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A 0-1 Program for Minimum Clustering in Downlink Base Station Cooperation

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A 0-1 Program for Minimum Clustering in Downlink Base Station Cooperation

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Abstract

Base station cooperation in the downlink of cellular systems has been recently suggested as a promising concept towards a better exploitation of the communication system physical resources. It may offer a high gain in capacity through interference mitigation. This however, comes at a cost of high information exchange between cooperating entities and a high computational burden. Clustering of base stations into subgroups is an alternative to guarantee such cooperation benefits in a lower scale. The optimal definition of clusters, however, and a systematic way to find a solution to such problem is not yet available. In this work, we highlight the combinatorial nature of the problem, exploit this to describe the system of users and base stations as a graph and formulate a pure 0-1 program. Its solution suggests a cost optimal way to form clusters and assign user subsets to them.

I. INTRODUCTION

An important issue in modern wireless communications is to develop techniques that mitigate co-channel intercell interference. This constitutes a major problem in the effort to optimally exploit the available physical resources such as frequency spectrum, time and energy. In the recent years, it has been analytically shown that cooperative transmission between base stations (BSs) of neighbouring cells can offer a high capacity gain [1], [2]. The costs of such a communications scenario approach are related to backhaul connections between the cooperating entities, increased signaling for information exchange and high computational effort.

The concept behind the performance optimization is that the entire system, including all base stations and user terminals, can be seen as a virtual (or network [3]) MIMO system and - provided the necessary information exchange - the transmission of all BSs can be *coordinated* in a way such that interference is minimized. An example of such an optimal cooperation can be to choose the precoding matrix in the downlink of all users in the system as a pseudo-inverse prefilter, also known as the Zero-Forcing precoder [1]. Such a choice results in interference free signal reception at all user ends. Alternative ways to chose the precoding matrix is by combining Zero-Forcing with Dirty Paper coding [1]. Such results however can be reached only through a huge information exchange, involving the estimated values of the channel coefficients between all users and all system BSs and further costs related to backhaul connections and channel bandwidth reservation.

Usually, the benefits of BS cooperation are considerable even in smaller subsets of the system BSs, which constitute *clusters*. In such clusters the required information available is reduced. On the other hand, users will still suffer inter-cluster interference. Suggestions of such limited cooperation can be found in [3], [4] and [5]. Clusters can be formed statically or dynamically and certain suggestions are found in the literature for both such approaches [6], [4], [7], [8], [9].

Since each user in a cluster can be served only by the base station subset that defines it, all entries of its precoding vector, related to base stations outside, should be set to zero, as shown in [9]. This leads to the conclusion, that the problem of optimal cluster formation is of combinatorial nature. The optimal user assignment to base stations should define which base stations form the serving clusters. On the other hand, it is important - due to cooperation costs - to keep the size of clusters as small as possible.

The current work is based on the above observation, in order to formulate and solve an exact 0-1 program, which defines the minimum cost BS clusters for cooperation within the cellular network. To do this, the global information over the user channel long term fading coefficients should be available at a central unit, where the optimization is considered to be performed. The importance of our contribution lies in the originality of the formulated optimization problem, as well as in the presentation of a systematic way to define clusters and treat problems of optimal clustering within the physical transmission framework.

The remainder of the paper is organized as follows. The general transmission scheme in the downlink of a cooperating virtual MIMO system is presented in section II, where the influence of clustering at the received SINR of the users is discussed. Section III provides a discription of the system as a graph and assignment variables, cooperation variables, cooperation scenarios and clusters are formally defined. Section IV begins with a statement of the optimization objective and introduces the assignment variables in the beamforming vector. A set of constraints for the problem is given, so that the outcome of the solution is well defined. Properties of the feasible and optimal solutions are presented. In section V, the solution software and results for example instances are provided. Section VI concludes our work.

II. MULTICELL DOWNLINK TRANSMISSION

We consider a set of users $\mathcal{V}^U : |\mathcal{V}^U| = N$, having a fixed position with respect to a set $\mathcal{V}^B : |\mathcal{V}^B| = M$ of single antenna base stations (BSs) throughout the optimization period. The signal vector to be transmitted is given by the $N \times 1$ complex-valued vector $\mathbf{s} = [s_1, \ldots, s_N]^T$, $s_u \in \mathbb{C}$. The user signals are considered independent realizations of a random process with a certain probability distribution. The expected power of each user signal equals p_u, whereas the signals of different users are uncorrelated, so that $\mathbb{E}[s_u \cdot s_u^*] = p_u$ and $\mathbb{E}[s_u \cdot s_n^*] = 0$, $n \neq u$.

Following [1], the set of BSs and users forms a generalized Multiple Input Multiple Output (MIMO) system, which implies that each user can potentially be served by each BS. The geographically remote BSs form altogether a virtual antenna array, which communicates with the user virtual array.

Each user's signal is mapped to BSs using a so called *beamforming or precoding vector* $\mathbf{w}_u := [w_{u,1} \ldots, w_{u,M}]^T$ with dimension $M \times 1$. The elements of such a vector are considered again complex numbers $w_{u,b} \in \mathbb{C}$.

After this mapping, the $M \times 1$ antenna signal vector $\mathbf{x}$ for transmission in the downlink is formed. For this, the signal vector is multiplied by the $M \times N$ precoding matrix $\mathbf{W} := [\mathbf{w}_1, \ldots, \mathbf{w}_N]$, that is

$$\mathbf{x} \;=\; \mathbf{W} \cdot \mathbf{s}$$

The power of the transmitted signal per antenna can be calculated as follows

$$\mathbb{E}\left[\mathbf{W}\cdot\mathbf{s}\cdot(\mathbf{W}\cdot\mathbf{s})^{\dagger}\right] = \mathbf{W}\begin{bmatrix} p_1 & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & p_N \end{bmatrix}\mathbf{W}^{\dagger}$$

where $\mathbf{W}^{\dagger}$ is the complex conjugate transpose of $\mathbf{W}$.

The per antenna power expenditure, which - from the above - equals

$$\sum_u |w_{u,b}|^2 \cdot p_u \tag{1}$$

describes the consumption of the transmission power physical resource of the system, for each b of the M BSs.

The complex signal x_b transmitted by each antenna to the serviced users experiences fading, with magnitude that depends on the user-BS distance and the stochastic behavior of the channel. It is assumed that each user $u \in \mathcal{V}^U$ has the ability to estimate the channel fading coefficient between itself and each of the BSs, thus forming a complex vector $\mathbf{h}_u := [h_{u,1} \ldots, h_{u,M}]$ of size $1 \times M$, with elements $h_{u,b} \in \mathbb{C}$. The $N \times M$ channel matrix is further denoted by $\mathbf{H} := \left[\mathbf{h}_1^T, \ldots, \mathbf{h}_N^T\right]^T$. In TDD systems e.g. the channel estimation can be done by using pilot symbols in the uplink and - assuming reciprocity of the uplink-downlink channel - each BS can inform the user of its current fading value.

The signals received in the downlink by the N users equal

$$\mathbf{y} = \mathbf{H}\cdot\mathbf{W}\cdot\mathbf{s} + \eta \tag{2}$$

where $\mathbf{y} := [y_1, \ldots, y_N]^T$ is the N-dimensional receive signal column vector and $\eta := [\eta_1, \ldots, \eta_N]^T$ is the N-dimensional noise column vector, with $\eta_u \in \mathbb{C}$ zero-mean additive Gaussian noise at user's u receiver end with variance $\mathbb{E}[\eta_u \cdot \eta_u^*] = \sigma_u^2$ and $\mathbb{E}[\eta_u \cdot \eta_n^*] = 0$, $n \neq u$. The per user received signal equals

$$y_u = \mathbf{h}_u \cdot (\mathbf{w}_1 s_1 + \ldots + \mathbf{w}_N s_N) + \eta_u \tag{3}$$

To calculate the received power at u we take expectation over transmitted signals and noise. Due to independence of signal and noise random realizations of different users

$$\mathbb{E}\left[\|y_u\|_2^2\right] = \mathbf{w}_u^{\dagger}\mathcal{R}_u\mathbf{w}_u \cdot p_u + \sum_{n\neq u}\mathbf{w}_n^{\dagger}\mathcal{R}_u\mathbf{w}_n \cdot p_n + \sigma_u^2$$

where

$$\mathcal{R}_u := \mathbf{h}_u^{\dagger}\cdot\mathbf{h}_u. \tag{4}$$

When interference is treated as noise, the Signal-to-Interference-Noise Ratio (SINR) for each user u is

$$SINR_u := \frac{\mathbf{w}_u^{\dagger}\mathcal{R}_u\mathbf{w}_u \cdot p_u}{\sum_{n\neq u}\mathbf{w}_n^{\dagger}\mathcal{R}_u\mathbf{w}_n \cdot p_n + \sigma_u^2} \geq \gamma_u \tag{5}$$

and in order for a level of Quality-of-Service (QoS) per user to be guaranteed, this should be above a predefined threshold, which depends on its receiver and the transmission modus.

In the above formulation, one can observe that each user $n \neq u$, depending on its assignment to the BSs by the beamforming vector, contributes $\mathbf{w}_n^\dagger \mathcal{R}_u \mathbf{w}_n \cdot p_n$ to the interference for user u. This term is written more clearly as

$$\mathbf{w}_n^\dagger \mathcal{R}_u \mathbf{w}_n \cdot p_n = \left(\sum_{b_i} \sum_{b_j} w_{n,b_i}^* h_{u,b_i}^* h_{u,b_j} w_{n,b_j} \right) p_n \tag{6}$$

When choosing the Zero-Forcing precoder

$$\mathbf{W} \;=\; \mathbf{H}^\dagger \left(\mathbf{H} \mathbf{H}^\dagger \right)^{-1}$$

it is possible for the user signals to be received interference free $y_u = s_u + \eta_u$, as implied by a simple substitution in (2). Such a choice however is prohibitive in systems with a large number of BSs, since the above precoding strategy would require an enormous amount of data exchange and computational effort due to the problem dimensioning, as well as frequent updates considering that the entries $h_{u,b}$ refer to fast fading coefficients.

These drawbacks can be partly mitigated by grouping the BSs into clusters that serve a specific subgroup of the user set. Such an approach can still provide the benefits of cooperative techniques and interference mitigation in a smaller scale and most importantly with lower costs. In such a case, interference can be avoided within the cluster and the users suffer only inter-cluster interference. To our best knowledge, how clusters should optimally be chosen within the network is not yet clear from the available literature. After the clusters are defined, zero-forcing or other types of precoders can be applied within the cluster BS subset.

Clusters should be formed based on long term channel fading coefficients, since the cooperation between base stations due to protocol signaling, requires a certain time interval to be established and should not be changed on the scale of instantaneous channel measurements. Considering long term measurements, the random effects of fast fading can be averaged out and the matrix $\mathcal{R}$ can be approximated by the channel covariance matrix for user u

$$\tilde{\mathcal{R}}_u := \; \mathbb{E}\left[\mathbf{h}_u^\dagger \cdot \mathbf{h}_u \right] \tag{7}$$

which is diagonal, since we consider the case of single antenna BSs and the channels from different BSs to the same terminal u are independent realizations of some random fading process, in other words $\mathbb{E}\left[h_{u,b_i}^* \cdot h_{u,b_j} \right] = 0$. In LTE systems, user terminals have the ability to gather so called RSRP measurements [10] over the instantaneous channel power, which can be averaged over a certain time window to get an unbiased estimator of the channel power expectation. Further information over the channel fading angle is not any more required in such case.

$$g_{u,b}^2 \approx \mathbb{E}\left[|h_{u,b_i}|^2 \right]. \tag{8}$$

By replacing $\mathcal{R}_u$ by (7) in the SINR, we get the following simplification

$$\mathbf{w}_n^\dagger \tilde{\mathcal{R}}_u \mathbf{w}_n \cdot p_n = \left(\sum_{b} |w_{n,b}|^2 \cdot g_{u,b}^2 \right) p_n. \tag{9}$$

In what follows, we first provide a graph description of the problem, and further formulate it - based on the scenario described above - as a 0-1 program, having as variables the cooperation between BSs and the assignment of users to clusters.

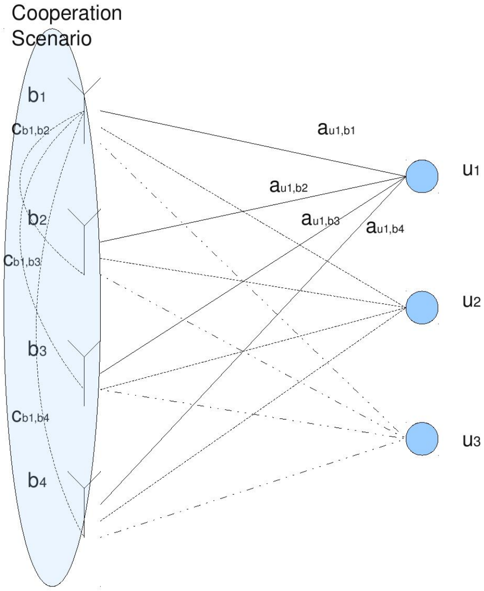


Figure 1: Example of a network topology with 4 Base Stations and 3 Users.

III. GRAPH MODEL AND CLUSTER DEFINITION

A. *Definitions*

The set of users and base stations, as well as their inbetween interaction, can be modelled as an undirected graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$, where $\mathcal{V}$ is the set of nodes and $\mathcal{E}$ is the set of edges. The set of nodes consists of two independent subsets: a) the set of User (U) nodes $\mathcal{V}^U$ with cardinality N and b) the set of Base Station (BS) nodes $\mathcal{V}^B$ with cardinality M. For these it holds $\mathcal{V}^U \cup \mathcal{V}^B = \mathcal{V}$ and $\mathcal{V}^U \cap \mathcal{V}^B = \emptyset$.

The set of edges further consists of two independent subsets: a) the set of U-BS edges

$$\mathcal{E}^U := \left\{(u,b)\,|u \in \mathcal{V}^U,\ b \in \mathcal{V}^B\right\} \tag{10}$$

as well as b) the set of BS-cooperation edges

$$\mathcal{E}^B \ := \ \left\{(b_i,b_j)\,|b_i,b_j \in \mathcal{V}^B,\ b_i \neq b_j\right\} \tag{11}$$

No edge between user nodes is considered and $\mathcal{E}^U \cup \mathcal{E}^B = \mathcal{E}$, $\mathcal{E}^U \cap \mathcal{E}^B = \emptyset$. An example is shown in Fig.1. A binary variable is assigned to each edge of the network:

- Variable $a_{u,b} \in \{0,1\}$ is assigned to edges in $\mathcal{E}^U$.
- Variable $c_{b_i,b_j} \in \{0,1\}$ is assigned to edges in $\mathcal{E}^B$.

We define the set of 'active' BS-cooperation edges such that

$$\mathcal{A}^B := \left\{(b_i,b_j) \in \mathcal{E}^B \mid c_{b_i,b_j} = 1\right\} \subseteq \mathcal{E}^B$$

which is named **Cooperation Set** in what follows and the set of 'active' U-BS edges such that

$$\mathcal{A}^U := \left\{(u,b) \in \mathcal{E}^U \mid a_{u,b} = 1\right\} \subseteq \mathcal{E}^U$$

which is named **Assignment Set**.

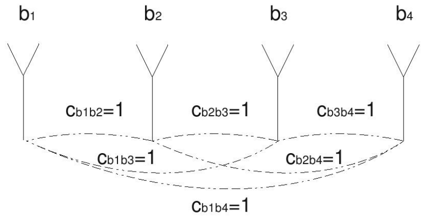


Figure 2: Cooperation Scenario under study: The cooperation set results in complete clusters.

B. Graph Partitions and Clusters

The number of possible partitions for the BS set with cardinality M is given by the so called Bell number, which satisfies the recursion $B_{n+1} = \sum_{k=0}^{n} \left(\begin{array}{c} n \\ k \end{array} \right) B_k$ and $B_0 = B_1 = 1$. For the case of $M = 4$ we have $B_4 = 15$.

On the other hand, given a total number M of BS's for the entire set $\mathcal{V}^B$, there are $\left(\begin{array}{c} M \\ 2 \end{array} \right) = \frac{M\cdot(M-1)}{2}$ binary variables c_{b_i,b_j} (one related to each edge (b_i, b_j)) which result in a number of $2^{\frac{M\cdot(M-1)}{2}}$ different cooperation sets. For the example of the Fig.1 where $M = 4$ this gives a total number of $64 > B_4 = 15$ sets.

From the above, there possibly exists more than one cooperation sets that define the same partition of the BS set into subsets, which we call here clusters. To restrict this, we put further constraints. These restrictions result from physical assumptions on the way the BSs cooperate with each other and we say they constitute a **cooperation scenario**. In what follows the following scenario is considered among the BSs.

Two BSs cooperate when there is a logical connection between them. If more than two BSs constitute a cluster, then there is a logical connection between any pair of BSs belonging to the cluster.

In the LTE standards for example, the logical connection between two BSs can be considered over the X2 interface [10].

An illustration for the $M = 4$ case is provided in Fig.2, where all BSs are considered in a single cluster formation. Note here, that such a cooperation pattern can be justified as in [2] and [3], where cooperating BSs exchange information through backhaul links and if we assume that cooperation between any pair of BSs within a cluster should be done *in a single hop*. This results in the following definition for the clusters under consideration:

Definition 1 *A* ***Complete Cluster*** $\mathcal{C}^o := \left\{ \mathcal{V}^{C^o}, \mathcal{A}^{C^o} \right\}$ *is defined to be a connected component of the subgraph* $\mathcal{G}^B := \left\{ \mathcal{V}^B, \mathcal{E}^B \right\}$*, which is also complete, that is every pair of the nodes of* $\mathcal{V}^{C^o}$ *is adjacent.*

Such a definition allows for a *bijection* $g : \pi \rightarrow \mathcal{A}^B$ from the set of all possible partitions π of $\mathcal{V}^B$ to the set of assignment sets. This matches each partition π to exactly one assignment set, while there exists no assignment set with complete clusters that remains unmapped.

IV. 0-1 PROGRAMMING FORMULATION

A. Objective Function

We aim in this work at minimizing the cooperation cost between BSs, which equals

$$\sum_{(b_i,b_j)\in\mathcal{E}^B} k_{b_i,b_j}\cdot c_{b_i,b_j} \tag{12}$$

and k_{b_i,b_j} are positive costs per connection (b_i, b_j). In what follows, the set of necessary constraints subject to which the minimization takes place will be presented.

B. Assignment Variables in the Precoding Vector

Based on the definitions in the previous section and the discussion on the downlink transmission in Section II, we will from now on replace the beamforming vector for user u by the binary assignment vector and formulate clusters considering average SINR. Precoding can be applied as a second step, after the clusters have been determined.

$$\mathbf{w}_u \ := \ [a_{u,1},\ldots,a_{u,M}]^T \tag{13}$$

The products in (9) take the form

$$\mathbf{w}_n^\dagger \tilde{\mathcal{R}}_u \mathbf{w}_n \cdot p_n \ = \ \sum_b a_{n,b}\cdot g_{u,b}^2 \cdot p_n \tag{14}$$

where we make use of the fact that $a_{n,b}^2 = a_{n,b}$. From this, user n contributes to the interference part of user u's receive signal, through all BSs b assigned ($a_{n,b}=1$). Assignment reserves p_n power from BS b.

C. Power Constraints

It was mentioned in Section II that the power expenditure per BS is given in (1). It is reasonable to consider an upper bound P_b on the power consumed per BS. Another, reasonable assumption (see also the Linear Wyner Model in [3]) is to consider equal power consumption per user served within a cell. Then instead of p_u, which is the signal power of user u, we can consider p_b to be the fixed power budget contributed by BS b to each user u assigned to it. Then the power constraint (1) is reformulated as

$$\sum_u a_{u,b}\cdot p_b \le P_b \Rightarrow \sum_u a_{u,b} \le \lfloor \frac{P_b}{p_b} \rfloor := K_b, \quad \forall b \tag{15}$$

which sets a bound K_b on the number of users served per cell. Furthermore, given K_b, the per BS contribution belongs to $p_b \in (\ \frac{P_b}{K_b+1}, \frac{P_b}{K_b}\ ]$.

D. SINR Constraints

Using the auxiliary variables and (14), and replacing p_u by p_b as explained above, we can reformulate the SINR constraint per user in (5) as a knapsack inequality with real valued coefficients.

$$\sum_{n\in\mathcal{V}^U}\sum_{b\in\mathcal{V}^B} r_{n,b}^u \cdot a_{n,b} \ge 1, \quad \forall u \tag{16}$$

where

$$r_{n,b}^u := \frac{g_{u,b}^2\cdot p_b}{\sigma_u^2}\cdot \begin{cases} \frac{1}{\gamma_u}, & \text{if } n = u \\ -1, & \text{if } n \neq u \end{cases} \tag{17}$$

E. User Assignment and Cooperation

Given the above definition of complete clusters, there should be a logical connection between any two BSs taking part in the service of a user. When there is no connection between two BSs, then these should belong to different clusters, and the user should not anymore be assigned to both of them. To formalize this idea we introduce the inequality

$$c_{b_i,b_j} + 1 \geq a_{u,b_i} + a_{u,b_j}, \quad \forall u, b_i, b_j \tag{18}$$

This has the following effect:

- When $c_{b_i,b_j} = 0$, the user can be assigned to at most one of the two BSs.
- When $a_{u,b_i} = a_{u,b_j} = 1$, the BSs should cooperate ($c_{b_i,b_j} = 1$).

F. Complete Cluster Constraints

Since we restrict the clustering to complete only clusters as defined in the previous section, extra constraints should be introduced.

$$c_{b_i,b_l} + 1 \geq c_{b_i,b_j} + c_{b_j,b_l}, \quad \forall b_i, b_j, b_l \tag{19}$$

Proposition 1 *The set of inequalities (16), (18) and (19) guarantee that:*

1) *All feasible clusters are complete by Def.1.*
2) *A feasible solution assigns a user to exactly one cluster.*

Proof: For Prop.1.1. it is sufficient to show that, the case where two BSs $b_i, b_l \in \mathcal{V}^B$ are connected and have distance equal to 2 is infeasible, since every path of length ≥ 2 has a subpath of length 2. Suppose, it can be feasible. Then, there exists a BS b_j with distance $c_{b_i,b_j} = c_{b_j,b_l} = 1$ from b_i and b_l respectively, so that the path has distance 2. Using (19), $c_{b_i,b_l} = 1$ and hence there exists a path from b_i to b_l with length 1, which is a contradiction.

For Prop.1.2. suppose it is feasible to assign user u to 2 clusters. Then there exist BSs b_i and b_j such that, $a_{u,b_i} = a_{u,b_j} = 1$ and $c_{b_i,b_j} = 0$ (complete clusters from Prop1.1.). By (18), $c_{b_i,b_j} = 1$, which is a contradiction. Assignment to at least one cluster comes from the fulfillment of the SINR constraint in (16). ■

The proposition that follows explains that, the optimal solution provides minimum size clusters, in the sense that another partition which results by reallocating a single BS from one cluster to another is either infeasible or suboptimal.

Proposition 2 *When $k_{b_i,b_j} := 1$, the solution of*

$$\begin{array}{ll} \min & \sum_{\mathcal{E}^B} c_{b_i,b_j} \\ \textbf{\textit{s.t.}} & \textit{(15)-(19)} \end{array} \tag{20}$$

satisfies Prop.1 and partitions $\mathcal{V}^B$ into clusters such that:

- *Reallocation of a single BS from a larger cluster to a smaller one is infeasible (unless the cardinality of the two clusters differs exactly by 1 in which case the cost is equal to the optimal).*
- *Reallocation of a single BS from a smaller cluster to a larger one is always suboptimal.*

Proof: Let the BS subset forming cluster e be denoted by $\mathcal{V}_e^{C^o}$. The optimal partition is π^* and consider elements (clusters) of it a and e, such that $|\mathcal{V}_a^{C^o}| > |\mathcal{V}_e^{C^o}|$.

For the first case, the partition is denoted by π^- and the resulting elements by a^- and e^- respectively. The change in total cost equals

$$\begin{aligned} \sum_{(b_i,b_j)\in\mathcal{A}_{a^-}^{C^o}} c_{b_i,b_j} + \sum_{(b_i,b_j)\in\mathcal{A}_{e^-}^{C^o}} c_{b_i,b_j} &= \\ \sum_{(b_i,b_j)\in\mathcal{A}_{a}^{C^o}} c_{b_i,b_j} + \sum_{(b_i,b_j)\in\mathcal{A}_{e}^{C^o}} c_{b_i,b_j} \quad -|\mathcal{V}_{a^-}^{C^o}| + |\mathcal{V}_{e}^{C^o}| & \end{aligned}$$

Since $|\mathcal{V}_e^{C^o}| - |\mathcal{V}_{a^-}^{C^o}| \leq 0$ with equality only in the case where $|\mathcal{V}_a^{C^o}| = |\mathcal{V}_e^{C^o}| + 1$ the cost will further reduce from the optimal, which implies that the new cluster set is infeasible (except from the equality case).

For the second case, the partition is denoted by π^+ and the resulting elements by a^+ and e^+ respectively. Following the above calculations, the change in cost will equal $|\mathcal{V}_a^{C^o}| - |\mathcal{V}_{e^+}^{C^o}| > 0$, and such a solution is always suboptimal. ■

V. SOLUTION AND NUMERICAL RESULTS

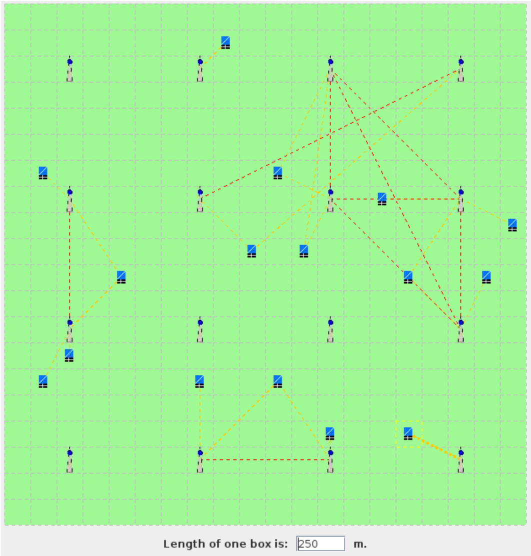
For the solution of the problem in (20), we have initially used the ZIMPL programming language [11] to translate the model into a 0-1 integer program. As a next step, the open source mixed integer problem solver SCIP [12], which implements Branch-and-Bound, was used to derive the 0-1 solution, for the optimal assignment and cooperation variables.

A. Scenario with 16 BSs and 16 Users

For simulation purposes we have created a platform, based on the Java programming language, which produces automatically a map of 16 fixed BSs and 16 users uniformly scattered on the 2D-plane. The per antenna constraint is set to $P_b = 40W$, $\forall b$ and each BS is able to serve at most $K_b = 3$ users. The noise variance is set to $\sigma^2 = -174dBm/Hz$. The long term channel fading coefficients are estimated using the COST-Walfish-Ikegami model for urban environments [13] depending on the distance of the user to each of the BSs. The SINR threshold is the same for all users and is allowed to vary within the interval $\gamma \in [0.2, 0.4]$. Fig.3 and Fig.4 provide two examples, where it is illustrated how the cooperation between BSs changes, as the per user SINR demand increases from $\gamma = 0.35$ to $\gamma = 0.37$. Clustering shows to behave in a very sensitive way related to the increase in overall demand.

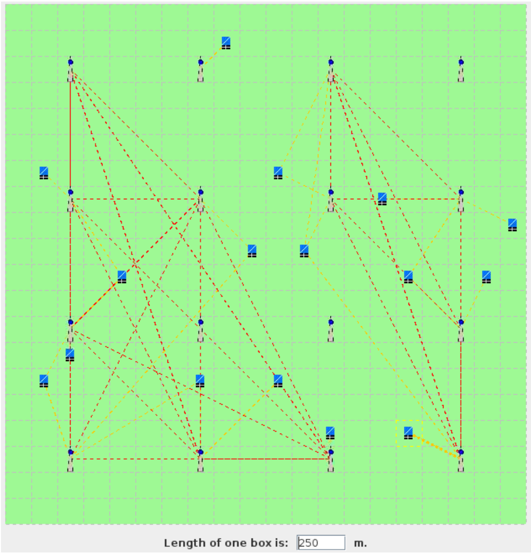
VI. CONCLUSIONS

In the current work, a 0-1 program for the optimal clustering of cooperating base stations in the downlink has been suggested. The cooperation depends on the assignment of users to BSs for fulfillment of their SINR requirement. To achieve this, the beamforming vector per user for the entire virtual MIMO system, has been treated as an assignment vector. Furthermore, the clusters are formed based on a cooperation scenario that requires a logical link connection between each cooperating BS pair, thus resulting in - so called - complete clusters. The objective is to define minimum cooperating groups of BSs among all feasible solutions. Major drawback of the approach is its centralized implementation, which requires information over the entire long term channel matrix for all users in the system and their demands. Furthermore, its feasibility depends on the chosen user QoS demands to be supported. Such a result can be used however as an optimal upper bound for all decentralized schemes suggested, while the formulation further gives insights to how BSs optimally cooperate and how clusters are formed.



Length of one box is: 250 m.

Figure 3: BS cooperation for global threshold $\gamma = 0.35$.



Length of one box is: 250 m.

Figure 4: BS cooperation for global threshold $\gamma = 0.37$.

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