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Abstract

Wireless networks have shown a rapid growth over the past two decades and now play an increasingly prominent role in different telecommunication systems. Consequently, scarce resources such as the radio spectrum and the physical sites that accommodate transmitters have become extremely congested and need to be allocated in more effective ways. Since the early 1980s several optimization models have been developed to design wireless networks, that is to localize and configure transmitters by assigning transmission frequencies and emission powers to them. Most such models represent emission powers as continuous decision variables. This choice typically yields ill-conditioned constraint matrices and requires the introduction of very large coefficients to model disjunctive relations. The corresponding relaxations are very weak and the solutions returned by Mixed-Integer Linear Programming solvers are typically far from the optimum and sometimes even infeasible. In order to overcome these difficulties, we introduce a pure 0-1 formulation for the problem that is obtained by considering only a finite set of power values. Basing on such formulation we also developed an iterative, row generation algorithm to solve wireless network design problems. The new approach presents many computational and modeling advantages. First, albeit considering only a subset of feasible solutions, it allows to find better solutions to large practical instances with less computational effort. Second, since the feasible powers are well spaced over the power spectrum, the final plans tend to be robust. Third, it directly models power restrictions that are often imposed by the technology and that sometimes permits two values only (i.e., on/off). Finally, it easily allows for generalizations, such as power consumption minimization.
Keywords: Wireless Network Design, 0-1 Linear Programming, Cover Inequalities

1 Introduction

Wireless communication systems are pervading everyday life. Television and Radio programs are distributed through broadcasting networks (both terrestrial and satellite), mobile communication is ensured by cellular networks, internet is provided through broadband access networks. Moreover, a number of security services are provided by ad-hoc wireless networks. All these networks have grown very rapidly and very large during the last decades, generating a dramatic congestion of all radio resources. Wireless networks provide different services and rely on different technologies and standards. Still, they share a common feature: they all need to reach users scattered over a target area with a radio signal that must be strong enough to prevail against other unwanted signals. The perceived quality of service thus depends on several signals, wanted and unwanted, generated from possibly a large number of transmitting devices. Due to the large size of the current networks, to an extremely congested radio spectrum, to local and international constraints, establishing suitable emission powers for all the transmitters has become a very difficult systemic task, which calls for sophisticated optimization techniques.

For our purposes, a wireless network can be described as a set of transmitters B that provide for a telecommunication service to a set of receivers T. A receiver is said to be *covered* (or *served*) by the network if it receives the service within a minimum level of quality. Transmitters and receivers are characterized by a number of physical and radio-electrical parameters. The *Wireless Network Design* problem (WND) consists in establishing suitable values for such parameters with the goal of maximizing the coverage (or a revenue associated with the coverage). In particular, each transmitter $b \in B$ emits a radio signal with power $p_b \in [0, P_{\max}]$. Typically a receiver $t \in T$ receives radio signals from a subset $B(t) \subseteq B$ of transmitters. Since each transmitter in $B(t)$ is associated to a unique received signal, in what follows we will also refer to $B(t)$ as the set of signals received by t.

To simplify the discussion, we assume here that all transmitters of the network operate at the same frequency (*Single Frequency Network* or simply *SFN*). This assumption is dropped in Section 4 where we describe the real-life application which motivated our developments. Among the received signals $B(t)$, receiver t can select a *reference signal* (or *server*), which is the one carrying the service. All the other signals are interfering (in digital broadcasting several signals can contribute to the overall wanted signal, but this case is not discussed here). A receiver t is regarded as served by the network with reference signal $\beta \in B(t)$ iff the following linear constraint in the emitted powers is satisfied:

$$a_{t\beta} \cdot p_\beta - \sum_{b \in B(t) \setminus \{\beta\}} e_{tb} \cdot p_b \geq \delta \tag{1}$$

where the *fading* coefficients $a_{t\beta}$ and e_{tb} with $b \in B(t) \setminus \{\beta\}$ are computed through suitable propagation models and depend on physical as on radio-electrical conditions. Inequality (1) is called *SIR inequality* and derives from the *Signal-to-Interference Ratio* (SIR) [21]. Parameter δ depends on the requested level of quality of service and is called *SIR threshold*. For any given $t \in T$, we have one inequality of type (1) for each potential server $\beta \in B(t)$. Receiver t is served iff at least one such inequality is satisfied, or, equivalently, if the following disjunctive constraint is satisfied:

$$\bigvee_{\beta \in B(t)} \left(a_{t\beta} \cdot p_\beta - \sum_{b \in B(t) \setminus \{\beta\}} e_{tb} \cdot p_b \geq \delta \right) \tag{2}$$

The above disjunction can be represented by a family of linear constraints in the p variables, by introducing, for each $t \in T$ and each $b \in B(t)$, a binary variable x_{tb}, with $x_{tb} = 1$ iff t is served by b. Then, for each $\beta \in B(t)$, the following constraint is introduced:

$$a_{t\beta} \cdot p_\beta - \sum_{b \in B(t) \setminus \{\beta\}} e_{tb} \cdot p_b + M \cdot (1 - x_{t\beta}) \geq \delta \tag{3}$$

where M is a large positive constant. Indeed, if $x_{t\beta} = 1$ then (3) reduces to (1); if instead $x_{t\beta} = 0$ and M is sufficiently large (for example, we can set $M = \delta + \sum_{b \in B \setminus \{\beta\}} e_{tb} \cdot P_{\max}$), then (3) is satisfied for any feasible power vector p and becomes redundant. Constraints of type (3) appear in the Mixed-Integer Linear Programs (MILPs) for the WND presented in several papers in different application contexts, such as Radio and Video Broadcasting (e.g. [16, 17]), GSM (e.g. [18]), UMTS (e.g. [3, 10, 15, 19]), WiMAX [7].

WND instances of some practical interest typically correspond to very large MILP formulations. In principle, such formulations can be solved by standard techniques like Branch&Cut and by means of effective commercial solvers such as ILOG Cplex [6]. However, it is well-known that the presence of a great number of constraints of type (3) results in ill-conditioned instances, due to the large variability of the fading coefficients, and weak bounds, due to the presence of the big-M coefficients. In practice, only small-sized instances can actually be solved to optimality.

In a paper from 1990 [9], Dyer and Wolsey introduced the *Time-Indexed* formulation (TI) for machine scheduling, obtained by discretizing the continuous time variables associated with jobs. Similarly, we introduce a new class of formulations for the WND by considering, for all $b \in B$, only a finite number of feasible values $\mathcal{P} = \{P_1, \ldots, P_{|\mathcal{P}|}\}$ for the power variable p_b, with $P_1 = 0$, $P_{|\mathcal{P}|} = P_{\max}$ and $P_i > P_{i-1}$, for $i = 2, \ldots, |\mathcal{P}|$: by analogy with the ($TI$), we call such formulations *Power-Indexed* (*PI*). A binary variable z_{bl} is introduced for each transmitter $b \in B$ and each power value $P_l \in \mathcal{P}$, with $z_{bl} = 1$ iff $p_b = P_l$, and $z_{bl} = 0$ otherwise. Like in the case of the (TI), this allows for a pure binary linear program, which, at the cost of increasing the number of binary variables and constraints, yields stronger relaxations and, consequently, tighter bounds. In contrast with the (TI), the number of new binary variables is a design parameter, which only affects the quality of the approximation error. This observation is exploited in the solution approach to the WND described in Section 3.

It is worth noting that, in the design of real-life wireless networks, it is common practice for network engineers to consider only a small number of possible power values. In particular, the useful power values are expressed by integer decibels w.r.t. a reference value (e.g., 1 milliwatt (mW) in mobile applications), that is the power is indicated by an integer P^{dBm} whereas the actual power in mW is computed as $P = 10^{P^{dBm}/10}$. Using such a subset of well spaced power values typically results in more robust plans and corresponds to a more cautious design approach. This conservative approach is also a practical response to the uncertainty characterizing radio-wave propagation, that is affected by several factors which are hard to estimate exactly. Also, in some real-life applications the number of feasible power values is limited by the technology. Indeed, another nice feature of the (PI) formulations is that it can immediately handle the case where powers are *actually discretized*, and easily model alternative objectives to coverage revenue (e.g., non-linear power consumption costs).

The Power-Indexed formulation for the WND is introduced in the next section, by deriving it from the classical big-M formulation. In Section 3 we describe our solution approach to the WND. In Section 4 we show a specific application to the planning of WiMAX networks. Finally, extensive computational results on realistic WND instances are presented in Section 5. Such results show that the new approach outperforms the one based on the big-M formulation in terms of quality of the solutions (i.e., coverage of receivers), upper bounds and running times.

2 A Power-Indexed formulation for the Wireless Network Design Problem

A classical and much exploited model for the WND belongs to the class of the so called *big-M formulations.* We now present such a type of formulation *(BM)* by introducing a parameter r_t to denote revenue (e.g., population, number of customers, expected traffic demand) associated with receiver $t \in T$. The notation introduced so far is summarized in Table 1.

$$\begin{array}{llll}
\max & \sum_{t \in T} \sum_{b \in B(t)} r_t \cdot x_{tb} & & (BM) \\
\text{s.t.} & a_{t\beta} \cdot p_\beta - \sum_{b \in B(t) \setminus \{\beta\}} e_{tb} \cdot p_b + M \cdot (1 - x_{t\beta}) \geq \delta & t \in T, \beta \in B(t) & (4) \\
& \sum_{b \in B(t)} x_{tb} \leq 1 & t \in T & (5) \\
& p_b \leq P_{max} & b \in B & \\
& p_b \geq 0 & b \in B & \\
& x_{tb} \in \{0,1\} & t \in T, b \in B(t) &
\end{array}$$

The objective function is to maximize total revenue, constraint (4) is the SIR inequality (3) introduced in Section 1, while constraint (5) ensures that each receiver is served at most once. Technology-dependent versions of (BM) can be obtained from the basic formulation by including suitable constraints or even new variables.

We now consider a different version of (BM) by restricting p_b to assume value in the finite set $\mathcal{P} = \{P_1, \ldots, P_{|\mathcal{P}|}\}$ of feasible power values, with $P_1 = 0$ (*switched-off value*), $P_{|\mathcal{P}|} = P_{max}$ and $P_i > P_{i-1}$, for $i = 2, \ldots, |\mathcal{P}|$. To this end, we introduce a binary variable z_{bl}, which is 1 iff b emits at power P_l. Since b is either switched-off or emitting at a positive value in $\mathcal{P}$, we have:

$$\sum_{l \in L} z_{bl} = 1 \qquad b \in B$$

where $L = \{1, \ldots, |\mathcal{P}|\}$ is the set of power value indices or simply *power levels*. Then we can write:

$$p_b = \sum_{l \in L} P_l \cdot z_{bl} \qquad b \in B \tag{6}$$

By substituting (6) in (4), we obtain the following SIR constraint that only involves 0-1 variables:

$$a_{t\beta} \cdot \sum_{l \in L} P_l \cdot z_{\beta l} - \sum_{b \in B(t) \setminus \{\beta\}} e_{tb} \cdot \sum_{l \in L} P_l \cdot z_{bl} + M \cdot (1 - x_{t\beta}) \geq \delta$$

So, a formulation for the WND with a finite number of power values is the following (DM):

$$\begin{aligned}
\max \quad & \sum_{t \in T} \sum_{b \in B(t)} r_t \cdot x_{tb} && && (DM) \\
\text{s.t.} \quad & a_{t\beta} \cdot \sum_{l \in L} P_l \cdot z_{\beta l} - \sum_{b \in B(t) \setminus \{\beta\}} e_{tb} \cdot \sum_{l \in L} P_l \cdot z_{bl} + M \cdot (1 - x_{t\beta}) \geq \delta && && \\
& && t \in T, \beta \in B(t) && (7) \\
& \sum_{b \in B(t)} x_{tb} \leq 1 && t \in T && \\
& \sum_{l \in L} z_{bl} = 1 && b \in B && \\
& x_{tb} \in \{0, 1\} && t \in T, b \in B(t) && \\
& z_{bl} \in \{0, 1\} && b \in B, l \in L &&
\end{aligned}$$

Note that, thanks to (6), p_b also satisfies $0 \leq p_b \leq P_{max}$. As a consequence, the box constraints on p_b and thus variable p_b is dropped from the formulation.

The Power-Indexed formulation is obtained from (DM) by substituting each constraint (7) with a suitable set of lifted cover inequalities. Before showing this, we recall some basic concepts related to cover inequalities:

given a knapsack constraint (KP) $\sum_{i \in N} a_i x_i \leq b$ with non-negative coefficients, a *cover* of (KP) is a subset of indices $C \subseteq N$ such that $\sum_{i \in C} a_i > b$. Denoting by $\mathcal{C}$ the set of covers of (KP), it is well known (see [20]) that the set of feasible solutions to $\{x \in \{0,1\}^n : \sum_{i \in N} a_i x_i \leq b\}$ coincides with the set of feasible solutions to $\{x \in \{0,1\}^n : \sum_{i \in C} x_i \leq |C| - 1, \forall C \in \mathcal{C}\}$.

We now define some notation that is necessary to introduce the specific form of the lifted cover inequalities. Consider the coverage condition (1) associated to a receiver $t \in T$ with server $\beta \in B(t)$. Let $\Gamma = \{b_1, \ldots, b_{|\Gamma|}\} \subseteq B(t) \backslash \{\beta\}$ be a subset of interferers. Suppose that the server β is emitting at power value $p_\beta = P_\lambda$, for some $\lambda \in L$. Denote by $L^I(t, \beta, \lambda, \Gamma)$ the set of $|\Gamma|$-tuples $q \in L^{|\Gamma|}$ such that $a_{t\beta} P_\lambda - e_{tb_1} P_{q_1} - \cdots - e_{tb_{|\Gamma|}} P_{q_{|\Gamma|}} < \delta$. In other words, receiver t is not served when t is assigned to server β emitting at power value P_λ, and the interferers $b_1, \ldots, b_{|\Gamma|}$ are emitting at power values $p_{b_1} = P_{q_1}, \ldots, p_{b_{|\Gamma|}} = P_{q_{|\Gamma|}}$, respectively. Finally, by letting $\bar{z}_{\beta l} = 1 - z_{\beta l}$ for $l \in L$, we rewrite (7) as the following knapsack constraint with non-negative coefficients:

$$a_{t\beta} \cdot \sum_{l \in L} P_l \cdot \bar{z}_{\beta l} + \sum_{b \in B(t) \backslash \{\beta\}} e_{tb} \cdot \sum_{l \in L} P_l \cdot z_{bl} + M \cdot x_{t\beta} \leq M + a_{t\beta} \cdot \sum_{l \in L} P_l - \delta \quad (8)$$

Theorem 1 *Every cover inequality of (8) is dominated by the following families of constraints:*

$$x_{t\beta} + \sum_{l=1}^{\lambda} z_{\beta l} + \sum_{i=1}^{|\Gamma|} \sum_{j=q_i}^{|\mathcal{P}|} z_{b_i j} \leq |\Gamma| + 1 \qquad t \in T, \beta \in B(t), \Gamma \subseteq B(t) \backslash \{\beta\},$$
$$\lambda \in L, q \in L^I(t, \beta, \lambda, \Gamma) \qquad (9)$$
$$\sum_{l \in L} z_{bl} = 1 \qquad b \in B \qquad (10)$$
$$x_{tb} \leq 1 \qquad t \in T, b \in B(t) \qquad (11)$$
$$z_{bl} \leq 1 \qquad b \in B, l \in L \qquad (12)$$

Proof. For M sufficiently large, variable $x_{t\beta}$ is trivially contained in every cover inequality of (8). In order to find the remaining variables of a cover inequality, we can thus consider covers of the following knapsack constraint:

$$a_{t\beta} \cdot \sum_{l \in L} P_l \cdot \bar{z}_{\beta l} + \sum_{b \in B(t) \backslash \{\beta\}} e_{tb} \cdot \sum_{l \in L} P_l \cdot z_{bl} \leq a_{t\beta} \cdot \sum_{l \in L} P_l - \delta \qquad (13)$$

Table 1: Summary of notation

T	set of receivers
B	set of transmitters
$B(t)$	set of transmitters (signals) received in $t \in T$
$\Gamma \subseteq B(t) \backslash \{\beta\}$	subset of interfering transmitters of $t \in T$
r_t	revenue of $t \in T$
$a_{tb},\ e_{tb}$	fading coefficients of signals sent to $t \in T$ from $b \in B(t)$
δ	SIR threshold
M	large coefficient (big-M)
$P_{\max}$	maximum emitted power
$\mathcal{P}$	set of power values ($P_1 = 0, ..., P_{\lvert\mathcal{P}\rvert} = P_{\max}$)
L	set of power levels
$L^I(t, \beta, \lambda, \Gamma)$	set of power level vectors of $\Gamma \subseteq B(t) \backslash \{\beta\}$ interfering $t \in T$ served by $\beta \in B(t)$ emitting at $\lambda \in L$
$p_b \in [0, P_{\max}]$	power variable of $b \in B$
$x_{tb} \in \{0, 1\}$	service variable of $t \in T$ served by $b \in B$
$z_{bl} \in \{0, 1\}$	discrete power variable of $b \in B$ emitting at $l \in L$

We use the standard notation denoting a cover of (13) by a set of indices $C \subseteq \{(b, l) : b \in B(t), l \in L\}$. In particular, we partition C into classes C_1 and C_2, where C_1 contains all the indices of type (β, l) (i.e., the indices relative to server β), whereas C_2 contains all the others. Then, $C = C_1 \cup C_2$ is a cover of (13) iff it satisfies:

$$\sum_{(\beta,l)\in C_1} a_{t\beta} \cdot P_l + \sum_{(b,l)\in C_2} e_{tb} \cdot P_l > a_{t\beta} \cdot \sum_{l \in L} P_l - \delta \tag{14}$$

and the corresponding cover inequality $I(C)$ (including variable $x_{t\beta}$) is:

$$\sum_{(\beta,l)\in C_1} \bar{z}_{\beta l} + \sum_{(b,l)\in C_2} z_{bl} + x_{t\beta} \leq |C| \tag{15}$$

Assume now that the cover inequality $I(C)$ is not dominated by the constraints (9)-(12).

Claim 1. $C_2 = \{(b_1, q_1), \ldots, (b_{|\Gamma|}, q_{|\Gamma|})\}$, with $\Gamma = \{b_1, \ldots, b_{|\Gamma|}\} \subseteq B(t) \setminus \{\beta\}$ subset of distinct interferers and $q = \{q_1, \ldots, q_{|\Gamma|}\} \in L^{|\Gamma|}$ a corresponding set of power levels. In other words, a non-dominated cover inequality contains exactly one variable for each interferer $b \in \Gamma$ and thus $|C_2| = |\Gamma|$). By contradiction, suppose that for some $b \in B(t)\backslash\{\beta\}$, we have $(b, q) \in C_2$ and $(b, r) \in C_2$, with $q \neq r$. It follows that $I(C)$ is implied by constraints (10)-(12). In fact, from (10) we have $z_{bq} + z_{br} \leq 1$ while all other variables in $I(C)$ are less than or equal to 1. By summing up all these box constraints to constraint $z_{bq} + z_{br} \leq 1$, we immediately obtain (15), and the claim is proved.

By Claim 1, if we denote by $L_1 = \{l \in L : (\beta, l) \in C_1\}$ the set of power levels of server β appearing in C, we can write (14) as:

$$a_{t\beta} \cdot \sum_{l \in L_1} P_l + \sum_{i=1}^{|\Gamma|} e_{tb_i} \cdot P_{q_i} > a_{t\beta} \cdot \sum_{l \in L} P_l - \delta$$

or equivalently as:

$$a_{t\beta} \cdot \sum_{l \in L \backslash L_1} P_l - \sum_{i=1}^{|\Gamma|} e_{tb_i} \cdot P_{q_i} < \delta$$

By letting $\lambda = \max\{l \in L\backslash L_1\}$, the above inequality implies:

$$a_{t\beta} \cdot P_\lambda - \sum_{i=1}^{|\Gamma|} e_{tb_i} \cdot P_{q_i} < \delta$$

which shows that $q \in L^I(t, \beta, \lambda, \Gamma)$, that is receiver t is not covered when server β emits at power level λ and $b_1, \ldots, b_{|\Gamma|}$ emit at power levels $q_1, \ldots, q_{|\Gamma|}$, respectively.

Now, by using this notation and complementing variables $\bar{z}$, we rewrite the cover inequality (15) as:

$$-\sum_{l \in L_1} z_{\beta l} + \sum_{i=1}^{|\Gamma|} z_{b_i q_i} + x_{t\beta} \leq |C_2| = |\Gamma|$$

which, by (10), can be rewritten as:

$$\sum_{l \in L \backslash L_1} z_{\beta l} + \sum_{i=1}^{|\Gamma|} z_{b_i q_i} + x_{t\beta} \leq |\Gamma| + 1$$

Since $L\backslash L_1 \subseteq \{1, \ldots, \lambda\}$, we thus obtain the contradiction that a generic non-dominated cover inequality of (13) is dominated by the inequality (9) corresponding with server β emitting at power level $\lambda = \max\{l \in L \backslash L_1\}$.

$\square$

Next theorem shows that constraints (9) are indeed valid for the 0-1 solutions to (DM). In fact, they can be derived as a conic combination of non-dominated cover inequalities followed by round down.

Theorem 2 *Constraints (9) are satisfied by every 0-1 solution to* (DM).

Proof. We use the same notation of the proof of Theorem 1. Let $t \in T$, $\beta \in B(t)$, $\Gamma = \{b_1, \ldots, b_{|\Gamma|}\}$, and consider a non-dominated cover $C = C_1 \cup C_2$ of (8), where $C_1 = \{(\beta, l) \in C\}$, and $C_2 = \{(b_1, q_1), \ldots, (b_{|\Gamma|}, q_{|\Gamma|})\}$ (Claim 1 of Theorem 1). Again we let $L_1 = \{l \in L : (\beta, l) \in C_1\}$ and $\lambda = \max\{l \in L \backslash L_1\}$. As we show in the proof of Theorem 1, it follows that $q = (q_1, \ldots, q_{|\Gamma|}) \in L^I(t, \beta, \lambda, \Gamma)$. Now, observe that if $\tilde{q} \in L^{|\Gamma|}$ and $\tilde{q} \geq q$ then $\tilde{q} \in L^I(t, \beta, \lambda, \Gamma)$. In fact, $P_{\tilde{q}_i} \geq P_{q_i}$, for $i = 1, \ldots, |\Gamma|$, and (14) is satisfied. As a consequence, the following constraints are also cover inequalities of (8):

$$\sum_{l \in L \backslash L_1} z_{\beta l} + z_{b_1 j} + \sum_{i=2}^{|\Gamma|} z_{b_i q_i} + x_{t\beta} \leq |\Gamma| + 1 \qquad j = q_1, q_1 + 1, \ldots, |\mathcal{P}| \tag{16}$$

From (10) we have:

$$\sum_{j=q_1}^{|\mathcal{P}|} z_{b_1 j} \leq 1 \tag{17}$$

If the previous inequality is multiplied by $|\mathcal{P}| - q_1$ and summed to constraints (16), we obtain:

$$(|\mathcal{P}| - q_1 + 1) \cdot \left(\sum_{l \in L \backslash L_1} z_{\beta l} + \sum_{j=q_1}^{|\mathcal{P}|} z_{b_1 j} + \sum_{i=2}^{|\Gamma|} z_{b_i q_i} + x_{t\beta} \right) \leq (|\mathcal{P}| - q_1 + 1) \cdot (|\Gamma| + 1) + |\mathcal{P}| - q_1 \tag{18}$$

Both members of the above inequality can be divided by the positive quantity $|\mathcal{P}| - q_1 + 1$ and then rounded down thus obtaining the following inequality, which is valid for the 0-1 solutions to (DM):

$$\sum_{l \in L \setminus L_1} z_{\beta l} + \sum_{j=q_1}^{|\mathcal{P}|} z_{b_1 j} + \sum_{i=2}^{|\Gamma|} z_{b_i q_i} + x_{t\beta} \leq |\Gamma| + 1 \tag{19}$$

By starting from the above inequality and by applying inductively the same argument to $(b_2, q_2), \ldots (b_{|\Gamma|}, q_{|\Gamma|})$, it is easy to obtain the following valid inequality:

$$\sum_{l \in L \setminus L_1} z_{\beta l} + \sum_{i=1}^{|\Gamma|} \sum_{j=q_i}^{|\mathcal{P}|} z_{b_i j} + x_{t\beta} \leq |\Gamma| + 1 \tag{20}$$

A similar reasoning can also be applied to terms associated to server β. Observe first that if $q \in L^I(t, \beta, \lambda, \Gamma)$ then $q \in L^I(t, \beta, l, \Gamma), \forall\, l \in L : l \leq \lambda$. In fact, $P_l \leq P_\lambda$, for $l \leq \lambda$, and (14) is satisfied. As a consequence, the following family of constraints are cover inequalities of (8):

$$z_{\beta l} + \sum_{i=1}^{|\Gamma|} z_{b_i q_i} + x_{t\beta} \leq |\Gamma| + 1 \qquad l = 1, \ldots, \lambda$$

Analogously to (20), the following inequalities are also valid for the 0-1 solutions to (DM):

$$z_{\beta l} + \sum_{i=1}^{|\Gamma|} \sum_{j=q_i}^{|\mathcal{P}|} z_{b_i j} + x_{t\beta} \leq |\Gamma| + 1 \qquad l = 1, \ldots, \lambda \tag{21}$$

Letting $L_2 = \{l \in L : l \leq \lambda,\ l \notin L \setminus L_1\}$, we can finally obtain (9) by: (i) summing up the constraints (21) associated to $l \in L_2$, the constraint (20) and the constraint $\sum_{j=1}^{\lambda} z_{\beta j} \leq 1$ multiplied by $|L_2|$; (ii) dividing both members by the quantity $|L_2| + 1$; (iii) rounding down.

□

We have already observed that inequality (7) can be replaced by the family of its cover inequalities. On the other hand, by Theorem 1 every cover inequality is dominated by constraints (9)-(12), which are also valid for the 0-1 solutions to (DM). So, the following is a valid (Power-Indexed) formulation for the WND problem (with finite set of power values):

$$\max \quad \sum_{t\in T}\sum_{b\in B(t)} r_t \cdot x_{tb} \tag{PI}$$

$$\text{s.t.} \quad x_{t\beta} + \sum_{l=1}^{\lambda} z_{\beta l} + \sum_{i=1}^{|\Gamma|}\sum_{j=q_i}^{|\mathcal{P}|} z_{b_i j} \leq |\Gamma| + 1 \qquad t \in T, \beta \in B(t), \Gamma \subseteq B(t)\backslash\{\beta\},$$

$$\lambda \in L, q \in L^I(t,\beta,\lambda,\Gamma) \tag{22}$$

$$\sum_{b\in B(t)} x_{tb} \leq 1 \qquad t \in T \tag{23}$$

$$\sum_{l\in L} z_{bl} = 1 \qquad b \in B \tag{24}$$

$$x_{tb} \in \{0,1\} \qquad t \in T, b \in B(t) \tag{25}$$

$$z_{bl} \in \{0,1\} \qquad b \in B, l \in L \tag{26}$$

3 Solution Algorithm

The solution algorithm is based on the (PI) formulation for the WND and consists of two basic steps: (i) the set $\mathcal{P}$ of feasible power values is established; (ii) the associated formulation is solved by row generation and Branch&Cut. We start by describing step (ii) and we come back to step (i) later in this section.

In the following, for a fixed power set $\mathcal{P}$, we denote the solution algorithm for the associated (PI) formulation as SOLVE-PI($\mathcal{P}$). Since the (PI) formulation has in general an exponential number (in the input size) of constraints of type (22), we adopt row generation. Namely, we start by considering only a suitable subset of constraints and we solve the associated relaxation. We then check if any of the neglected rows is violated by the current fractional solution. If so, we add the violated row to the formulation and solve again, otherwise we proceed with standard Branch&Cut (as implemented by the commercial solver ILOG Cplex [6]). The separation of violated constraints is repeated in each branching node.

At node 0, the initial formulation includes only a subset of constraints (22), namely the constraints (22) including at most one interferer (i.e. $\Gamma \leq 1$).

3.1 Separation

We now proceed to show how violated constraints are separated. Let (x^*, z^*) be the current fractional solution. In Section 2 we have showed that constraints (22) are indeed lifted cover inequalities of (8). In order to separate a violated inequality of type (22), we extend the standard heuristic approach to the separation of cover inequalities described in [20].

To this end, let us first select a receiver $t \in T$ and one of its servers, say $\beta \in B(t)$. We want to find a constraint of type (22) that is associated with t and β, and is violated by the current solution (x^*, z^*). In other words, we want to identify a power level $\lambda \in L$ for β, a set of interferers $\Gamma = \{b_1, \ldots, b_{|\Gamma|}\} \subseteq B(t)\backslash\{\beta\}$ and an interfering $|\Gamma|$-tuple of power levels $q = (q_1, \ldots, q_{|\Gamma|}) \in L^I(t, \beta, \lambda, \Gamma)$, such that:

$$x^*_{t\beta} + \sum_{l=1}^{\lambda} z^*_{\beta l} + \sum_{i=1}^{|\Gamma|} \sum_{j=q_i}^{|\mathcal{P}|} z^*_{b_i j} > |\Gamma| + 1 \tag{27}$$

Recall that $q \in L^I(t, \beta, \lambda, \Gamma)$ iff

$$a_{t\beta} \cdot P_\lambda - \sum_{i=1}^{|\Gamma|} e_{tb_i} \cdot P_{q_i} < \delta \tag{28}$$

We solve the above separation problem by defining a suitable 0-1 linear program. In particular, in order to identify a suitable pair (β, l) we introduce, for all power levels $l \in L$, a binary variable $u_{\beta l}$, which is 1 iff $l = \lambda$. Similarly, we introduce binary variables u_{bl} for all $b \in B(t)\backslash\{\beta\}$ and $l \in L$, with $u_{bl} = 1$ iff $(b, l) = (b_i, q_i)$, where $b_i \in \Gamma$ and q_i is the corresponding interfering power level. Then $u \in \{0, 1\}^{|B(t)| \times |L|}$ satisfies the following system of linear inequalities:

$$a_{t\beta} \sum_{l \in L} P_l \cdot u_{\beta l} - \sum_{b \in B(t)\backslash\{\beta\}} e_{tb} \sum_{l \in L} P_l \cdot u_{bl} < \delta \tag{29}$$

$$\sum_{l \in L} u_{bl} = 1 \qquad b \in B(t) \tag{30}$$

Constraint (29) ensures that the combination of emitted power values is not sufficient to allow transmitter β to serve receiver t, whereas constraint (30) states that each transmitter emits at exactly one power level.

Observe now that $|\Gamma| = \sum_{b\in B(t)\setminus\{\beta\}} \sum_{l\in L} u_{bl}$. So, if u identifies a violated constraint of type (27), we also have:

$$\sum_{l\in L} u_{\beta l} \sum_{k=1}^{l} z^*_{\beta k} + \sum_{b\in B(t)\setminus\{\beta\}} \sum_{l\in L} u_{bl} \sum_{k=l}^{|\mathcal{P}|} z^*_{bk} > \sum_{b\in B(t)\setminus\{\beta\}} \sum_{l\in L} u_{bl} + 1 - x^*_{t\beta} \quad (31)$$

In order to (heuristically) search for a violated inequality, we proceed in a way which resembles the classical approach for standard cover inequalities (see [20]), by writing the following linear program (SEP):

$$\begin{aligned} Z \;=\; \max \quad & \sum_{l\in L} u_{\beta l} \sum_{k=1}^{l} z^*_{\beta k} + \sum_{b\in B(t)\setminus\{\beta\}} \sum_{l\in L} u_{bl} \cdot \left(\sum_{k=l}^{|\mathcal{P}|} z^*_{bk} - 1 \right) & & (SEP) \\ \text{s.t.} \quad & a_{t\beta} \sum_{l\in L} P_l \cdot u_{\beta l} - \sum_{b\in B(t)\setminus\{\beta\}} e_{tb} \sum_{l\in L} P_l \cdot u_{bl} \le \delta & & (32) \\ & \sum_{l\in L} u_{bl} = 1 & b \in B(t) & \\ & u_{bl} \ge 0 & b \in B(t), l \in L & \end{aligned}$$

It is easy to notice that the feasible region of (SEP) contains all binary vectors satisfying (29) and (30). Let Z be the optimum value to (SEP). If $Z \le 1 - x^*_{t\beta}$ then no binary vector u satisfies (31) and consequently no violated constraint exists. If $Z > 1 - x^*_{t\beta}$ then a violated constraint may exist, and we resort to a heuristic approach to find it. In particular, observe first that Z can be computed by relaxing the knapsack constraint (32) in a Lagrangian fashion and then by solving the resulting Lagrangian dual, namely:

$$\begin{aligned} Z \;=\; & \min_{\eta\ge 0} \quad Z(\eta) \\ & \text{s.t.} \quad \sum_{l\in L} u_{bl} = 1 \qquad b \in B(t) \end{aligned}$$

where $\eta \in \mathbb{R}$ is the Lagrangian multiplier and:

$$\begin{aligned} Z(\eta) &= \max_{u \geq 0} \sum_{l \in L} u_{\beta l} \sum_{k=1}^{l} z^*_{\beta k} + \sum_{b \in B \setminus \{\beta\}} \sum_{l \in L} u_{bl} \cdot \left(\sum_{k=l}^{|\mathcal{P}|} z^*_{bk} - 1 \right) + \\ &+ \eta \cdot \left(\delta - a_{t\beta} \sum_{l \in L} P_l \cdot u_{\beta l} + \sum_{b \in B \setminus \{\beta\}} e_{tb} \sum_{l \in L} P_l \cdot u_{bl} \right) \end{aligned}$$

For fixed $\eta \geq 0$, the objective $Z(\eta)$ can be easily computed by inspection. In particular, we introduce the following coefficient for every $b \in B(t), l \in L$:

$$c_{bl}(\eta) = \begin{cases} \sum_{k=1}^{l} z^*_{\beta k} - \eta \cdot a_{t\beta} \cdot P_l & \text{if } b = \beta \\ \\ \sum_{k=l}^{|\mathcal{P}|} z^*_{bk} - 1 + \eta \cdot e_{tb} \cdot P_l & \text{if } b \in B(t) \setminus \{\beta\} \end{cases}$$

then $Z(\eta)$ rewrites as:

$$\begin{aligned} Z(\eta) &= \delta \cdot \eta + \max_{u \geq 0} \sum_{b \in B(t)} \sum_{l \in L} c_{bl}(\eta) \cdot u_{bl} \\ & \qquad \text{s.t.} \quad \sum_{l \in L} u_{bl} = 1 \qquad\qquad b \in B(t) \end{aligned} \tag{33}$$

For fixed $\eta \geq 0$, an optimal solution $u(\eta)$ to the inner maximization problem can be found by inspection as follows:

(i) for each $b \in B(t)$, identify a power level $l_b \in L$ which maximizes the coefficient in the objective function, namely $c_{bl_b}(\eta) = \max_{l \in L} c_{bl}(\eta)$.

(ii) for each $b \in B(t)$ and each $l \in L$, let

$$u_{bl}(\eta) = \begin{cases} 1 & \text{if } l = l_b \\ 0 & \text{otherwise} \end{cases}$$

It is straightforward to see that, for all $\eta \geq 0$, $u(\eta) \geq 0$ satisfies all constraints (33) and maximizes the function $\sum_{b \in B(t)} \sum_{l \in L} c_{bl}(\eta) \cdot u_{bl}(\eta)$. For $\eta = \bar{\eta}$, we have $Z(\bar{\eta}) = \delta \cdot \bar{\eta} + \sum_{b \in B(t)} \sum_{l \in L} c_{bl}(\bar{\eta}) \cdot u_{bl}(\bar{\eta})$.

For $\eta \geq 0$, the function $Z(\eta)$ is convex and unimodal (see [20]), and the optimum solution η^* can be found efficiently by applying the *Golden Section Search Method* (see [11]). Suppose now that $Z(\eta^*) > 1 - x^*_{t\beta}$ (otherwise no violated constraints exist). If, in addition, $u(\eta^*)$ also satisfies (29), then the positive components of the binary solution $u(\eta^*)$ are in one-to-one correspondence with the variables of a violated constraint. Otherwise the oracle returns no violated cover.

3.2 The Algorithm

We come back now to the first step in our algorithm, namely the choice of the set of admissible power values $\mathcal{P}$. Large sets are in principle more likely to produce better quality solutions. However, the ability of the solution algorithm to find optimal or simply good-quality solutions is strongly affected by $|\mathcal{P}|$, as we will show in more detail in the computational section. Thus, the size and the elements of $\mathcal{P}$ should represent a suitable compromise between these two opposite behaviors. Moreover, the effectiveness of the Branch&Cut is typically affected by the availability of a good initial feasible solution. In order to take all these questions into account, we decided to iteratively apply SOLVE-PI($\mathcal{P}$) to a sequence of feasible power sets $\mathcal{P}_0 \subset \mathcal{P}_1 \subset \cdots \subset \mathcal{P}_r$. Each invocation inherits all the generated cuts, the best solution found so far and the corresponding lower bound from the previous invocation. More precisely, if we denote by -99 the switched-off state (in dBm), and $P_{\min}^{dBm}$, $P_{\max}^{dBm}$ are the (integer) minimum and maximum power values (in dBm), then we have $\mathcal{P}_0 = \{-99, P_{\max}^{dBm}\}$, $\mathcal{P}_1 = \{-99, P_{\min}^{dBm}, \left\lfloor \frac{P_{\max}^{dBm} - P_{\min}^{dBm}}{2} \right\rfloor, P_{\max}^{dBm}\}$ and $\mathcal{P}_r = \{-99, P_{\min}^{dBm}, P_{\min}^{dBm} + 1, \ldots, P_{\max}^{dBm}\}$. The structure of the intermediate power sets will be described in Section 5.

The overall approach, denominated *WPLAN*, is summarized in Algorithm 1, where i denotes the current iteration, along with the associated best solution found x_i, the corresponding value LB_i, and the set of feasible powers $\mathcal{P}_i$. If SOLVE-PI($\mathcal{P}_i$) is executed in less than the iteration time limit TL_i then the residual time τ_i is used to increase the time limit of the following iteration (i.e., TL_{i+1}:= $\mathrm{TL}_{i+1} + \tau_i$). The initial incumbent solution x_{-1} corresponds with all transmitters switched off and no receiver served ($LB_{-1} = 0$).

Algorithm 1 WPLAN

Input: the power sets $\mathcal{P}_0, \mathcal{P}_1, \ldots, \mathcal{P}_r$, the iteration time limit TL_i for $i = 0, \ldots, r$
Output: the best solution x_r
$LB_{-1} := 0$
for $i = 0$ to r **do**
  1. Invoke SOLVE-PI($\mathcal{P}_i$) with LB_{i-1} and TL_i
  2. Get x_i, LB_i and τ_i
  3. $TL_{i+1} := TL_{i+1} + \tau_i$
end for
Return x_r

4 WiMAX network planning

The model introduced so far to solve the WND is a basic one and applies to most technologies, both in cellular and in broadcasting network design. Each technology is characterized by specific values for the constants appearing in the model. Also, each technology may require additional constraints and/or variables to model specific features.

In this section we introduce the technological elements and the modeling assumptions characterizing the specific technology addressed in this paper, namely the IEEE Standard 802.16, better known as *WiMAX* [13]. The major amendments concern the introduction of different frequency channels, channel capacity and traffic demand. In particular, each antenna emits at a specific frequency channel, and only iso-channel signals are considered as interfering. Also, a traffic demand is associated to each receiver, and the amount of total traffic served by an antenna is limited by the channel capacity. We note that the resulting formulation incorporates the common features of the so-called *Next Generation Networks*, which adopt *Orthogonal Frequency Division Multiplexing* (*OFDM*) [22].

Specifically, we consider the design of a *Fixed WiMAX Network* [13]: it consists of a set of installations - the *base stations* (*BS*) - distributed over a number of *sites* in order to provide connectivity to a set of customers' equipments - the *subscriber stations* (*SS*) - located in a target area. The target area is decomposed into a grid of approximately squared elementary areas called *testpoints* (TPs). All SSs located in a TP are aggregated in a single fictitious SS located in the centre of the TP. Each TP thus corresponds to a single receiver and the set of all the TPs corresponds to the set of receivers T in the basic model. For each TP $t \in T$ we introduce the quantity d_t to represent the joint bandwidth request (traffic demand) of all the SSs located in t.

A BS typically consists of a pylon accommodating a number of *transceivers (TRXs).* The set of all the TRXs that can be deployed in the target area corresponds to the set of transmitters B of our basic model. Every TRX $b \in B$ is characterized by a position (the TP in which the TRX is located) and by two radio-electrical parameters: *i*) *frequency channel* f, which belongs to a finite set of available channels F, each having a constant bandwidth D; *ii*) *emitted power* $P_b^f \in [P_{\min}, P_{\max}]$ on frequency $f \in F$.

Just like other Next Generation Networks, WiMAX supports the so-called *Adaptive Modulation and Coding* (AMC), which allows to change transmission scheme (*burst profile*) according to radio channel condition [4]. Each TRX can select a specific burst profile to serve each testpoint. The selected burst profile affects both the SIR threshold and the fraction of channel capacity exploited to fulfil the traffic demand of a testpoint. So, by denoting the set of available burst profiles as H, we introduce two new parameters for every $h \in H$: the *SIR threshold* δ_h that must be satisfied to ensure service coverage according to (1), and the spectral efficiency s_h, which is the bandwidth required to satisfy one unit of demand.

We are now able to write a modified version of the SIR inequality that takes into account the WiMAX specific features. In particular, TP $t \in T$ is served by $\beta \in B(t)$ if the following constraint is satisfied:

$$a_{t\beta} \cdot p_\beta^{f(\beta)} - \delta_{h(t)} \sum_{b \in B(t) \setminus \{\beta\}} a_{tb} \cdot p_b^{f(\beta)} \geq \delta_{h(t)}. \tag{34}$$

where $f(\beta) \in F$ is the transmission frequency assigned to β, whereas $h(t) \in H$ is the burst profile used to serve t. If we denote by $T(\beta)$ the family of testpoints served by $\beta \in B$, the limited channel capacity is expressed by the following constraint:

$$\sum_{t \in T(\beta)} d_t \cdot \frac{1}{s_h} \leq D \tag{35}$$

In order to represent these new features into our basic 0-1 program, we need to introduce new binary variables, obtained by slightly modifying the original ones to take into account multiple frequencies and burst profiles. We thus let:

$$x_{tb}^{fh} = \begin{cases} 1 & \text{if testpoint } t \in T \text{ is served by TRX } b \in B \\ & \text{on channel } f \in F \text{ with burst profile } h \in H \\ 0 & \text{otherwise} \end{cases}$$

$$z_{bl}^{f} = \begin{cases} 1 & \text{if TRX } b \in B \text{ emits at power level } l \in L \text{ on frequency } f \in F \\ 0 & \text{otherwise} \end{cases}$$

We also need to introduce a new version of the set of interfering levels $L^I(t, \beta, \lambda, \Gamma)$, that now depends also on the used burst profile $h \in H$, in addition to the TP $t \in T$, the server $\beta \in B(t)$, the server emitted power $\lambda \in L$ and the set of interferers $\Gamma \subseteq B(t) \backslash \{\beta\}$:

$$L^I(t, \beta, h, \lambda, \Gamma) = \{q \in L^{|\Gamma|} : a_{t\beta} \cdot P_\lambda - \delta_h \sum_{i=1}^{|\Gamma|} a_{tb_i} \cdot P_{q_i} < \delta_h\}$$

We can finally state the Power-Indexed formulation for WiMAX network design:

$$\max \quad \sum_{t \in T} \sum_{b \in B(t)} \sum_{f \in F} \sum_{h \in H} r_t \cdot x_{tb}^{fh} \qquad (WiMAX-PI)$$

$$\text{s.t.} \quad x_{tb}^{fh} + \sum_{k=1}^{\lambda} z_{\beta k}^{f} + \sum_{i=1}^{|\Gamma|} \sum_{j=q_i}^{|\mathcal{P}|} z_{b_i j}^{f} \leq |\Gamma| + 1 \qquad t \in T, \beta \in B(t), f \in F, h \in H, \; \lambda \in L, \Gamma \subseteq B(t) \backslash \{\beta\}, \; q \in L^I(t, \beta, h, \lambda, \Gamma) \qquad (36)$$

$$\sum_{b \in B(t)} \sum_{f \in F} \sum_{h \in H} x_{tb}^{fh} \leq 1 \qquad t \in T \qquad (37)$$

$$\sum_{l \in L} z_{bl}^{f} = 1 \qquad b \in B, f \in F \qquad (38)$$

$$\sum_{t \in T} \sum_{h \in H} d_t \cdot \frac{1}{s_h} \cdot x_{tb}^{fh} \leq D \qquad b \in B, f \in F \qquad (39)$$

$$x_{tb}^{fh} \in \{0, 1\} \qquad t \in T, b \in B(t), f \in F, h \in H \qquad (40)$$

$$z_{bl}^{f} \in \{0, 1\} \qquad b \in B, l \in L, f \in F \qquad (41)$$

Note that constraint (39) models the capacity constraint (35). All other constraints are simple generalizations of the basic ones.

5 Computational Results

In this section we present computational results over a set of realistic instances, developed with the Technical Strategy & Innovations Unit of British Telecom (BT).

The target of these tests is manyfold. First, we compare the new (PI) formulation to the (BM) formulation and show that (PI) outperforms (BM) in terms of quality of produced bounds and solutions found. Then, we illustrate specific features of the solution algorithm WPLAN, in particular we motivate the iterative approach with increasing power sets. Finally, we assess the ability of WPLAN to tackle realistic WiMAX network design instances. The tests were performed under Windows XP 5.1 operating system, with 1.80 GHz Intel Core 2 Duo processor and 2×1024 MB DDR2-SD RAM. The algorithm is implemented in C++ (under Microsoft Visual Studio 2005 8.0), whereas the commercial MILP solver ILOG Cplex 10.1 is invoked by ILOG Concert Technology 2.3.

5.1 The test-bed

All our instances correspond to an urban area located in the North Eastern part of Rome (Italy) selected in agreement with the engineers at BT, who considered it as a representative residential traffic scenario. All instances are available online [8].

Physical data of the target area are provided by a *Digital Elevation Model* (DEM) that represents the territory as a raster with a resolution of about 100 meters. The set of instances refers to an area of about 2.5 Km $\times$ 2.5 Km, corresponding to a residential neighborhood of Rome: according to the DEM resolution, the area is decomposed into a 25×25 testpoints grid. Nine instances of increasing dimension are drawn out of this basic area, that is classified as an urban environment.

In conformity with the regulations established by the *Italian Communications Regulatory Authority (Agcom)* for the deployment of WiMAX networks in Italy [2], we carry out the planning study for one of the provided transmission licenses. The frequency set F thus includes three 7 Mhz channels in the $(3.4 \div 3.6)$ GHz band.

A set H of four burst profiles is available for transmissions and the bandwidth demand d_t of each testpoint $t \in T$ is estimated according to the methodology described in [23], considering an urban scenario where customers are mainly residential.

On the basis of the target area size and considering an average spacing of about 0.6 Km, a number of potential BSs can be activated: each BS may install up to 3 directional TRXs with 120° antennas emitting in the power range [20,40] dBm. We refer to commercial devices operating in the 3.5 GHz frequency band. The azimuth of each antenna may vary in the range $[0°, 360°]$ with a step of 10°, thus allowing 36 distinct orientations for each TRX. So, in principle, we may have up to 12 different orientations associated with each directional TRX. However, similarly to [10], in order to limit the size of our instances, we choose to reduce the number of possible installations by selecting one most promising orientation in advance: for each directive antenna, we select the direction which maximizes coverage (an exact description of the selection strategy can be found at our WiMAX web page [8]).

The fading coefficients a_{tb} are computed by means of the path loss model *COST-231 Hata* [5], that is widely used and taken as reference for predictions in WiMAX networks [1, 12, 14]. However, we remind that the optimization model is independent of the particular propagation model that is used, as it only affects the coefficients of the fading matrix.

We define two types of instances, denoted by Sx with $x = \{1, \ldots, 5\}$ and Rx with $x = \{1, \ldots, 4\}$. For the Sx instances, the traffic is uniformly distributed among the testpoints and we assign unitary revenue to each TP (i.e. $r_t = 1$). Finding optimal coverage plan thus corresponds to define the plan with the maximum number of covered TPs. Only one frequency and one burst modulation are allowed.

For the Rx instances, we consider a traffic distribution based on the actual distribution of the buildings. We also introduce multiple frequencies and modulations. In this case, the revenue of each testpoint is proportional to the traffic generated.

5.2 Numerical Results and Comparisons

In order to evaluate the quality of our approach we compare the iterative procedure WPLAN with the direct application of Cplex to the (BM) formulation. In the first experiment we focus on a single instance of our test-bed and detail the behaviour of WPLAN for each invocation of SOLVE-PI($\mathcal{P}$).

Table 2: Description of the test-bed instances

ID	|T|	|B|	|F|	|H|
S1	100	12	1	1
S2	169	12	1	1
S3	225	12	1	1
S4	289	12	1	1
S5	361	12	1	1
R1	400	18	3	4
R2	441	18	3	4
R3	484	27	3	4
R4	529	27	3	4

The table refers to the 225 testpoints instance introduced in Table 2 (instance S3), with 12 candidate TRXs, one frequency and one burst profile. The sets of power values in the first three invocations of SOLVE-PI($\mathcal{P}$) are (in dBm) $\mathcal{P}_1 = \{-99, 40\}, \mathcal{P}_2 = \{-99, 20, 30, 40\}, \mathcal{P}_3 = \{-99, 20, 25, 30, 35, 40\}$, respectively. Then, in each of the following invocations, the set of power values is expanded by including two more values (suitably spaced).

In Table 3 we report, for each invocation of SOLVE-PI($\mathcal{P}$), the number |L| of considered power levels, the number of constraints of type (22) included in the initial formulation (node 0), the number of constraints of type (22) separated during the current iteration, the upper bound at node 0, the value |T*| of the final solution and the final gap. On the first line of the table, the third column shows the number of SIR (big-M) constraints (24) included in (BM). For the solution of (BM) we set a time limit of 3 hours. In order to evaluate the behaviour of the single iterations of WPLAN, and establish the correct sequence of power sets, in these experiments we set, for each invocation of SOLVE-PI($\mathcal{P}$), a time limit of 1 hour.

The figures in Table 3 are actually representative of the typical behaviour of WPLAN on all instances of our test-bed. In particular, some relevant observations can be derived from the results. First, the size of the (PI) formulation grows quickly with the number of power levels, and is typically much larger than that of (BM). This is counterbalanced by the quality of the upper bounds, which are constantly better for (PI) and, most important, the quality of the solutions found. Interestingly, the best solution is found quite early in the iterative procedure, namely for $|\mathcal{P}| \leq 6$. A similar behaviour is

Table 3: Behaviour of WPLAN

	|T|=225,	|B|=12,	|F|=1,	|H|=1		
Method	|L|	No. coverage constraints	No. added cuts	UB (node 0)	|T*|	gap%
(BM)	-	1170	-	221.3925	93	97.18
	2	5743	17	199.2193	106	0.00
	4	9035	7	204.2500	111	0.00
	6	14312	13	206.6261	111	59.03
	8	17142	45	209.4200	111	67.51
	10	24638	6	210.0000	111	79.99
WPLAN	12	27799	1	211.7000	111	82.05
	14	35944	0	212.0000	111	83.46
	16	38496	10	214.5930	111	85.48
	18	45425	2	215.8000	111	86.44
	20	48918	2	218.0000	111	89.99
	22	57753	3	218.0000	111	90.83

observed for the other instances reported in Table 4 as well. This motivated our choice of the sequence of feasible power values in the final version of WPLAN, where most of the computational effort is concentrated on small cardinality power sets, and only one large set. In particular, there will be only 4 iterations, corresponding to 2, 4, 6 and 22 power levels, respectively. Concerning the number of generated cuts, it is interesting to observe that in general they are not too many. Also, in most cases they include only two interferers, and in any case never more than three. In other words, even if several interferers can reach a given testpoint, still only very few of them gives a significant contribution to the actual overall interfering signal.

Table 4 reports the full set of results over our benchmark instances. In this case, we set a time limit of 3 hours both for the solution of (BM) and for WPLAN. The value of the best solutions found within the time limit are shown in column |T*|: two values are presented for (BM), namely the nominal value of the best solution returned by Cplex (in brackets) and its actual value computed by re-evaluating *off-line* the solution. The *gap* columns report the gap between upper and lower bound at termination, whereas the last column |L*| is the number of power levels used in the iteration in which WPLAN obtains the best solution.

Table 4: Comparisons between (BM) and WPLAN formulations

ID	|T|	(BM) |T*|	(BM) gap%	(BM) time (sec)	WPLAN |T*|	WPLAN time (sec)	WPLAN |L*|
S1	100	63 (78)	43.72	10698	74	10565	6
S2	169	99 (100)	56.18	10705	107	5591	4
S3	225	93 (93)	103.43	10761	111	7935	4
S4	289	77 (77)	202.24	10002	86	10329	6
S5	361	154 (154)	130.76	8110	170	8723	4
R1	400	370 (370)	7.57	10626	400	1579	2
R2	441	302 (303)	45.03	3595	441	1244	4
R3	484	99 (99)	385.86	10757	427	3472	2
R4	529	283 (286)	84.96	10765	529	2984	2

The results show that WPLAN outperforms (BM) in terms of quality of the solutions found and running times to obtain them. Even if in principle the reduced and quite small number of power values considered by WPLAN could result in poorer coverage w.r.t. the (BM), the figures clearly show that this is not the case. On one hand, this happens as a small number of well-spaced power values suffices in practice to obtain good coverage; indeed, it is common practice in WiMAX network planning to neglect intermediate values, i.e. a device is either switched-off or activated at its maximum power.

On the other hand, the size of the (BM) formulation and the ill-conditioning of the constraint matrix, along with the presence of the big-M coefficient, makes the solution process quite unstable, the solutions found unreliable and the branching tree extremely large. Due to rounding errors, Cplex tends to overestimate the actual value of the solutions found and is often lower than the one returned by the solver. All these difficulties are overcome by the new formulation (PI) and the overall solution approach WPLAN. This is particularly apparent for the R-instances, which seem to be quite easy for WPLAN but very difficult for (BM). Indeed, when no time limit is imposed to the solution of (BM), Cplex runs out of memory after about ten hours of computation without getting sensible improvements in the bounds. On the contrary, for $R1, R2$ and $R4$ less than 1 hour suffices to SOLVE-PI($\mathcal{P}$)

to terminate with the optimum solution (when $|\mathcal{P}| = 2$).

Table 5 shows the impact of the iterative approach on the quality of the solutions found. In particular we compare *cold starts*, which correspond to invoking SOLVE-PI($\mathcal{P}$) without benefiting from cuts and lower bounds obtained at former invocations, with *warm starts* which, in contrast, make use of such information. The value of the best solutions found during successive invocations of SOLVE-PI both under warm and cold starts are shown in the columns identified by $|L| = n$, where n denotes the number of corresponding power levels. The value of the best solution found at the first invocation is in column $|L| = 2$, while the value of the best solution and the number of levels used to find it are shown in column $|T^*|$ and $|L^*|$, respectively.

Table 5: Comparisons between warm and cold starts

ID	|T*|	|L*|	|L|=2	WARM START		COLD START	
				|L|=4	|L|=6	|L|=4	|L|=6
S1	74	6	69	72	74	71	58
S2	107	4	72	107	107	80	63
S3	111	4	75	111	111	100	97
S4	86	6	76	84	86	83	81
S4	170	4	127	170	170	110	127
R1	400	2	400	\	\	399	304
R2	441	4	416	441	\	394	355
R3	484	2	427	427	427	414	*
R4	529	2	529	\	\	512	*

For all S-instances the best solution can be found only thanks to warm start. Apparently SOLVE-PI encounters increasing difficulties to find good solutions as the number of power levels increases. This is mainly due to the large size of the corresponding instances, that, in two cases denoted by *, makes Cplex run out of memory while building the model. However, when a good initial solution is provided to SOLVE-PI, then this solution can be improved in most cases. We have already observed that for a larger number of levels (i.e. > 6), no improved solutions can be found for all instances in our test-bed. Finally, for $R1$ and $R4$ a solution covering the entire target area is found already with $|L| = 2$, while for $R2$ such a solution is found with $|L| = 4$ (and warm-start).

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