignment arcs associated with a facility that can serve u is activated on technology t. The constraints (3) establish a link between the opening of a facility f on technology t and the activation of assignment arcs involving f and t: users can be assigned to a facility through some technology only when the facility is opened and uses that technology. The coverage requirement for each user technology are expressed by the constraints (4) (we remark that, in such constraints, the weighted sum of users that access to the service through the "better" technology $t = 1$ also contributes to satisfy the coverage requirement of the "worse" technology $t = 2$). The fiber connectivity between offices and facilities of the core network is jointly modelled by the constraints (5) and (6): the constraints (5) model the flow conservation rule in root and core nodes, whereas the constraints (6) are variable upper bound constraints that link the activation of a root or core arc with the activation of the arc.

An important difference of the present formulation and that proposed in Leitner et al. [45] is that we model connectivity within the core network through a multicommodity flow formulation whose size is polynomial in the size of the problem input (i.e., our formulation is *compact*). Conversely, the formulation of Leitner et al. [45] models connectivity by cut-set inequalities and therefore its size is potentially exponential in the size of the problem input. We made this choice since a compact formulation is more suitable to the needs of our new heuristic, where we do not want to execute additional time consuming routines for separating valid inequalities.

3. 3-Architecture connected facility location

In this section, we derive our new and original generalization of the 2-ConFL, which also includes the *FTTA architecture* and thus also considers the wireless technology. A distinctive feature of our new model is to include the mathematical formulas that are commonly used for evaluating the wireless coverage of a user.

To take into account also the wireless technology, we add a further element $t = 3$, representing wireless, to the set of available technologies. We therefore have $T := T \cup \{3\}$. From now on, we thus assume that in each facility $f \in F$ we can also install a *wireless transmitter*, which is able to guarantee service coverage to a subset of users *without cables*.

Wireless transmitters are characterized by several radio-electrical parameters: just to make a few major examples, we can consider the power emission, the frequency that is used to transmit, the modulation and coding scheme adopted to encode the information sent over the wireless link, the tilt of the antennas – see [19,43,58] for a more detailed overview. In principle, the value of all these parameters could be established in an optimal way by solving a properly defined (very large-scale) optimization problem. However, in practice, only a limited number of there parameters are subject to optimization in a wireless network design problem, as discussed in [19] and [43].

A very crucial decision which is included in practically all wireless network design problems is establishing the power emission of each transmitter: a good setting of this parameter is indeed critical to guarantee a good service coverage of the users (see e.g., [19–21,26,32,43,51,52]). The problem of establishing the power emission in an optimal way is commonly known by the name of *Power Assignment Problem* and can be seen as a basic problem in a hierarchy of wireless network design problems (see [19,52]). We can model the power emission of a wireless facility $f \in F$ by introducing a semi-continuous power variable $p_f \in [P_{\min}, P_{\max}]\ \forall f \in F$, representing that a facility is either turned off or emits with a power lying in the range $[P_{\min}, P_{\max}]$, which depends upon the technical features of the transmitter. A user u picks up wireless signals from all the wireless facilities in the network and the power $P_f(u)$ that u gets from f can be expressed as the product of the power emitted by f and a coefficient $a_{fu} \in [0, 1]$, i.e. $P_f(u) = a_{fu} \cdot p_f$. The factor a_{fu} is called *fading coefficient* and synthesizes the decrease in power that a signal propagating from f to u experiences [58]. A user $u \in U$ is said *covered* or *served* when it receives the signal providing the wireless telecommunications service within a minimum level of quality. The service is typically provided by one single transmitter, which constitutes the *server* of the user. All the remaining transmitters are interferers and contribute to deteriorate the quality of the signal received from the server and thus the quality of the service. Assessing whether a user receives a signal of sufficiently high quality from the server is done through the *Signal-to-Interference Ratio (SIR)*, a measure that compares the power received from the server with the sum of the power received by the interfering transmitters [58]:

$$\frac{a_{fu}p_f}{N + \sum_{k \in F \setminus \{f\}} a_{ku}p_k} \geq \delta. \tag{7}$$

The user is served when the SIR is not lower than a threshold $\delta > 0$, which depends upon the wanted quality of service. We note that, in the denominator of (7), a constant $N > 0$ is included to represent the noise of the system, a (very) weak but unavoidable source of interference. Through straightforward linear algebra operations, inequality (7) can be reorganized in the so-called *SIR inequality*:

$$a_{fu}p_f - \delta \sum_{k \in F \setminus \{f\}} a_{ku}p_k \geq \delta N.$$

It is now important to highlight that we do not know a priori which wireless facility $f \in F$ will be the server of user $u \in U$, since identifying the server f for u is a decision that we must take in the optimization problem. As a consequence, for each user $u \in U$ we must introduce one SIR inequality for each potential server $f \in F$ and such inequalities must be activated or deactivated depending upon the facility-user assignment.

User u is served through a wireless connection when one of the SIR inequalities defined for every of its possible servers is satisfied. Therefore, we are actually facing a disjunctive constraint, which, following a standard approach in Mixed Integer Programming (see [26,54] and the discussion specific for SIR inequalities in [22]), can be represented in a linear way through a modified version of the SIR inequality. Specifically, we add the product of a sufficiently large positive constant M (commonly called *big-M coefficient*) with the assignment variable y^3_{fu} (representing the assignment of user u to facility f using the wireless technology $t = 3$), obtaining what we call a *SIR constraint*:

$$a_{fu}p_f - \delta \sum_{k \in F \setminus \{f\}} a_{ku}p_k + M(1 - y^3_{fu}) \geq \delta N. \tag{8}$$

It is easy to check that if $y^3_{fu} = 1$, then u is served by f through a wireless connection and (8) reduces to a SIR inequality to be satisfied. On the contrary, if $y^3_{fu} = 0$, then u is not served by f and M comes into play, making the SIR constraint (8) satisfied by any power emission vector p and thus redundant for the problem. The MILP formulation for the 3-ConFL that we obtain is:

$$\min \sum_{(i,j) \in A^{\text{R-C}}} c_{ij}x_{ij} + \sum_{f \in F}\sum_{t \in T} c^t_f z^t_f + \sum_{u \in U}\sum_{t \in T}\sum_{f \in F^t_u} c^t_{fu} y^t_{fu} \qquad \text{(3-ConFL-MILP)}$$

$$\sum_{t \in T} z^t_f \leq 1 \qquad f \in F$$

$$\sum_{f \in F^t_u} y^t_{fu} = v^t_u \qquad u \in U, t \in T$$

$$y^t_{fu} \leq z^t_f \qquad u \in U, f \in F, t \in T$$

$$\sum_{u \in U}\sum_{\tau=1}^{t} w_u v^t_u \geq W_t \qquad t \in T \tag{9}$$

$$\sum_{(j,i) \in A^{\text{R-C}}} \phi^f_{ji} - \sum_{(i,j) \in A^{\text{R-C}}} \phi^f_{ij} = \begin{cases} -\sum_{t \in T} z^t_f & \text{if } i = r \\ 0 & \text{if } i \neq r, f \\ +\sum_{t \in T} z^t_f & \text{if } i = f \end{cases} \qquad i \in V^C \cup \{r\}, f \in F$$

$$0 \leq \phi^f_{ij} \leq x_{ij} \qquad (i,j) \in A^{\text{R-C}}, f \in F$$

$$a_{fu}p_f - \delta \sum_{k \in F \setminus \{f\}} a_{ku}p_k + M(1 - y^3_{fu}) \geq \delta N \qquad f \in F, u \in U$$

$$0 \leq P^{\min} z^3_f \leq p_f \leq P^{\max} z^3_f \qquad f \in F \tag{10}$$

$$x_{ij} \in \{0, 1\} \qquad (i,j) \in A^{\text{R-C}}$$

$$z^t_f \in \{0, 1\} \qquad f \in F, t \in T$$

$$y^t_{fu} \in \{0, 1\} \qquad u \in U, t \in T, f \in F^t_u$$

$$v^t_u \in \{0, 1\} \qquad u \in U, t \in T$$

Comparing 2-ConFL-MILP and 3-ConFL-MILP, we can notice that the modifications that we made consist in: (1) introducing the SIR constraints (9) that model the wireless coverage conditions; (2) introducing the variable bound constraints (10) that model the semi-continuous variables p_f. Concerning these latter constraints, we observe that if $z^3_f = 0$, facility f does not install a wireless transmitter and the power p_f is forced to be 0; if instead $z^3_f = 1$, then the facility hosts an operative wireless transmitter and its power must belong to the range $[P^{\min}, P^{\max}]$.

3.1. Strengthening the 3-ConFL-MILP

The new optimization algorithm developed in this study to quickly solve the 3-ConFL-MILP is based on a probabilistic variable fixing procedure that combines an a-priori and an a-posteriori measure of fixing attractiveness based on linear relaxations of the 3-ConFL-MILP. Specifically, we derive the a-priori measure by solving a *tighter formulation* (informally speaking, a problem presenting a "mathematically stronger" structure) of the 3-ConFL-MILP obtained by adding two class of valid inequalities: 1) superinterferer inequalities; 2) conflict inequalities. These two families of valid inequalities have been introduced in [19] and [25] and we refer the reader to these two papers for a detailed description. Here, we provide a concise description of their main features.

The first class of inequalities captures the existence of so-called *superinterferers*: a superinterferer is a particularly strong interfering transmitter that, alone, is able to deny service coverage to a user even when it emits at minimum power $P^{\min}$ and the serving transmitter emits at maximum power $P^{\max}$. More formally, if facility $k \in K \setminus \{f\}$ is a superinterferer for user u served by facility f then:

$$a_{fu}P^{\max} - \delta a_{ku}P^{\min} < \delta N$$

If such condition holds, then the following logical constraint is a valid inequality for 3-ConFL-MILP:

$$y^3_{fu} \leq 1 - z^3_k \tag{11}$$

and expresses the fact that if the superinterfer k is activated, then the variable assigning user u to f installing wireless technology is forced to 0, since the corresponding SIR constraint cannot be satisfied.

More generally, given a user-facility couple (f, u), if we denote by K the subset of facilities that are superinterferers for u served by f, then we can add to the problem the following valid inequalities:

$$y^3_{fu} \leq 1 - z^3_k \; \forall k \in K \tag{12}$$

and in the SIR constraint corresponding to the couple (f, u), we can eliminate the superinterferers in K from the summation over the interfering facilities [19], i.e.:

$$a_{fu}p_f - \delta \sum_{k \in F \setminus (\{f\} \cup K)} a_{ku}p_k + M(1 - y^3_{fu}) \geq \delta N$$

The second class of valid inequalities captures the existence of couples of SIR constraints that involve just two wireless facilities and that cannot be satisfied at the same time. More formally, consider the two SIR constraints corresponding to two users u_1, u_2 served by two distinct wireless facilities f_1, f_2, namely:

$$a_{f_1 u_1} p_{f_1} - \delta a_{f_2 u_1} p_{f_2} \geq \delta N \quad (13)$$

$$a_{f_2 u_2} p_{f_2} - \delta a_{f_1 u_2} p_{f_1} \geq \delta N, \quad (14)$$

respectively representing u_1 served by f_1 and interfered by f_2 and u_2 served by f_2 and interfered by f_1. If there is no power vector (p_1, p_2) that satisfies the power bounds (10) and the two SIR constraints ((13)–(14)), then the following is a *valid inequality* for 3-ConFL-MILP stating that both SIR constraints cannot be activated simultaneously:

$$y^3_{f_1 u_1} + y^3_{f_2 u_2} \leq 1 \quad (15)$$

Such valid inequalities can be easily identified in a pre-processing phase and can be added to the formulation to get remarkable strengthening (see [19,25]). In the next section, we denote by *Strong-3-ConFL-MILP*, the problem 3-ConFL-MILP suitably strengthened by inequalities (12) and (15). We remark that this is just one of the possible way of strengthening 3-ConFL-MILP and that in another short conference paper [28], we have started to investigate the possibility of using an alternative strengthening approach that exploits power discretization, according to the Power-Indexed paradigm introduced in [26].

4. Robust optimization for 3-architecture connected facility location

After having introduced the 3-architecture problem 3-ConFL-MILP, we derive its robust version that takes into account the uncertainty of wireless signal propagation. In order to do this, we first define signal propagation uncertainty, then we introduce fundamental principles of Robust Optimization, the methodology that we use for dealing with data uncertainty in our problem. Finally, we present a Robust Optimization model for the wireless propagation-uncertain version of 3-ConFL-MILP.

4.1. Wireless propagation uncertainty and robust optimization

The fading coefficients a_{fu} appearing in the signal-to-interference ratios (7) and in the wireless coverage constraints (8) depend upon many factors of different nature (e.g., the distance between the transmitter and the receiver, the presence of obstacles, the terrain features and even the weather – see [58] for a thorough discussion). Giving a high precision estimation of such coefficients is impractical and may result very costly and time consuming. The coefficients are thus canonically derived by *propagation models*, namely mathematical formulas that express signal attenuation, mainly as a function of the distance between the transmitter and the receiver, the frequency used to transmit and the general features of the territory where the propagation takes place (urban, suburban, rural, etc.) [58]. The propagation models offer estimations of the actual values of the fading coefficients that are of good quality, but in general (even deeply) different from the actual values. Hence, in real-world wireless network design problems, it is natural to assume that fading coefficients are *uncertain data*, i.e. their value is not exactly known when the design problem is solved (see e.g., [14,19,21]).

The presence of uncertain data in an optimization problem requires to act cautiously: as well-known from sensitivity analysis, even small deviations in the value of input data may turn an optimal solution into a solution of bad quality, whereas a feasible solution may instead reveal to be infeasible and thus completely meaningless in practice. For such reasons, robustness against deviations in the input data and against failures has become a major topic in (telecommunications) network design (see, e.g., [6,15,30,36,53]). We refer the reader to [7,9] for a detailed discussion on the consequences of data uncertainty in mathematical optimization.

For the wireless coverage problem considered in this study, the network should not be designed referring to nominal fading coefficients returned, for example, by a propagation model. It is instead rational to consider that the actual value may not correspond to the nominal value. Indeed, neglecting deviations of fading coefficients may lead to obtain solutions that do not achieve expected wireless coverage of users. In particular, users supposed to be served could reveal to be not served, because of (bad) deviations of the nominal fading coefficients that lead to a violation of the corresponding SIR constraints.

Since the seminal work of Dantzig [31], many methodologies such as *Stochastic Programming* and *Robust Optimization* were proposed over the years to deal with data uncertainty in mathematical optimization. We refer the reader to [7,9] for an overview of the most successful methodologies. In this paper, we adopt Robust Optimization (RO), a methodology that, thanks especially to its accessibility and computational tractability, has received very high attention in the last decade (see [7,9] for a detailed introduction to theory and applications of RO). RO is essentially based on the following principles:

- the decision maker does not know the *actual value* of an uncertain coefficient when the problem is solved. However, he possesses a good estimation of it, in the form of a *nominal value*. The actual value is then equal to the summation of the nominal value and an unknown deviation;
- the decision maker defines an uncertainty set, which identifies the deviations with respect to nominal values, against which the decision maker wants to be protected;
- the decision maker solves a so-called *robust counterpart* of its optimization problem, namely a modified version of the problem that only considers robust feasible solutions, i.e. feasible solutions protected against all deviations specified by the uncertainty set. An optimal robust solution grants the best objective value under the worst data deviations.

In RO, protection against the deviations of the uncertainty set is guaranteed at a price: the so-called *price of robustness* [10]. This is a worsening in the optimal value of the problem, due to ensuring protection against worst data realizations under the form of hard constraints that reduce the set of feasible solutions maintaining only robust solutions. In general, uncertainty sets guaranteeing more protection lead to a higher price of robustness, since the hard constraints imposing robustness are stricter and define a more constrained version of the problem, associated with smaller feasible sets.

We can express more formally the essential principles of RO by referring to a generic mixed-integer linear program:

$$v = \min c^T x \text{ with } x \in \mathcal{F} = \{Ax \geq b, x \in \mathbb{R}^n \times \mathbb{Z}^q\}$$

in which the coefficient matrix A is uncertain. We can then identify a family $\mathcal{A}$ of coefficient matrices that represent possible valorizations of the uncertain matrix A. This family corresponds to the uncertainty set of the robust problem. A robust optimal solution is an optimal solution of the following *robust counterpart* of the original problem:

$$v^{\text{ROB}} = \min c^T x \text{ with } x \in \mathcal{F}^{\text{ROB}} = \{\tilde{A}x \geq b \, \forall \tilde{A} \in \mathcal{A}, x \in \mathbb{R}^n \times \mathbb{Z}^q\}$$

which considers robust feasible solutions, i.e. solutions that satisfy the constraints for all the realizations of the coefficient matrix A belonging to $\mathcal{A}$.

This is a very general definition of robust counterpart and we must now introduce a detailed description of how we model data uncertainty and structure the uncertainty set. Specifically, we model the uncertainty of the fading coefficients through the famous Γ-Robustness model (Γ-ROB) by Bertsimas and Sim [10], which is based on a *cardinality-constrained* uncertainty set combined with an *interval deviation model*.

In Γ-ROB, a generic uncertain coefficient a_j is assumed to vary in a symmetric interval $[\bar{a}_j - d_j, \bar{a}_j + d_j]$, where $\bar{a}_j$ and d_j are respectively the nominal value and the maximum deviation of the uncertain coefficient specified by the uncertainty set. Additionally, given an uncertain constraint of the problem including a number K of uncertain coefficients of which we know the nominal value and the maximum deviation with respect to the nominal value, Γ-ROB defines a robust counterpart for an uncertainty set that guarantees protection against at most $0 \leq \Gamma \leq K$ coefficients deviating from their nominal value. Remarkably, the Γ-ROB counterpart maintains the nature of the original uncertain problem for a vast class of optimization problems (e.g., an uncertain mixed integer linear problem has a mixed integer linear counterpart). The parameter Γ is used to control the conservatism and robustness of solutions: when $\Gamma = 0$, there is no protection and the price of robustness is zero; when Γ increases, the protection and the price of robustness increase, until reaching the highest protection for $\Gamma = K$.

We now proceed to define precisely the robust counterpart of an uncertain SIR constraint under Γ-Robustness

4.2. Γ-Robust SIR constraints

Following the interval deviation model provided by Γ-ROB, we assume that the decision maker exactly knows a *nominal value* $\bar{a}_{fu}$ of each fading coefficient and a *maximum deviation* $d_{fu} \geq 0$ from it. The (unknown) *actual value* of a fading coefficient a_{fu} then belongs to the interval:

$$a_{fu} \in [\bar{a}_{fu} - d_{fu}, \bar{a}_{fu} + d_{fu}] \quad ,$$

which is centered on $\bar{a}_{fu}$. As an example, the nominal value could be the value returned by the propagation model, while the maximum deviation could be set on the basis of data derived from field measurements.

For each facility $f \in F$ and user $u \in U$, we can write the uncertain version of a SIR constraint taking into account fading deviations as:

$$a_{fu} p_f - \delta \sum_{k \in F \setminus \{f\}} a_{ku} p_k + M(1 - y^3_{fu}) - DEV_{fu}(\Gamma, p) \geq \delta N \quad , \qquad (16)$$

which is the SIR constraint (8) with the additional term $-DEV_{fu}(\Gamma, p)$, which represents the worst deviation that the left-hand-side of the constraint may experience under Γ-ROB for a power vector p, when at most Γ coefficients deviate from their nominal value $\bar{a}_{fu}$.

Before giving a precise characterization of DEV_{fu} as the optimal value of a suitable optimization problem, we notice that to identify the worst deviation, we must make a distinction between the serving wireless facilities f of u and all other interfering facilities $k \in F \setminus \{f\}$. Indeed, a pejorative deviation for the serving facility occurs when the corresponding fading coefficient decreases (indeed, the serving power decreases), while for an interfering facility a pejorative deviation occurs when the corresponding fading coefficient increases (indeed, the interfering power increases). Given a power vector p specifying the power p_f emitted by each facility $f \in F$, for the serving facility the worst deviation is then:

$$-d_{fu} p_f$$

while for an interfering facility $k \in F \setminus \{f\}$ is: then:

$$-\delta d_{ku} p_k$$

Both deviations correspond with a reduction in the value of the left-hand-side of a SIR constraint. In order to simplify notation, for a SIR constraint defined for the serving facility f and the user u, we can introduce the modified deviation coefficients $\bar{d}_{\ell u}$ for each facility $\ell \in F$:

$$\bar{d}_{\ell u} = \begin{cases} d_{\ell u} & \text{if } \ell = f \\ \delta d_{\ell u} & \text{if } \ell \neq f \text{ (i.e., } \ell \in F \setminus \{f\}) \end{cases}$$

Under these premises, the value $DEV_{fu}(\Gamma_{fu}, p)$ corresponds to the optimal value of the following 0–1 linear program:

$$\begin{aligned} DEV_{fu}(\Gamma_{fu}, p) = \max \quad & \sum_{\ell \in F} (d_{\ell u} p_\ell) \lambda_{fu\ell} \\ & \sum_{\ell \in F} \lambda_{fu\ell} \leq \Gamma_{fu} \\ & \lambda_{fu\ell} \in \{0, 1\} \; \ell \in F. \end{aligned}$$

In this problem, (1) a binary variable $\lambda_{fu\ell}$ is equal to 1 if, in the SIR constraint (f, u), the fading coefficient of the wireless facility ℓ deviates from its nominal value and experiences the worst deviation $d_{\ell u} p_\ell$, whereas it is equal to 0 otherwise; (2) the single constraint imposes an upper bound $0 \leq \Gamma_{fu}$ on the number of fading coefficients which may deviate in the considered constraint; (3) the objective function maximizes the deviation from the nominal value for a given power vector $p = (p_1, \ldots, p_{|F|})$.

The robust version of 3-ConFL-MILP including the terms $DEV_{fu}(\Gamma_{fu}, p)$ thus actually includes inner maximization problems which contain the products of variables $p_\ell \lambda_\ell$. However, as proved in [10], such non-linearities can be linearized according to the following procedure. First, we note that for a fixed power vector p, the value $DEV_{fu}(\Gamma_{fu}, p)$ is equal to the optimal value of its *linear relaxation*:

$$\begin{aligned} DEV_{fu}(\Gamma_{fu}, p) = \max \quad & \sum_{\ell \in F} (d_{\ell u} p_\ell) \lambda_{fu\ell} \qquad \text{(DEV-primal)} \\ & \sum_{\ell \in F} \lambda_{fu\ell} \leq \Gamma_{fu} \\ & 0 \leq \lambda_{fu\ell} \leq 1 \qquad \ell \in F. \end{aligned}$$

We can then define the *dual problem* of the previous linear program, i.e.:

$$\begin{aligned} \min \quad & \Gamma_{fu} \varphi_{fu} + \sum_{\ell \in F} \psi_{fu\ell} \qquad \text{(DEV-dual)} \\ & \varphi_{fu} + \psi_{fu\ell} \geq d_{\ell u} p_\ell \qquad \ell \in F \\ & \varphi_{fu} \geq 0 \\ & \psi_{fu\ell} \geq 0 \qquad \ell \in F. \end{aligned}$$

Since the problem DEV-primal is feasible and bounded, on the basis of strong duality we can conclude that also its dual problem DEV-dual is feasible and bounded and their optimal values are equal. We can then substitute each (non-linear) uncertain version (16) of a SIR constraint with the following family of linear constraints and decision variables:

$$\begin{aligned} & a_{fu} p_f - \delta \sum_{k \in F \setminus \{f\}} a_{ku} p_k + M(1 - y^3_{fu}) - \left(\Gamma_{fu} \varphi_{fu} + \sum_{\ell \in F} \psi_{fu\ell} \right) \geq \delta N \\ & \varphi_{fu} + \psi_{fu\ell} \geq d_{\ell u} p_\ell \qquad \ell \in F \\ & \varphi_{fu} \geq 0 \\ & \psi_{fu\ell} \geq 0 \qquad \ell \in F. \end{aligned}$$

It should be noted that the increase in problem size, due to the presence of the additional variables and constraints used in the dualization approach, is "not big" (the new formulation is *compact*, i.e. its size is polynomial in the size of the input).

The deterministic version 3-ConFL-MILP of the design problem can then be replaced by the following linear and compact robust counterpart, which we denote by ROB-3-ConFL-MILP, taking account of the uncertainty of fading coefficients through a Γ-ROB uncertainty model.

$$\begin{aligned}
\min \quad & \sum_{(i,j)\in A^{\text{R-C}}} c_{ij}x_{ij} + \sum_{f\in F}\sum_{t\in T} c_f^t z_f^t + \sum_{u\in U}\sum_{t\in T}\sum_{f\in F_u^t} c_{fu}^t y_{fu}^t && \text{(ROB-3-ConFL-MILP)} \\
& \sum_{t\in T} z_f^t \le 1 && f \in F \\
& \sum_{f\in F_u^t} y_{fu}^t = v_u^t && u\in U, t\in T \\
& y_{fu}^t \le z_f^t && u\in U, f\in F, t\in T \\
& \sum_{u\in U}\sum_{\tau=1}^{t} w_u v_u^t \ge W_t && t\in T
\end{aligned} \qquad (17)$$

Comparing 2-ConFL-MILP and 3-ConFL-MILP, we can notice that the modifications that we made consist in: 1) introducing the SIR constraints (9) that model the wireless coverage conditions; 2) introducing the variable bound constraints (10) that model the semi-continuous variables p_f. Concerning these latter constraints, we observe that if $z_f^3 = 0$, facility f does not install a wireless transmitter and the power p_f is forced to be 0; if instead $z_f^3 = 1$, then the facility hosts an operative wireless transmitter and its power must belong to the range $[P^{\min}, P^{\max}]$.

5. A primal heuristic algorithm for the 3-ConFL-MILP

The robust problem ROB-3-ConFL-MILP is a mixed integer linear program and in principle can be solved by using a commercial MILP solver, such as IBM ILOG CPLEX [41]. However, the presence of the SIR constraints (9) combined with the additional Γ-Robustness constraints makes ROB-3-ConFL-MILP a really challenging extension of the 2-ConFL problem that is very difficult to solve even using a modern state-of-the-art MILP solver like CPLEX. According to our direct experience on real-sized instances, CPLEX often experiences difficulties in finding good quality solutions even after hours of computations.

As an alternative to trying to solve ROB-3-ConFL-MILP by directly using a MILP solver, we thus propose a new algorithm combining:

- a *probabilistic fixing procedure*, guided through information provided by the solution of peculiar linear relaxations of 3-ConFL-MILP;
- an MIP heuristic, corresponding to an *exact very large neighborhood search*, namely a very large neighborhood search formulated as an MIP problem solved exactly through an MIP solver.

Our probabilistic fixing procedure is partially inspired by the algorithm ANTS (*Approximate Nondeterministic Tree Search*), proposed in [50] to refine the canonical version of an ant colony algorithm through the exploitation of information coming from bounds available for the optimization problem. In contrast to ANTS, the new heuristic does not use bounds that are specifically available for the considered problem, but is founded on using suitable (tight) linear relaxations and their corresponding optimal solutions, taking into account considerations that have been first made for multiperiod network design in [23,24] and then extended to other application contexts, such as the energy-efficient design of sensor networks (e.g., [29,30]). Since our procedure uses information from linear relaxations, in contrast to "simple" heuristics, we can also provide a certificate of quality for the best solution produced by our heuristic: the certificate is produced under the form of an *optimality gap*, which measures the distance of the best solution from the best lower bound given by Strong-3-ConFL-MILP or ROB-3-ConFL-MILP.

Considering similarity of ANTS and Ant Colony Optimization (ACO), it is useful to briefly recall here the major features of ACO. ACO is a metaheuristic algorithm mimicking foraging behavior of ants, introduced by Dorigo and colleagues in a series of papers (e.g., [33,34,50]). An exhaustive overview of ACO is given in [12,13,35].

The general structure of an ACO algorithm (ACO-alg) is presented in Algorithm 1. An ACO-alg is essentially based on executing a cycle where a number of feasible solutions are built in an iterative way, using information about the quality of solutions built in previous executions of the cycle.

Algorithm 1. General ACO Algorithm (ACO-alg)

```
1: while an arrest condition is not satisfied do
2:    ant-based solution construction
3:    pheromone trail update
4: end while
5: local search
```

The first step of the while-cycle of Algorithm 1 provides for defining a number of *ants* and each ant builds a feasible solution in an iterative way. At every iteration, the ant is in a *state* that actually corresponds with a *partial solution* to the problem. The ant can make a further step towards completing the partial solution by executing a *move*. The move corresponds to fixing the value of a variable whose value has not yet been fixed. The variable to fix is chosen probabilistically, evaluating a measure that combines two measures of fixing attractiveness, an *a-priori* and an *a-posteriori* measure. The a-priori attractiveness measure is commonly called *pheromone trail value* in an ACO-alg context and is updated at the end of each construction phase: the update aims at rewarding fixing that led to good quality solutions and at penalizing fixing that instead led to bad quality or infeasible solutions. The execution of the cycle is interrupted once that an arrest condition is satisfied (it is common to adopt a time limit) and then a local search is started to improve the best solution found by exploring some specified neighborhood.

It should be emphasized that the new algorithm developed here for the ROB-3-ConFL-MILP, though presenting similarities with an ACO-alg is actually *not* an ACO-alg, but is rather an evolution and refinement of the ANTS algorithm, which we strengthen by the use of suitable linear relaxations. Specifically, in our case, the a-priori measure is provided by the optimal value of the linear relaxation of the robust problem ROB-3-ConFL-MILP, whereas the a-posteriori measure is provided by the linear relaxation of the strengthened formulation Strong-3-ConFL-MILP including partial fixing of the facility opening variables. As discussed above, another important difference of our algorithm with respect to an ACO-alg is the possibility of deriving an optimality gap, thanks to the use of optimal solutions of linear relaxations.

The new algorithm developed in this study is now described in detail.

5.1. Feasible solution construction

In order to illustrate how we build feasible solutions for the ROB-3-ConFL-MILP problem, we first introduce the concept of *Facility Opening state*:

Definition 1. Facility opening state (FOS):

Let $F \times T$ be the set of couples (f, t) that represent the activation of a facility f on a technology t. An FOS specifies an opening of a subset of facilities $\bar{F} \subseteq F$ on some technologies and excludes that the same facility is opened on more than one technology (i.e., $FOS \subseteq F \times T : \nexists (f_1, t_1), (f_2, t_2) \in FOS : f_1 = f_2$ and $t_1 \neq t_2$).

Given a FOS and a facility-technology couple $(f, t) \in FOS$, we denote by W_{ft}^{POT} the total weight of users that can be potentially served by f activated on technology t, i.e. $W_{ft}^{\mathrm{POT}} = \sum_{u \in U_f^t} w_u$. We introduce this measure to distinguish between a partial and complete FOS for a technology $t \in T$. We say that a FOS is *partial for technology t* when the total weight of potential users that can be served by facilities appearing in the *FOS* using technology t does not reach the minimum coverage requirements W_t for t, i.e.:

$$\sum_{f \in F:(f,t) \in FOS} \sum_{u \in U_f^t} w_u < W_t. \tag{18}$$

On the contrary, we say that a FOS is *complete for technology t* when the total weight is not lower than W_t. Additionally, we call *fully complete* a FOS that is complete for all technologies $t \in T$, i.e. such that:

$$\sum_{f \in F:(f,t) \in FOS} \sum_{u \in U_f^t} w_u \geq W_t \ \forall \ \ t \in T.$$

In our algorithm, a complete FOS is used as basis to try to derive a feasible design solutions for the problem. We introduce the concept of completeness and the formula (18) in order to guide and limit the probabilistic fixing of facility opening variables during the construction phase of feasible solutions.

Given a *partial* FOS for technology t, the probability p_{ft}^{FOS} of operating an additional fixing $(f, t) \notin FOS$, thus making a further step towards reaching a complete FOS, is set according to the formula:

$$p_{ft}^{\mathrm{FOS}} = \frac{\alpha \tau_{ft} + (1-\alpha)\eta_{ft}}{\sum_{(k,t) \notin FOS} \alpha \tau_{kt} + (1-\alpha)\eta_{kt}} \quad , \tag{19}$$

which provides for a convex combination of the a-priori attractiveness measure τ_{ft} and the a-posteriori attractiveness measure η_{ft} through factor $\alpha \in [0, 1]$. In the present case, for each facility-technology couple $(f, t) \notin FOS$, τ_{ft} is equal to the optimal value of the linear relaxation of ROB-3-ConFL-MILP including the single additional fixing $z_f^t = 1$, while η_{kt} is the optimal value of the linear relaxation of Strong-3-ConFL-MILP obtained for the current partial FOS, which specifies a partial fixing of the facility opening variables z. Furthermore, the measures τ_{kt} and η_{kt} appearing in the summation of the denominator consider a couple (k, t) not included in the current partial FOS and respectively correspond to the optimal value of the linear relaxation of ROB-3-ConFL-MILP including the fixing $z_k^t = 1$ and to the optimal value of the linear relaxation of Strong-3-ConFL-MILP obtained for the current partial FOS. The way measures τ_{ft}, η_{ft}, τ_{kt} and η_{kt} are defined follows the principles of ANTS, according to which: (1) a linear relaxation that is "stronger" and provides a higher quality bound, but requires more time to be solved (in the present case, the robust formulation) should be used for initializing the a-priori measure of attractiveness τ_{ft}; (2) a "weaker" linear relaxation that can be however computed faster should be used for setting the a-posteriori measure η_{kt} (in our case, the tighter formulation of the non-robust formulation). The probability formula (19) adopted here is a revised formula that was proposed in [50] to improve the efficiency of the canonical ACO formula, by eliminating the need for products and powers of attractiveness measures and by reducing the number of parameters.

Once that a solution construction phase ends, the algorithm provides for updating the a-priori measures τ on the basis of how good the variable fixing resulted in the obtained solutions. In our algorithm, we update the attractiveness measures through a revised version of the improved formula proposed for ANTS in Maniezzo [50], which does not include the pheromone evaporation parameter, whose setting may result tricky.

To define the new formula, we first introduce the concept of *optimality gap* (*Gap*) for a feasible solution of value v and a lower bound L that is available on the optimal value v^* of the problem (note that it holds $L \leq v^* \leq v$): the *Gap* provides a measure of the quality of the feasible solution, comparing its value to the lower bound and is formally defined as $Gap(v, L) = (v - L)/v$. The a-priori attractiveness measure that we use is:

$$\begin{aligned} \tau_{ft}(h) &= \tau_{ft}(h-1) + \sum_{\sigma=1}^{\Sigma} \Delta\tau_{ft}^{\sigma} \text{ with } \Delta\tau_{ft}^{\sigma} \\ &= \tau_{ft}(0) \cdot \left(\frac{Gap(\bar{v}, L) - Gap(v_\sigma, L)}{Gap(\bar{v}, L)} \right) \end{aligned} \tag{20}$$

where $\tau_{ft}(h)$ is the a-priori attractiveness of fixing (f, t) at fixing iteration h, L is a lower bound on the optimal value of the problem (as lower bound, we use the optimal value of the robust formulation ROB-3-ConFL-MILP), v_σ is the value of the σ-th feasible solution built in the last construction cycle and $\bar{v}$ is the (moving) average of the values of the Σ solutions produced in the previous construction phase. $\Delta\tau_{ft}^{\sigma}$ represents the penalization/reward factor for a fixing and depends upon the initialization value $\tau_{ft}(0)$ of τ (in our case, based upon the linear relaxation of ROB-3-ConFL-MILP), combined with the relative variation in the optimality gap that v_σ implies with respect to $\bar{v}$. It should be noted that the use of a relative gap difference in Eq. (20) allows us to reward or penalize fixing adopted in the last solution making a comparison with the average quality of the last Σ solutions constructed.

After having obtained a *fully complete FOS*, we have identified an opening of facilities that can *potentially* meet the weighted coverage requirements for the technologies T. The term "potentially" indicates that the facility opening associated with the FOS may not admit a feasible completion in terms of connectivity variables and assignment of users to facilities: hence, not all of the SIR constraints (9) corresponding to open wireless facilities can be satisfied simultaneously because of interference effects. In other words, a complete FOS may correspond to an infeasible solution. Such a scenario requires us to set up a *check-and-repair phase* after that we have built a complete FOS: this phase must verify that the complete FOS actually leads to a feasible solution and, if this is not the case, try to repair the FOS, so that a feasible solution can be obtained. The reparation phase is based on using the same MIP heuristic employed at the end of the construction phase to try to improve a feasible solution (see Section 5.2 for further details).

Given a FOS that is complete for all technologies, we check its feasibility and try to obtain a feasible solution for the complete problem ROB-3-ConFL-MILP by considering a restricted version of ROB-3-ConFL-MILP, where we set $z_f^t = 1$ if $(f, t) \in FOS$. We solve this restricted problem through the MIP solver with a time limit: if this problem is recognized as infeasible by the solver, we run the MIP heuristic for reparation. Otherwise, we run the solver to possibly find a solution that is better than the best incumbent solution.

5.2. MIP-VLNS – an MIP-based repair/improvement exact search heuristic algorithm

To repair an infeasible partial fixing of the variables z induced by a complete FOS or to improve an incumbent feasible solution, we rely on an MIP heuristic that conducts a very large neighborhood search *exactly*, by formulating the search as a mixed integer linear program solved through an MIP solver [13]. Specifically, given a (feasible or infeasible) and possibly incomplete fixing $\bar{z}$ of variables, we define the neighborhood $\mathcal{N}$ including all the feasible solutions of ROB-3-ConFL-MILP that can be obtained by modifying at most $n > 0$ components of $\bar{z}$ and leaving the remaining variables free to vary. This condition can be expressed in ROB-3-ConFL-MILP by adding an *hamming distance constraint* imposing an upper limit n on the number of variables in z that change their value w.r.t. $\bar{z}$:

$$\sum_{(f,t) \in F \times T:\ \bar{z}_f^t = 0} z_f^t + \sum_{(f,t) \in F \times T:\ \bar{z}_f^t = 1} (1 - z_f^t) \leq n$$

The modified problem is then solved with an MIP solver like CPLEX, setting a certain time limit. The solver chooses the variables to be modified in order to repair or improve the incumbent solution including $\bar{z}$. Imposing a time limit is essential from a practical point of view: optimally solving the exact search can take a very high amount of time to close the optimality gap; additionally, a state-of-the-art MIP solver is usually able to quickly find solutions of good quality for large problems whose size has been conveniently reduced by fixing. In what follows, we denote the overall procedure for repair/improvement that we have discussed by MIP-VLNS.

5.3. The complete algorithm

The complete algorithm for solving the ROB-3-ConFL-MILP is presented in Algorithm 2. The algorithm is based on running two nested loops: the outer loop runs until reaching a global time limit and contains an inner loop whose task is to try to build Σ feasible solutions by first defining a complete FOS and then applying the MIP heuristic to repair or complete the fixing associated with the FOS.

More in detail, the first task of the algorithm is to solve the linear relaxation of ROB-3-ConFL-MILP for each single fixing $z_f^t = 1$, getting the corresponding optimal value and using it to initialize the a-priori measure of attractiveness $\tau_{ft}(0)$. This is followed by the definition of a solution X^* that represents the best solution found during the execution of the algorithm. Each run of the inner loop provides for building a complete FOS by considering the technologies according to the following order: fiber, copper and wireless (notice that such construction is not subject to a time limit). Choosing this order is in line with the coverage requirement constraints (17), according to which the weighted sum of users that access to the service through a "better" technology also contributes to satisfy the coverage requirement of the "worse" technology. In contrast, a "worse" technology does not contribute to reach the coverage requirement for a "better" technology. The complete FOS is built according to the procedure using the probability measures (19) and update formulas (20) that we have discussed before. The complete FOS provides a (partial) fixing of the facility opening variables $\bar{z}$ and the MIP solver uses it as a basis for finding a complete feasible solution X^* to the problem. If the MIP solver identifies the fixing $\bar{z}$ as infeasibile, then we run the heuristic MIP-VLNS in a reparation mode. If instead $\bar{z}$ is feasible and leads to find a feasible solution to ROB-3-ConFL-MILP that has a better value than the best solution found X^B in the current run of the inner loop, then X^B is updated. The inner loop is then iterated.

After that the execution of the inner loop is concluded, the a-priori measures τ are updated according to formula (20), considering the quality of the produced solutions, and we check the necessity of updating the global best solution X^*. After having reached the global time limit, the heuristic MIP-VLNS is in the end run with the aim of improving the best solution found X^*.

Algorithm 2. - Heuristic for ROB-3-ConFL-MILP

```
compute the linear relaxation of ROB-3-ConFL-MILP for every
    z_f^t = 1 and initialize the values τ_ft(0) with the corresponding
    optimal values
let X* be the best feasible solution found
while a global time limit is not reached do
   let X^B be the best solution found in the inner loop
   for σ := 1 to Σ do
build a complete FOS
solve ROB-3-ConFL-MILP imposing the fixing z̄ specified by the FOS
if ROB-3-ConFL-MILP with fixing z̄ is infeasible then
   run MIP-VLNS for repairing the fixing z̄
end if
if a feasible solution X̄ is found by the MIP solver and c(X̄) < c(X^B)
then
   update the best solution found X^B := X̄
end if
   end for
   update τ according to (20)
   if c(X^B) < c(X*) then
update the best solution found X* := X^B
   end if
end while
run MIP-VLNS for finding an improvement of X*
return X*
```

6. Computational results

The new algorithm was tested on 40 instances based on realistic network data defined within past consulting and industrial projects for a major telecommunications company. The computations were performed on a 2.70 GHz Windows machine with 8 GB of RAM and using IBM ILOG CPLEX 12.5 as MILP solver. The code was written in C/C++ and is interfaced with CPLEX through Concert Technology. As usually done in mathematical optimization literature, default setting of all parameters of CPLEX was adopted: in this case, CPLEX autonomously decides the setting of parameters that identifies to be the best for solving the problem (see the official CPLEX documentation). The optimization runs were performed setting a time limit of 3600 s (quite common in mathematical optimization literature).

All the instances refer to a urban district in the metropolitan area of Rome (Italy) and consider different traffic generation and user location scenarios. The considered area has been discretized into a grid of about 450 pixels, following the *testpoint model* recommended by international telecommunications regulatory bodies for wireless signal evaluation (see [1,19,26]). The nominal values and deviations of the uncertain fading coefficients were derived from previous studies carried out on real-world problems by one of the authors of this paper (see, e.g., [19,26]). We considered 30 potential facility locations that can accommodate any of the 3 technology considered in the study and can be connected to 5 potential central offices. On the basis of past computational experience and preliminary tests, we imposed the following setting of the parameters of the heuristic: $\alpha = 0.5$ (a-priori and a-posteriori attractiveness are balanced), $\Sigma = 5$ (number of solutions built in the inner loop before updating the a-priori measure and width of the moving average), $n = 0.05\ \bar{n}$ (number of variables z_f^t in an incumbent solution whose value can be modified by the reparation/improvement MIP heuristic – here $\bar{n}$ is the total number of variables z_f^t included in the problem). Furthermore, we imposed a time limit of 3000 s to the execution of the outer loop of Algorithm 2 and a limit of 600 seconds to the execution of the improvement heuristic MIP-VLNS.

The computational results are presented in Tables 1 and 2. Table 1 reports the cost of feasible solutions found by CPLEX and by our heuristic, which we denote by *HEU*. In this table, from left to right we report: (1) the ID of the instance; (2) the cost of the best solution found by CPLEX (column named "$c(X^*)$ – CPLEX"); (3) the cost of the worst solution found in the construction phase, i.e. before applying the improvement heuristic (column "$c(X^*)$ – HEU – Ph1 – worst" – note that we indicate by 'Ph1' the construction phase of our heuristic); (4) the average cost of solutions found in the construction phase (column "$c(X^*)$ – HEU – Ph1 – avg"); (5) the cost of the best solution found in the construction phase (column "$c(X^*)$ – HEU – Ph1 – best"); (6) the cost of the best solution found in the improvement phase (column "$c(X^*)$ – HEU – Ph2" – note that we indicate by 'Ph2' the construction phase of our heuristic); (7) the percentage cost decrease granted by the worst solution found in

Table 1
Computational results – cost.

ID	$c(X^*)$					$\Delta c(X^*)\%$			
	CPLEX	HEU				HEU			
		Ph1			Ph2	Ph1			Ph2
		Worst	Avg	Best		Worst	Avg	Best	
I1	1.373.800	1.300.450	1.295.543	1.212.940	1.080.440	5.63	6.04	13.26	27.15
I2	1.439.340	1.367.210	1.305.831	1.238.630	1.119.800	5.27	10.22	16.20	28.53
I3	1.456.030	1.358.090	1.311.675	1.274.660	1.110.660	7.21	11.01	14.23	31.09
I4	1.507.750	1.450.730	1.432.811	1.380.970	1.180.000	3.93	5.23	9.18	27.77
I5	1.372.090	1.303.510	1.269.963	1.228.500	1.094.150	5.26	8.04	11.69	25.40
I6	1.462.910	1.347.940	1.248.302	1.176.740	1.039.040	8.52	17.19	24.32	40.79
I7	1.263.690	1.218.640	1.210.004	1.155.660	1.104.400	3.69	4.44	9.35	14.42
I8	1.459.210	1.429.680	1.350.561	1.291.620	1.218.680	2.06	8.04	12.98	19.73
I9	1.374.390	1.317.740	1.290.305	1.248.010	1.174.800	4.29	6.52	10.13	16.98
I10	1.552.309	1.468.380	1.370.079	1.309.830	1.200.630	5.71	13.30	18.51	29.29
I11	2.267.900	2.105.010	2.005.864	1.895.860	1.745.090	7.73	13.06	19.62	29.95
I12	2.703.450	2.618.300	2.407.734	2.131.400	2.009.810	3.25	12.28	26.84	34.51
I13	2.454.270	2.396.660	2.327.862	2.150.390	1.850.540	2.40	5.43	14.13	32.62
I14	1.900.630	1.760.040	1.728.613	1.553.700	1.552.000	7.98	9.95	22.33	22.46
I15	2.670.000	2.560.730	2.405.091	2.256.520	2.190.600	4.26	11.01	18.32	21.88
I16	2.405.000	2.310.710	2.299.604	2.160.460	1.898.000	4.08	4.58	11.32	26.71
I17	2.236.180	2.088.210	2.055.591	1.990.050	1.900.630	7.08	8.79	12.37	17.65
I18	2.644.900	2.558.190	2.460.023	2.411.340	2.294.090	3.38	7.52	9.69	15.29
I19	1.863.090	1.814.060	1.753.427	1.680.420	1.500.270	2.70	6.25	10.87	24.18
I20	2.823.630	2.707.000	2.635.435	2.584.190	2.356.000	4.30	7.14	9.27	19.84
I21	2.128.000	2.044.010	1.963. 017	1.921.500	1.885.340	4.10	8.40	10.75	12.87
I22	2.559.810	2.331.680	2.224.058	2.110.380	1.979.020	9.78	15.10	21.30	29.34
I23	2.588.540	2.391.160	2.190.354	2.071.500	1.890.090	8.25	18.18	24.96	36.95
I24	2.992.000	2.720.460	2.667.010	2.543.420	2.333.540	9.98	12.19	17.64	28.21
I25	2.436.900	2.335.940	2.291.432	2.178.230	1.985.000	4.32	6.35	11.88	22.76
I26	1.825.270	1.791.690	1.713.066	1.669.300	1.567.630	1.8	6.55	9.34	16.43
I27	1.760.000	1.610.830	1.510.341	1.408.100	1.344.000	9.26	16.53	24.99	30.95
I28	1.959.090	1.814.600	1.723.012	1.599.420	1.478.800	7.96	13.70	22.49	32.47
I29	1.972.540	1.841.030	1.793.028	1.639.360	1.554.990	7.14	10.01	20.32	26.85
I30	1.886.360	1.720.040	1.629.835	1.497.140	1.406.720	9.66	15.74	26.00	34.09
I31	2.785.000	2.479.660	2.321.493	2.185.700	2.058.470	12.31	19.97	27.42	35.29
I32	3.100.240	2.837.440	2.675.026	2.414.910	2.090.610	9.26	15.90	28.38	48.29
I33	2.966.700	2.738.190	2.595.802	2.328.660	2.108.200	8.34	14.29	27.40	40.72
I34	2.860.040	2.550.340	2.190.674	2.086.000	1.987.300	12.14	30.56	37.11	43.91
I35	2.798.300	2.496.370	2.330.061	2.229.580	2.165.000	12.09	20.10	25.51	29.25
I36	3.289.400	2.983.910	2.789.953	2.505.060	2.260.730	10.23	17.90	31.31	45.50
I37	2.833.060	2.490.790	2.366.859	2.191.800	2.095.040	13.75	19.70	29.26	35.22
I38	3.000.670	2.766.630	2.449.732	2.318.530	2.100.220	8.45	22.49	29.42	42.87
I39	3.010.340	2.573.900	2.350.497	2.226.870	2.030.650	16.95	28.07	35.18	48.24
I40	2.700.410	2.414.720	2.190.325	2.070.120	1.997.480	11.83	23.29	30.45	35.19

the construction phase with respect to (w.r.t.) the best solution of CPLEX (column "$\Delta c(X^*)\%$ – HEU – Ph1 – worst"); (8) the percentage cost decrease granted on average by solutions found in the construction phase w.r.t. the best solution of CPLEX (column "$\Delta c(X^*)\%$ – HEU – Ph1 – avg"); (9) the percentage cost decrease granted by the best solution found in the construction phase w.r.t. the best solution of CPLEX (column "$\Delta c(X^*)\%$ – HEU – Ph1 – best"); (10) the percentage cost decrease granted by the best solution found in the improvement phase w.r.t. the best solution of CPLEX (column "$\Delta c(X^*)\%$ – HEU – Ph2").

Table 2 reports the optimality gaps associated with solutions found by CPLEX and by *HEU*. In this table, from left to right we report: (1) the ID of the instance; (2) the optimality gap of the best solution found by CPLEX (column named "*Gap*% – CPLEX"); (3) the optimality gap associated with the worst solution found in the construction phase (column "*Gap*% – HEU – Ph1 – worst"); (4) the average optimality gap of solutions found in the construction phase (column "*Gap*% – HEU – Ph1 – avg"); (5) the optimality gap associated with the best solution found in the construction phase (column "*Gap*% – HEU – Ph1 – best"); (6) the optimality gap associated with the best solution found in the improvement phase (column "*Gap*% – HEU – Ph2"); (7) the percentage optimality gap decrease granted by the worst solution found in the construction phase with respect to (w.r.t.) the best solution of CPLEX (column "Δ*Gap*% – HEU – Ph1 – worst"); (8) the percentage optimality gap decrease granted on average by solutions found in the construction phase w.r.t. the best solution of CPLEX (column "Δ*Gap*% – HEU – Ph1 – avg"); (9) the percentage optimality gap decrease granted by the best solution found in the construction phase w.r.t. the best solution of CPLEX (column "Δ*Gap*% – HEU – Ph1 – best"); (10) the percentage optimality gap decrease granted by the best solution found in the improvement phase w.r.t. the best solution of CPLEX (column "Δ*Gap*% – HEU – Ph2"). In the case of the heuristic, we note that the optimality gaps are obtained combining the feasible solutions found by HEU with the best lower bound obtained by CPLEX using the robust formulation ROB-3-ConFL-MILP.

It can be seen from the table that the ROB-3-ConFL-MILP problem is very challenging even for a modern state-of-the-art solver like CPLEX, as indicated by the large optimality gaps computed for the best solutions returned when the time limit is reached: almost all CPLEX gaps are well beyond 100% and can even exceed 200%. If we do not impose a time limit, for all the considered instances, CPLEX runs out of memory within 3 h of computations without being able to improve the best feasible solution found within the 1 h time limit that we adopted in our tests. By increasing the time limit, CPLEX is only able to get negligible improvements in the lower bound, obtaining at most a 3.65% improvement when running out of memory.

Table 2
Computational results – optimality gaps.

ID	*Gap%*					Δ*Gap%*			
	CPLEX	HEU				HEU			
		Ph1			Ph2	Ph1			Ph2
		Worst	Avg	Best		Worst	Avg	Best	
I1	84.67	74.82	74.16	63.05	45.24	11.64	12.42	25.53	46.57
I2	84.46	75.22	67.36	58.75	43.51	10.94	20.25	30.45	48.48
I3	105.00	91.22	84.68	79.47	56.37	13.13	19.35	24.31	46.30
I4	114.37	106.27	103.72	96.35	67.77	7.08	9.31	15.76	40.74
I5	97.98	88.09	83.25	77.27	57.87	10.09	15.03	21.14	40.93
I6	102.95	87.00	73.18	63.25	44.14	15.49	28.91	38.56	57.11
I7	85.29	78.69	77.42	69.45	61.93	7.74	9.22	18.57	27.38
I8	105.25	101.11	89.98	81.68	71.42	3.94	14.51	22.39	32.14
I9	96.90	88.79	84.86	78.80	68.30	8.37	12.43	18.68	29.50
I10	108.92	97.63	84.40	76.29	61.58	10.37	22.51	29.96	43.45
I11	115.97	100.46	91.02	80.54	66.18	13.37	21.51	30.54	42.92
I12	156.21	148.15	128.19	102.00	90.47	5.16	17.94	34.70	42.08
I13	113.33	108.33	102.35	86.92	60.85	4.41	9.69	23.30	46.30
I14	100.05	85.25	81.95	63.54	63.35	14.79	18.09	36.49	36.67
I15	164.50	153.68	138.26	123.54	117.01	6.58	15.95	24.90	28.86
I16	78.13	71.15	70.33	60.02	40.58	8.93	9.99	23.18	48.06
I17	101.41	88.08	85.15	79.24	71.18	13.14	16.03	21.86	29.80
I18	163.28	154.65	144.88	140.04	128.36	5.28	11.27	14.23	21.38
I19	119.16	113.30	106.27	97.68	76.48	4.84	10.82	18.03	35.81
I20	182.27	170.61	163.46	158.34	135.52	6.39	10.32	13.13	25.64
I21	122.97	114.17	105.69	101.34	97.54	7.15	14.05	17.59	20.67
I22	104.77	86.52	77.91	68.82	58.31	17.41	25.635	34.31	44.34
I23	133.13	115.36	97.27	86.57	70.22	13.35	26.93	34.97	47.24
I24	117.12	97.42	93.54	84.57	69.34	16.82	20.13	27.79	40.79
I25	103.06	94.66	90.95	81.51	65.41	8.16	11.76	20.91	36.53
I26	85.19	81.79	73.81	69.37	59.05	3.99	13.36	18.57	30.68
I27	78.17	63.07	52.90	42.55	36.05	19.31	32.33	45.57	53.87
I28	105.22	90.09	80.50	67.55	54.91	14.38	23.50	35.80	47.81
I29	100.27	86.92	82.05	66.45	57.88	13.31	18.17	33.73	42.27
I30	108.84	90.43	80.44	65.75	55.74	16.91	26.09	39.59	48.78
I31	151.14	123.61	109.34	97.10	85.62	18.21	27.65	35.75	43.34
I32	140.25	119.89	107.30	87.15	62.01	14.52	23.49	37.86	55.78
I33	167.53	146.93	134.09	110.00	90.11	12.29	19.96	34.34	46.21
I34	171.11	147.10	112.26	102.11	92.55	16.94	36.61	42.34	47.74
I35	131.29	106.34	92.59	84.29	78.95	19.00	29.47	35.80	39.86
I36	227.87	197.42	178.09	149.69	125.34	13.36	21.84	34.30	44.99
I37	155.78	124.85	113.69	97.89	89.14	19.85	27.01	37.16	42.77
I38	187.86	165.31	135.01	122.42	101.48	11.95	28.13	34.83	45.98
I39	174.13	134.39	114.05	102.79	84.92	22.82	34.50	40.97	51.23
I40	144.76	118.87	98.53	87.64	81.05	17.88	31.93	39.46	44.01

We believe that such difficulty encountered by CPLEX is especially due to the fact that ROB-3-ConFL-MILP contains the (robust) SIR constraints (9) associated with wireless coverage, which lead to a difficult generalization of the 2-ConFL-MILP. Indeed, pure wireless coverage problems are themselves challenging optimization problems – see, for example, [19,26] and the book [43] for a discussion.

The second critical observation is that all solutions returned from our heuristic are better than the best solutions found by CPLEX, both in terms of objective values and optimality gaps. A strong indication of the good performance of our heuristic is given by the fact that even the worst solutions found during the construction phase have a better value of those returned by CPLEX: on average, these worst solutions can grant a reduction in cost of about 2%. The improvement in cost that is granted on average by all solutions built in the construction phase reaches about 12% and, in the case of some instances like I38 and I37, can reach high improvements of about 20% and 30%. The best solutions found grant on average a reduction in cost of about 20% with respect to CPLEX. All these results indicate that our algorithm presents a solid and effective construction phase, which on its own can already produce (much) better solutions than CPLEX, thanks to exploiting the valuable information coming from (strengthened) linear relaxation in the variable fixing routines. We think that this is a strongpoint of our algorithm: ant-colony-like algorithms are commonly characterized by quite weak construction phases that really needs to be followed by a local search improvement phase to find good quality solutions (see, for example, the discussion in [35]).

The solutions obtained in the construction phase can be further effectively refined through the improvement phase, which leads to the identification of solutions leading to a very satisfying reduction in cost of about 30% on average. The very good performance of our heuristic is particularly evident in the case of the more challenging instances from I31 to I40, where we can obtain a percentage cost improvement that exceeds 40% for a good number of instances, leading to savings of several hundreds thousands of euros.

Looking at Table 2, it can be observed that all the solutions returned by the heuristics grant also a very satisfying improvement in the optimality gaps: for the construction phase, the average percentage reduction ranges from about 10%, for the worst case, to about 30%, for the best case. In the case of the improvement phase, the advantage in terms of optimality gaps is even more evident: we obtain an average percentage reduction of about 40%, which can even be well over 50%, as in the case of instance I6. Both the improvement in the objective values and the optimality gaps are really remarkable, leading to solutions that are sensibly closer to the optimum (we remember that decreasing the optimality gap is crucial to "move towards" the identification of an optimal solution of the problem, see [54]). This shows the effectiveness of the proposed heuristic algorithm, which constitutes a very valid alternative to CPLEX to get solutions of higher quality.

7. Conclusion and future work

We considered the problem of optimally designing a 3-architecture telecommunication access networks, combining the use of wired optical fiber- and copper-based connections with wireless connections. In literature, it has been suggested that this problem can be modeled as a simple generalization of 2-architecture Connected Facility Location Problems, by including an additional technology index. However, this is a generalization that neglects the interaction between signals emitted by distinct wireless transmitters and that may possibly lead to service coverage plans not implementable in practice. In order to overcome this limitation, we have proposed a new optimization model that also includes the variables and constraints modeling the power emissions of wireless transmitters and the signal-to-interference formulas that are recommended for evaluating wireless service coverage. Furthermore, we have derived a robust optimization model for designing the network while taking into account the uncertainty that naturally affects signal propagation in a real environment. Since the resulting MILP problem is very challenging even for a state-of-the-art commercial solver like CPLEX, we have proposed a new heuristic algorithm combining a probabilistic variable fixing procedure, guided by suitable linear relaxations of the problem, and an exact very large neighborhood search. Computational tests performed on a set of realistic instances indicated that our heuristic algorithm can provide solutions of much higher quality than those returned by CPLEX.

The work presented in this paper could be further developed along several distinct axes. First, it would be desirable to reduce the optimality gap and reach an optimal solution, by increasing the lower bound and/or decreasing the upper bound. The lower bound could be increased by studying and including additional families of valid inequalities to further strengthen the mathematical model and get tighter linear relaxations. For example, *power-indexed formulations* [26] could be adopted for modelling the signal-to-interference constraints of wireless facilities and new more efficient separation algorithms for power-indexed inequalities could be developed. Other families of conflict inequalities, as in [11], and the generation of more-effective cuts leading to a maximal lower bound improvement, as discussed in [18], could also be investigated. The upper bound could be decreased by studying refinements of the current heuristic or exploring new forms of integration between exact and heuristic optimization approaches. For example, it could be investigated the adoption of other bio-inspired algorithmic approaches, such as an evolutionary algorithm, both as improvement heuristic and standing-alone solution algorithms. A genetic algorithm could be developed using as basis the genetic algorithm of [20] for pure wireless network based on power-indexed formulations. An alternative heuristic approach could be to break the integrated design problem into the sequential solution of separated problems (e.g., we first assign user to facilities and then solve the connectivity flow problem), similarly to the approaches proposed for wireless network design in [32] and [61]. Last but not least, we plan to address refined robust optimization models, such as Multiband Robust Optimization (e.g., [14]) and Two-Stage Robustness (e.g., [42]), to reduce conservatism of solutions without reducing their protection.

Acknowledgements

This work was partially supported by the *Einstein Center for Mathematics Berlin* through Project ROUAN (MI4) and by BMBF through Project VINO (Grant no. 05M13ZAC) and Project *ROBUKOM* (Grant no. 05M10ZAA). The authors would like to thank the three anonymous reviewers for their very valuable comments and suggestions, which helped to improve the readability of the paper.

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