A MILP formulation for WiMAX Network Planning

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Abstract

We describe a Mixed Integer Linear Programming formulation to optimize base stations location and configuration of a wireless network implementing the IEEE standard 802.16 (WiMAX). The system elements relevant to the optimization model are discussed in detail.

Keywords : WiMAX, Network Planning, Testpoint Model, Mixed Integer Linear Programming.

1 Introduction

The extraordinary growth of demand for wireless connections resulted in highly congested frequency spectrums and in a drastic reduction of available sites to accommodate transmitting antennas.

This stimulated the development of optimization models and algorithms to support planning decisions. In this chapter we describe an optimization model for the WiMAX network planning problem. In order to do this, the physical and radio-electrical parameters relevant to the model are first identified and described. Such parameters are then associated with binary and semi-continuous decision variables. Logical relations, coverage and capacity requirements are represented by linear inequalities in the decision variables. We finally give an example of practical application of the resulting model to the planning of a WiMAX network in a large district of the city of Rome.

2 Optimization in wireless network design

A standard model, suitable for planning purposes, identifies a wireless network with a set of transmitting and receiving antennas scattered over a territory. Such antennas are characterized by a position (geographical coordinates and elevation) and by a number of radio-electrical parameters. The network design process consists in establishing locations and suitable radio-electrical parameters of the antennas. The resulting network is evaluated by means of two basic performance indicators: *i*) network *coverage*, that is the quality of the wanted signals perceived in the target region and *ii*) network *capacity*, that is the ability of the network to meet traffic demand. On the basis of quality requirements and projected demand patterns, suitable target thresholds are established for both indicators. In principle, coverage and capacity targets should be pursued simultaneously, as they both depend on the network configuration. However, in order to handle large real-life instances, conventional network planning resorts to a natural decomposition approach, which consists in performing coverage and capacity planning at different stages (see [28]). In particular, the network is designed by first placing and configuring the antennas so as to ensure the coverage of a target area, and then by assigning a suitable number of frequencies in order to meet (projected) capacity requirements. The final outcome can be simulated and evaluated by an expert, and the whole process can be repeated until a satisfactory result is obtained (see Fig. 1). Future change in demand patterns can be met by increasing sectorization (i.e. mounting additional antennas in a same site), by selecting new sites and by assigning additional transmission frequencies (see [25]).

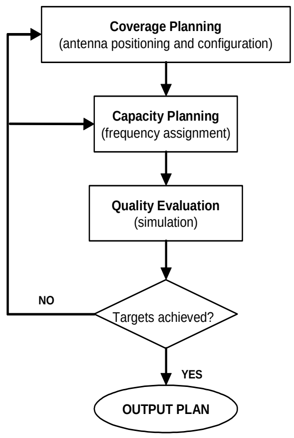

Figure 1: Phases of the conventional planning approach

The network planning process requires an adequate representation of the territory. In past years, the standard approach was to subdivide the territory into equally sized hexagons (see [27]) and basic propagation laws were implemented in order to calculate field strengths. By straightforward analytical computations these simplified models could provide the (theoretical) position of the antennas and their transmission frequencies. Unfortunately, the approximations introduced by this approach were in most cases unacceptable for practical planning, as the model does not take into account several fundamental factors (e.g., orography of target territories, equipment configurations, actual availability of frequencies and of geographical sites to accommodate antennas, etc.). Furthermore, the extraordinary increase of wireless communication quickly resulted in extremely large networks and congested frequency spectrum, and asked for a better exploitation of the available band. It was soon apparent that effective automatic design algorithms were necessary to handle large instances of complex planning problems, and to improve the exploitation of the scarce radio resources. These algorithms were provided by mathematical optimization. Indeed, already in the early 80's, it was recognized that the frequency assignment performed at the second stage of the planning process is equivalent to the *Graph Coloring Problem* (or to its generalizations). The graph coloring problem consists in assigning a color (= frequency) to each vertex (= antenna) of a graph so that adjacent vertices receive different colors and the number of colors is minimum. The graph $G = (V, E)$ associated with the frequency assignments of a wireless network is called *interference graph*, since edge $uv \in E$ represents interference between nodes $u \in V$ and $v \in V$ and implies that u and v cannot be assigned the same frequency (see [6]). The graph coloring problem is one of the most known and well studied topics in combinatorial optimization. A remarkable number of exact and heuristic algorithms have been proposed over the years in order to obtain optimal or sub-optimal colorings. Some of these methods were immediately at hand to solve the frequency assignment problem.

The development of mathematical optimization methods triggered the introduction of more accurate representations of the target territories. In particular, also inspired by standard *Quality of Service* (*QoS*) evaluation methodologies, the coarse hexagonal cells were replaced with (the union of) more handy geometrical entities, namely the *demand nodes* introduced by Tutschku [28], and with the now universally adopted *testpoints*. In the testpoints model, a grid of approximately squared cells is overlapped to the target area. Antennas are supposed to be located in the center of testpoints: all information about customers and QoS in a testpoint, such as traffic demand and received signals quality, are aggregated into single coefficients. The testpoints model allows for smarter representations of the territory, of the actual antennas position, of the signal strengths and of the demand distributions. This in turn permits a better evaluation of the QoS and, most important, makes it possible to construct more realistic interference graphs, thus leading to improved frequency assignments. Indeed, by means of effective coloring algorithms, it was possible to improve the design of large real-life mobile networks (see the FAP-web [13]), and also of analogue [18] and digital broadcasting networks [19].

Finally, basing on the testpoints model, it was also possible to develop accurate models and effective optimization algorithms to accomplish the first stage of the planning process, namely the coverage phase, in order to establish suitable positions and radio-electrical parameters for the antennas of a wireless network [8].

In recent years, thanks to the development of more effective optimization techniques and to the increase of computational power, a number of models integrating coverage and capacity planning have been developed and applied to the design of GSM ([26]), UMTS ([9, 12]), Analog and Digital Video Broadcasting ([19]) networks.

The optimization models above mentioned are defined by associating suitable decision variables $x \in R^n$ with the physical and radio-electrical antenna parameters (i.e. candidate locations, power values, activation statuses, transmission frequencies, service and coverage requirements, etc). Such variables must satisfy a number of constraints, which are represented by inequalities of the form $g(x) \leq b$, where $g : R^n \rightarrow R$ and $b \in R$. The set of the feasible values is defined as $X = \{x \in R^n : g_i(x) \leq b_i, i = 1, \ldots, m\}$. Finally, the general optimization problem can be written as:

$$\min_{x \in X} f(x). \tag{1}$$

where $f : R^n \rightarrow R$ is the *objective function* and may represent, for example, unsatisfied traffic demand or installation costs to be minimized. When f and g_i, for $i = 1, \ldots, m$, are linear functions, Problem (1) is a *Linear Program* (LP) [22]. If, in addition, some of the variables can only assume integer values, Problem (1) becomes a *Mixed Integer Linear Program* ($MILP$). We will show that the WiMAX network planning problem can be reduced to the solution of a suitable MILP.

Several optimization models (most often MILPs) have been proposed for wireless network planning, in broadcasting, mobile, military and civil communications. Such models can be solved by ad hoc algorithms or by commercial solvers. However, the large size of most instances of practical relevance and the notorious difficulty of the corresponding optimization problems, makes it often necessary to resort to heuristic procedures which typically yield suboptimal solutions.

In the following sections we formalize the WiMAX network planning problem and formulate it as a MILP. In Section (3) we describe the technological features which will be modelled by our decision variables. The overall MILP model is then shown in Section (4). An example of application is given in Section (5), by considering the design of a fixed WiMAX network in a district of the city of Rome.

3 Systems elements

In this section we introduce the technological elements and the modelling assumptions which provide the basis of the optimization model presented in Section (4). We consider the design of a *Fixed WiMAX Network* [1]: it consists of a set of installations - the *base stations* (*BS*) - distributed over a number of *sites* in order to provide connectivity to a set of customers' equipments - the *subscriber stations* (*SS*) - located in a portion of territory called *target area* (Fig. 2).

3.1 Representation of territory

The definition of an appropriate territory representation is an essential requirement for effective wireless network planning. The coverage is in fact evaluated on the basis of predicted propagation conditions and these are in turn calculated by a propagation model which takes into account the characteristics of the area. The choice of the model for the territory representation has therefore to be made very carefully as it deeply affects the quality of the planning.

Up to a few years ago, the target area was commonly modelled by superimposing a regular hexagonal grid to the region: a transceiver station was supposed to be placed in the centre of every hexagon.

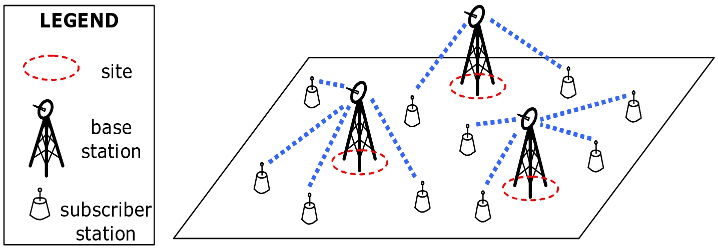

Figure 2: Scheme of a Fixed WiMAX Network

The *hexagonal grid model* was mainly suitable for a gross analysis, oriented to frequency reuse evaluation [14]. However, the assumptions of the model are far from being realistic and the resulting plans are of little or no use at all for practical implementations.
A more accurate representation, the so-called *Demand Node Concept (DNC)*, was introduced by Tutschku in [28]. A *demand node* is (the center of) an area in which users generate a quantum of traffic usually expressed in Erlang. On the basis of the traffic demand distribution over the territory, the DNC subdivides the target area in a set of demand nodes. The obtained discretization of the territory is therefore "traffic based" and the elementary components may have very different sizes. Demand nodes are in fact denser in high traffic areas and become sparser as traffic concentration decreases. As a consequence, the area of a demand node with low traffic density, such as a rural one, may be large. Now, since signal strength is evaluated only in the centre of the demand nodes, as the size increases the prediction quality gets worse.
In recent years, the more accurate *testpoints model* has been universally adopted for quality evaluation and planning purposes. In contrast to the DNC, the elementary areas are equally sized squares overlapped to the target area and traffic is aggregated

consequently by summing up all requests within a square. By choosing a suitable dimension of the basic square - the *testpoint* (TP) - signal strength in the centre of the corresponding area can be considered as an acceptable approximation for the whole square. In the sequel, the set of all testpoints is denoted by T.
Every SS located in a testpoint originates a demand for WiMAX services that turns into a bandwidth request. Such demand can be estimated on the basis of demographic information and economical considerations. For modeling purposes, the SSs located in a testpoint $t \in T$ are aggregated in a single fictitious SS located in the centre of t. The bandwidth request d_t of testpoint t is equal to the sum of the bandwidth requests of all the SSs in t. A parameter r_t is also introduced to represent the revenue that the network operator obtains from supplying services to customers in t.

3.2 Base stations and transceivers

The backbone of every wireless network is constituted by the base stations: these are the key elements for providing telecommunication services over the target area and generally consist of a pylon accommodating a number of *transceivers (TRX)*. A TRX is the basic device for radio transmissions management and operates by means of an antenna. In the following, the set of all the TRXs that can be deployed in the potential sites is denoted by B.
Every TRX is characterized by a position, represented by geographical coordinates, height and orientation (*azimuth* and *tilt*). Generally, a BS with its TRXs may be placed only in a limited number of locations because of technological, economical and environmental constraints. On the basis of these constraints the network operator may determine a set of potential installation sites. As the dimension of a testpoint is chosen to be small in comparison with the target area, a candidate site is conventionally identified with the centre of a testpoint, analogously to the fictitious subscriber station. A TRX is also characterized by a set of radio-electrical parameters that are strictly related to the size and the geometrical structure of its antenna. Most optimization models typically take into account the following features:

- *frequency channel* - the channel on which the TRX transmits. It belongs to a finite set of available channels F, each having a bandwidth D.
- *emitted power* - the power level P_b^f at which TRX $b \in B$ transmits on frequency $f \in F$. It belongs to a fixed range of feasible values $[P_b^{min}, P_b^{max}]$.
- *radiation pattern* - the spatial distribution of radiated energy as a function of the observer's position along a path or surface of constant radius [11]. In practice, instead of using a complete tridimensional pattern, a couple of bidimensional patterns is usually derived from two perpendicular planes crossed along the direction of maximum radiation [15].

In every site and for every frequency channel, it is possible to install either a single TRX, mounting an omnidirectional antenna, or several TRXs, each with a directive antenna. Note that the activation of a TRX may prevent the simultaneous activation of other potential TRXs in the same site and operating on the same frequency. This is the case, for example, of TRXs with a small difference in azimuth. A family $\mathcal{G} = \{G_1, \ldots, G_{|\mathcal{G}|}\}$ is therefore introduced, where $G_i \subseteq B$, $i = 1, \ldots, |\mathcal{G}|$, is a set of mutually exclusive TRXs. Finally, a parameter c_b is introduced to represent the overall cost of installation and activation of TRX b.

3.3 Propagation models

Quality and reliability of a radio connection depend, besides on radio-electrical parameters of base and subscriber stations, also on propagation conditions experienced by signals. The planning phase requires therefore the adoption of a *propagation model* that is able to predict these conditions and to calculate the overall strength attenuation. This is not a simple task, as the easy computation of the *free space loss* must be adjusted by taking into account additional loss and degradation phenomena that result from propagation in a real environment. The knowledge of landscape orography and human infrastructures is therefore an essential requirement. Nowadays, morphological data of a geographical area are usually collected in large databases as *Digital Elevation Model (DEM)* files, the most widespread and used format. A DEM generally represents the surface of a region by means of a raster, whose elements specify relevant features (elevation and composition in terms of vegetation, buildings, etc.) of the corresponding elementary portion of territory. By considering these characteristics, a propagation model provides an attenuation coefficient that can be composed to other relevant contributions, such as antenna gain and connector loss, in order to obtain the total radio link budget. In particular, the signal power $P_b^f(t)$ received by the SS in TP t from TRX b on frequency f is proportional by a coefficient $a_{tb} \in [0,1]$ to the power P_b^f emitted by TRX b on frequency f, namely:

$$P^f(t) = a_{tb} \cdot P_b^f \tag{2}$$

3.4 Service coverage

A SS is said to be *covered* by the network if the wanted signal is received with suitable quality. Coverage depends not only on the wanted signal strength, but also on the strength of other unwanted (interfering) signals. Specifically, the quality of the received signal is measured by means of the *Signal to Interference Ratio (SIR)*, which is defined as the ratio between the wanted signal power and the sum of the interfering signals powers, also including thermal noise (see [24]). A SS is regarded as covered if the SIR value is above a given threshold δ. Recalling that the power received by the SS located in TP $t \in T$ is given by (2) and denoting by β the TRX serving t on a frequency $f \in F$, the above requirement can be expressed by the following inequality:

$$\frac{a_{t\beta} \cdot P_\beta^f}{\sum_{b \in B \setminus \{\beta\}} a_{tb} \cdot P_b^f + N} \geq \delta \tag{3}$$

where N is the thermal noise and $\sum_{b \in B \setminus \{\beta\}} a_{tb} \cdot P_b^f$ is the cumulative interference generated by all other TRXs.

3.5 Adaptive Modulation and Coding

In a digital wireless system, the aim of modulation is to transmit a digital bit stream over an analogue bandpass channel. This can be done in conformity to a series of techniques, each defining a modulation scheme (see [25]). A scheme is characterized by a value of *spectral efficiency*, which expresses the amount of information that can be transmitted over a bandwidth unit. By defining more complex modulation schemes, a higher number of bits per symbol can be transmitted and thus higher spectral efficiency can be reached [29].

In a classical wireless system, a single modulation scheme is used to support all transmissions: its selection is oriented to meet worst case propagation conditions and resorts to power control to fit changes in channel characteristics over time. In order to provide higher capacity using limited bandwidth, it would be desirable to select more efficient schemes. However, these schemes are more vulnerable to interference and thus require a higher signal-to-noise ratio to ensure a fixed bit error ratio. Consequently, the adoption of complex modulation schemes can considerably reduce the network coverage area. The *Adaptive Modulation* overcomes limitations arising in a single scheme system: it grants flexibility by allowing a TRX to select the best modulation, among those supported by the system. The choice is made on the basis of the current propagation conditions by applying the following simple rule: *transmit at as high as possible data rate when channel state is good and transmit at lower rate when the state is poor* [10]. Higher data rates are achieved by using higher *order* schemes.

WiMAX also supports *Forward Error Correction (FEC)*, a technique for error control in data transmissions which relies on adding redundancy information to transmitted data: the redundant component allows the receiver to detect and correct errors without need of data retransmission. A FEC code operates at a *code rate*, typically a fractional number, that expresses the useful and non redundant portion of the total amount of transmitted information. Thus a higher code rate corresponds to transmit user information at a lower rate because of redundancy, but at the same time achieves a greater robustness to errors and consequently to interference. The implementation of both Adaptive Modulation and Forward Error Correction realizes an overall form of adaptive transmission called *Adaptive Modulation and Coding (AMC)* [10]. The combination of a modulation scheme with a FEC code rate defines a so called *burst profile.*

For modeling purposes, a set H is introduced to represent the available burst profiles. For every profile $h \in H$ two parameters are introduced:

- δ_h representing the *SIR threshold* that must be fullfilled to ensure service coverage according to (3)
- s_h representing the spectral efficiency associated with the combined modulation scheme and FEC rate

In Table (1) we resume the notation introduced so far.
We are finally able to state the (WPP):

Problem 3.1 (WPP) *Given sets B, T, F, H, $\mathcal{G}$, the attenuation matrix $[\mathbf{a}]_{t\in T,b\in B}$, the minimum and maximum power vectors $\mathbf{P}^{min}, \mathbf{P}^{max} \in R^{|B|}$ the demand vector $\mathbf{d} \in R^{|T|}$, the capacity vector $\mathbf{D} \in R^{|B|}$, the testpoint revenue vector $\mathbf{r} \in R^{|T|}$ and the TRXs installation costs $\mathbf{c} \in R^{|T|}$, and associated radio-electrical constants, the* WiMAX Network Planning Problem *is the one of finding the position and the emission powers of the TRXs so as to maximize the overall revenue.*

In [21], a hierarchy of major wireless network planning problems is identified. The classification takes into account a wide literature and is based on the decision parameters included in the optimization model. The root of the hierarchy is a general planning problem, namely the *Power and Frequency Assignment Problem* (PFAP), which asks for establishing suitable emission powers and transmission frequencies of the antennas so as to maximize the total revenue. Interestingly, by considering capacity constraints and multiple burst profiles, (WPP) generates a new class of problems, which further generalizes (PFAP).

Table 1: Summary of notation

T	set of testpoints (TP)
B	set of transceivers (TRX)
F	set of frequency channels
H	set of burst profiles
D	channel bandwidth
d_t	bandwidth demand of testpoint t
r_t	revenue from service supply of testpoint t
$\mathcal{G}$	family of sets of mutually exclusive TRXs
c_b	cost of installation and activation of TRX b
P_b^{min}	minimum emitted power of TRX b
P_b^{max}	maximum emitted power of TRX b
a_{tb}	total attenuation of signal power transmitted by TRX b to TP t
N	thermal noise
δ_h	SIR threshold of burst profile m
s_h	spectral efficiency of burst profile m

4 The formulation

In this section we describe a MILP formulation for (WPP) along with reduction techniques to improve the quality of the coefficient matrix. We concentrate on downlink transmission. Uplink can be modelled similarly and easily included. We use the notation introduced in the previous section and summarized in Table 1.
We introduce three sets of boolean variables:

$$z_t = \begin{cases} 1 \text{ if testpoint } t \in T \text{ is served} \\ 0 \text{ otherwise} \end{cases}$$

$$y_b^f = \begin{cases} 1 \text{ if TRX } b \in B \text{ transmitting on channel } f \in F \text{ is} \\ \qquad \text{activated} \\ 0 \text{ otherwise} \end{cases}$$

$$x_{tb}^{fh} = \begin{cases} 1 \text{ if testpoint } t \in T \text{ is served by TRX } b \in B \text{ on} \\ \qquad \text{channel } f \in F \text{ with burst profile } h \in H \\ 0 \text{ otherwise} \end{cases}$$

We introduce also a set of semi-continuous variables:

$$p_b^f \in \{0, [P_b^{min}, P_b^{max}]\}$$

representing the power emitted by TRX $b \in B$ on channel $f \in F$. The aim is to maximize total profit, which corresponds to the difference between total revenue from served testpoints and total TRXs activation costs. This can be expressed by the following objective function:

$$\max \quad \sum_{t \in T} r_t \cdot z_t - \sum_{b \in B} \sum_{f \in F} c_b \cdot y_b^f \tag{4}$$

A testpoint $t \in T$ is served if there exists at least one TRX b serving it on a frequency f with burst profile h, that is at least one variable x_{tb}^{fh} is equal to 1, for $b \in B, f \in F, h \in H$. This can be expressed by the following linear constraints:

$$z_t \leq \sum_{b \in B} \sum_{f \in F} \sum_{h \in H} x_{tb}^{fh} \qquad t \in T \tag{5}$$

In fact, if $x_{tb}^{fh} = 0$ for all $b \in B, f \in F, h \in H$, then $z_t = 0$ and t is not served.
If $x_{tb}^{fh} = 1$, for some $b \in B, f \in F, h \in H$, then TRX b must be activated on frequency f. This is ensured by the following:

$$x_{tb}^{fh} \leq y_b^f \qquad t \in T, b \in B, f \in F, h \in H \tag{6}$$

In fact, $y_b^f = 0$ implies $x_{tb}^{fh} = 0$.
Observe that if $x_{t\beta}^{fh} = 1$, for some $t \in T, \beta \in B, f \in F, h \in H$, then the SIR inequality:

$$a_{t\beta} \cdot p_\beta^f - \delta_h \sum_{b \in B \setminus \{\beta\}} a_{tb} \cdot p_b^f \geq \delta_h \cdot N \tag{7}$$

must be satisfied. This can be modelled by introducing a large costant M and by including in the MILP model the following constraint for all $t \in T, b \in B, f \in F, h \in H$:

$$a_{t\beta} \cdot p_\beta^f - \delta_h \sum_{b \in B \setminus \{\beta\}} a_{tb} \cdot p_b^f + M \cdot (1 - x_{t\beta}^{fh}) \geq \delta_h \cdot N \tag{8}$$

Note that when $x_{t\beta}^{fh} = 1$, then (8) reduces to the SIR inequality (7). On the other hand, when $x_{tb}^{fh} = 0$, constraint (8) reduces to the following inequality:

$$a_{t\beta} \cdot p_\beta^f - \delta_h \sum_{b \in B \setminus \{\beta\}} a_{tb} \cdot p_b^f \geq \delta_h \cdot N - M \tag{9}$$

which, for $M \geq \delta_h \sum_{b \in B \setminus \{\beta\}} a_{tb} \cdot P_b^{max} + \delta_h \cdot N$, is satisfied by every feasible power vector $\mathbf{p}$ and is therefore redundant.

When a $x_{tb}^{fh} = 1$, testpoint t consumes a portion $d_t \cdot s_h$ of the bandwidth of channel f. The sum of the bandwidth consumed by all testpoints served by b on f must therefore not exceed the total bandwidth D of the channel. This is expressed by the following set of constraints:

$$\sum_{t \in T} \sum_{h \in H} d_t \cdot s_h \cdot x_{tb}^{fh} \leq D \qquad b \in B \tag{10}$$

In order to prevent the activation of mutually exclusive TRXs, we introduce the following family of constraints:

$$\sum_{b \in G} y_b^f \leq 1, \qquad f \in F, G \in \mathcal{G} \tag{11}$$

Finally, observe that if TRX b is not activated on frequency f then $p_b^f = 0$, otherwise $P_b^{min} \leq p_b^f \leq P_b^{max}$. This can be expressed by:

$$p_b^f \leq y_b^f \cdot P_b^{max} \qquad b \in B, f \in F \tag{12}$$

and

$$p_b^f \geq y_b^f \cdot P_b^{min} \qquad b \in B, f \in F \tag{13}$$

We are finally able to summarize the overall MILP formulation for *(WPP)*:

$$\max \quad \sum_{t\in T} r_t \cdot z_t - \sum_{b\in B}\sum_{f\in F} c_b \cdot y_b^f \qquad (WPP-LP)$$

$$\text{s.t.} \quad a_{t\beta}\cdot p_\beta^f - \delta_h \sum_{b\in B\setminus\{\beta\}} a_{tb}\cdot p_b^f + M\cdot(1-x_{tb}^{fh}) \geq \delta_h \cdot N \qquad t\in T, \beta\in B, f\in F, h\in H \quad (14)$$

$$z_t \leq \sum_{b\in B}\sum_{f\in F}\sum_{h\in H} x_{tb}^{fh} \qquad t\in T \quad (15)$$

$$\sum_{t\in T}\sum_{h\in H} d_t\cdot s_h\cdot x_{tb}^{fh} \leq D \qquad b\in B, f\in F \quad (16)$$

$$\sum_{b\in G} y_b^f \leq 1 \qquad f\in F, G\in\mathcal{G} \quad (17)$$

$$x_{tb}^{fh} \leq y_b^f \qquad t\in T, b\in B, f\in F, h\in H \quad (18)$$

$$p_b^f \leq y_b^f \cdot P_b^{max} \qquad b\in B, f\in F \quad (19)$$

$$p_b^f \geq y_b^f \cdot P_b^{min} \qquad b\in B, f\in F \quad (20)$$

$$y_b^f \in \{0,1\} \qquad b\in B, f\in F \quad (21)$$

$$x_{tb}^{fh} \in \{0,1\} \qquad t\in T, b\in B, f\in F, h\in H \quad (22)$$

$$z_t \in \{0,1\} \qquad t\in T \quad (23)$$

$$p_b^f \in \{0,[P_b^{min}, P_b^{max}]\} \qquad b\in B, f\in F \quad (24)$$

4.1 Strengthening

It is common experience that the above formulation cannot be solved to optimality when applied to large real-life instances. In some cases, even finding feasible solutions can represent a difficult task, also for effective commercial MILP solvers such as CPLEX [30]. These difficulties are mainly determined by the following reasons: i) the general wireless network planning problem belongs to the class of *NP-hard* problems [19], and no polynomial time algorithm is known to the solution of (WPP-LP). This implies that the solution time can grow very fast as the number of (binary) variables grows. ii) the presence of the notorious "BIG M" coefficient M makes (WPP-LP) a *weak formulation*, that is the solution to its *linear programming relaxation*, obtained by removing the integrality stipulation on the variables, yields poor quality upper bounds. This in turn drives standard MILP solution algorithms to generate larger search trees. iii) the coefficient matrix of (WPP-LP) is (very) ill conditioned, because of the large range of feasible power values and overall attenuation coefficients of most real-life instances. Indeed, the ratio between the largest and the smallest coefficient in a SIR constraints (14) can be up to 10^{12}.

In order to overcome these difficulties, two re-formulation techniques have been recently investigated for DVB and UMTS network planning. In particular, the *Dantzig-Wolfe decomposition* can be applied to generate a set packing reformulation of (WPP-LP) with exponentially many columns [20] while an analogous of *Benders' decomposition* yields a set covering formulation with exponentially many rows [21]. Such non-compact reformulations have (0,1) coefficient matrices and must be solved by applying appropriate row and column generation mechanisms.

Another promising coefficient reduction approach has been proposed in [20], allowing to improve the quality of the coefficient matrices and reduce the value of M. It is based on the simple observation that the activation of an interfering TRX at its minimum emission power may be sufficient to prevent service in a testpoint. Analogously, the activation of a serving TRX at its minimum emission power may be sufficient, in special cases, to ensure coverage for any possible emission value of the interfering TRXs. We show now how to extend these ideas to reduce (WPP-LP).

Definition 4.1 *Let* $SIR(t, \beta, f, h)$ *be a constraint of set* (7)*. Then* $\gamma \in B - \{\beta\}$ *is a* superinterferer *for* $SIR(t, \beta, f, h)$ *if* $\delta_h \cdot a_{t\gamma} \cdot P_\gamma^{min} > a_{t\beta} \cdot P_\beta^{max} - \delta_h \cdot N$.

It is worth noting that a superinterfer γ is likely to be associated with a large coefficient $\delta_h \cdot a_{t\gamma}$ of constraint (7). If a superinterferer γ for a constraint $SIR(t, \beta, f, h)$ is activated on frequency f (i.e. $y_\gamma^f = 1$), then the corresponding SIR inequality (7) cannot be satisfied. This implies that $x_{t\beta}^{fm} = 0$, and the corresponding constraint (14) in (WPP-LP) becomes redundant. On the other hand, if γ is *not* activated on frequency f ($y^f_\gamma = 0$), then $p^f_\gamma = 0$ and the term $a_{t\gamma} \cdot p^f_\gamma$ can be removed from constraint (14). In other words, the constraint (14) associated with $SIR(t, \beta, f, h)$ can be replaced with the pair of inequalities:

$$y^f_\gamma \leq 1 - x^{fh}_{t\beta} \tag{25}$$

and

$$a_{t\beta} \cdot p^f_\beta - \delta_h \sum_{b \in B \setminus \{\beta,\gamma\}} a_{tb} \cdot p^f_b + \bar{M} \cdot (1 - x^{fh}_{t\beta}) \geq \delta_h \cdot N \tag{26}$$

where $\bar{M} = \delta_h \sum_{b \in B \setminus \{\beta,\gamma\}} a_{tb} \cdot P^{max}_b + \delta_h \cdot N$.
If fact, when $y^f_\gamma = 1$, then inequality (25) implies $x^{fh}_{t\beta} = 0$ and inequality (26) becomes redundant. When $y^f_\gamma = 0$ then $p^f_\gamma = 0$ and inequality (14) reduces to (26).

It is easy to see that the two new inequalities have *"nicer"* coefficients than the original one. In particular, inequality (25) is a standard *packing* constraint (all non-zero coefficients are 1). The large coefficient $a_{t\gamma}$, which appears in (14), does not appear in (26). Finally, we also have $\bar{M} \leq M - a_{t\gamma}$.

If the set of superinterferers of $SIR(t, \beta, f, h)$ is denoted by $Super(t, \beta, f, h)$, then the corresponding inequality (14) can be replaced with the following family:

$$y^f_\gamma \leq 1 - x^{fh}_{t\beta} \qquad \gamma \in Super(t, \beta, f, h) \tag{27}$$

and

$$a_{t\beta} \cdot p^f_\beta - \delta_h \sum_{b \in \hat{B}} a_{tb} \cdot p^f_b + \hat{M} \cdot (1 - x^{fh}_{t\beta}) \geq \delta_h \cdot N \tag{28}$$

where $\hat{B} = B \setminus Super(t, \beta, f, h) \cup \{\beta\}$ and $\hat{M} = \delta_h \sum_{b \in \hat{B}} a_{tb} \cdot P^{max}_b + \delta_h \cdot N$.

Now, suppose $a_{t\beta} \cdot P^{min}_\beta \geq \delta_h \sum_{b \in \hat{B}} a_{tb} \cdot P^{max}_b + \delta_h \cdot N$, i.e. β is a *superserver*. This implies that if all superinterferers are not activated on frequency f while the superserver β is active, then the SIR inequality is satisfied. Then (28) can be replaced by the following logic constraint:

$$y^f_\beta - \sum_{\gamma \in Super(t,\beta,f,h)} y^f_\gamma \leq x^{fh}_{t\beta} \tag{29}$$

In fact, when $y^f_\gamma = 0$ for all $\gamma \in Super(t, \beta, f, h)$ and $y^f_\beta = 1$, then $x^{fh}_{t\beta} = 1$. Note that in the above constraint, all coefficients are in the set $\{-1, 0, 1\}$.

5 An application scenario

The MILP formulation presented in Section 4 has been applied to the planning of a WiMAX network in a large district of the City of Rome. The MILP was solved by means of the commercial solver *ILOG CPLEX 10.0* and runs on an *Intel Core 2 Duo* 1.80 Ghz machine with 2 Gb RAM.

In the following, we describe more precisely how the reference scenario is built up.

The scenario refers to *Montesacro* district, in the Northeastern part of Rome. Originally a residential district, Montesacro has been recently pervaded by a network of small and medium enterprises. For this reason, it appears to be suitable for the penetration of WiMAX services and has been selected to build up a reference scenario.

Figure 3: Satellite image of Montesacro district

The target area is approximately 3 Km $\times$ 3 Km wide (Fig. 3), and is decomposed into a grid of 900 hundred testpoints (each testpoint is about 100 m $\times$ 100 m).

The bandwidth demand d_t of each testpoint $t \in T$ is estimated according to the methodology described in [3]. In particular, the projected demands are obtained by considering the extent and composition of the population, the offered range of services and the expected market penetration rate. The district of Montesacro is classified as an urban environment in which two groups of customers, namely *residential* and *small and medium enterprises (SME)*, generate an average demand of 30 Mbps/Km^2. This demand is then distributed over the testpoints taking into account the particular area demographics. For each customer class, two types of services are supposed to be available: a slower and cheaper basic service and a faster and more expensive premium service. The revenue r_t of a testpoint is computed by assuming a particular distribution of the demand d_t among the four possible combinations of customers and service classes. We also define positive transceivers activation costs c_b, according to the WiMAX Forum analysis [3]. Observe that, by considering no activation costs and unitary revenues, the objective function (4) reduces to $\max \sum_{t \in T} z_t$, corresponding to maximizing the number of served testpoints. This well represent the objective of a public authority, which is mostly interested in ensuring broadband services to the largest possible area.

On the basis of the territory configuration and the demand distribution, 5 potential sites (Fig. 4) were selected to accommodate candidate TRXs. One site is located in the court of an industrial service centre: a 15 meter high pylon is supposed to be directly installed on the soil. The other sites correspond to rooftops of buildings that are placed in elevated locations: in these cases, transceivers are mounted on 5 meter high pylons.

The notification of public auction for WiMAX licenses of the *Italian Communications Regulatory Authority (Agcom)* provides for the assignment of three licenses in the $(3.4 \div 3.6)$ GHz band [4]. The set F coincides with the three 7 Mhz frequency channels that a licence grants in downlink. In every site and for every frequency channel, it is possible to install either a single TRX, equipped with an omnidirectional antenna (see Table 2 for the radio-electrical parameters), or up to three TRXs, each equipped

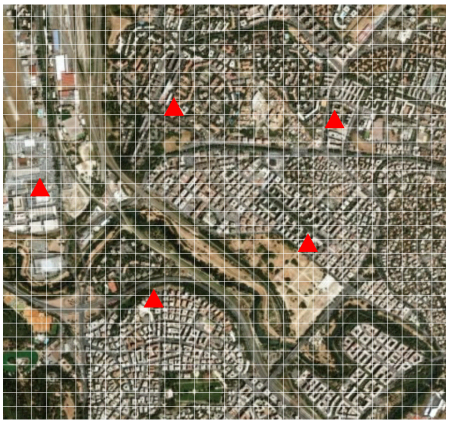

Figure 4: The target area: testpoints and sites

with a 120° directive antenna (Table 3).

Table 2: Omnidirectional antenna parameters

Gain	8.5 dBi
Power	[10,50] dBm

Table 3: Directive antenna parameters

Gain	15 dBi
Power	[10,50] dBm
Azimuth	$[0°, 360°]$ 10° step

The azimuth of a directive antenna may vary in the range $[0°, 360°]$ with a step of 10° thus allowing 36 distinct orientations. For every orientation, a TRX b is added to set B. In these tests, we did not take into account tilt. However, a discrete number of tilts, say k, can be easily considered in the model by including k copies of each $b \in B$. If the set of all candidate sites is denoted by S, then for each site $s \in S$ 37 potential TRXs are introduced:

- one TRX corresponding to the omnidirectional antenna and denoted by $(s, 0)$
- 36 directive TRXs, each corresponding to one specific orientation and denoted by (s, i), for $i = 1, 2, \ldots, 36$

The activation in a same site and on a same frequency of overlapping directive (120°) transceivers is not allowed: in other words, the activation of TRX (s, i) prevents the activation of TRX $(s, i-k)$, for $k = 1, \ldots, 11$, and of TRXs $(s, i+k)$, for $k = 1, \ldots, 11$, associated with the preceding and the following 11 orientations. Analogously, the activation of the omnidirectional antenna $(s, 0)$ prevents the activation of any other TRX (s, i), for $i = 1, \ldots, 36$. In other words, for each site $s \in S$, exactly 2×36 mutually exclusive sets of TRXs are identified, namely the sets $G^-_{s,i} = \{(s, 0), (s, i-11), \ldots, (s, i)\}$ and $G^+_{s,i} = \{(s, 0), (s, i), \ldots, \ldots, (s, i+11)\}$, for $i = 1, \ldots, 36$. Thus, the family of mutually exclusive TRXs is given by $\mathcal{G} = \{G^+_{s,i} : s \in S, i = 1, \ldots, 36\} \cup \{G^-_{s,i} : s \in S, i = 1, \ldots, 36\}$.

The cost c_b of a TRX is obtained by summing up a fixed and a variable component [3]. The fixed component includes all the elements that precede the installation of any WiMAX equipment (acquisition of sites and pylons positions, backhaul interface equipment, etc.). The variable component is strictly related to the mechanical and radio-electrical specifications of the TRX.

In order to predict propagation conditions, the *COST-231 Hata Model* [5] is adopted. However, it must be noted that the optimization model is independent from the particular propagation model that is used as it only affects coefficients of the input attenuation matrix. COST-231 Hata is a path loss model developed as an extension of the *Hata-Okomura* model [17], [23]: it is designed for transmissions on the band from 500 to 2000 MHz and includes a correction term in order to take into account the typology of the area (urban, suburban or rural). It is widely used and taken as reference for predictions in WiMAX networks [2]. Though out of its frequency range of definition, it is also used in WiMAX networks operating in the 3.5 GHz band [7], [16]. In the case of Montesacro, the correction factors for an urban environment are applied. Elevation data required by the model are provided by a digital terrain database of the area.

In most WiMAX implementations, the number of offered burst profiles is typically a small fraction of all the possible [10]. For the Montesacro network a set of six profiles involving the four possible modulation scheme are provided (Table 4). The reference scenario

Table 4: Directive antenna parameters

Modulation scheme	Code rate
BPSK	1/2
QPSK	1/2
16-QAM	1/2
16-QAM	3/4
64-QAM	1/2
64-QAM	3/4

was solved by applying the commercial solver ILOG CPLEX 10.0. The output plan is shown in Fig.5. The grey squares are associated with zero demand and zero revenue testpoints, which in most cases correspond to unbuilt areas. Three out of the five potential installations are activated (darker squares in the figure): these correspond to the most central and higher sites. The testpoints served by each installation are represented by different colors. The white testpoints are unserved. Observe that some of the white testpoints are completely surrounded by served areas: they correspond to residential areas with low revenue customers. In this case, the limited bandwidth capacity is allocated to more profitable testpoints that generate business premium traffic. The unserved testpoints at the boundaries of the target area are due to signal fading. Each base station transmits on all available channels and thanks to quite fair propagation conditions is able to use in most cases QAM-64 based burst profiles.

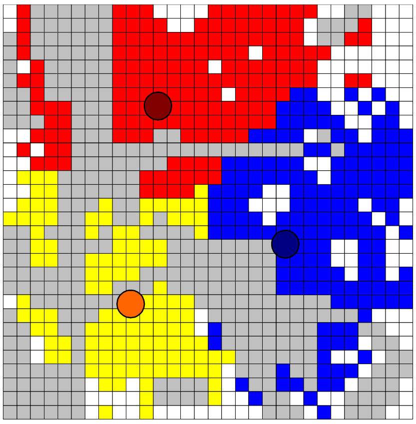

Figure 5: Testpoints assignment and activated base stations

6 Conclusions

This chapter presented an optimization model for the WiMAX network planning problem. The major physical and radio-electrical parameters are identified and represented by the decision variables of a suitable Mixed Integer Linear Program. An example of application to the planning of a large Roman district is finally shown.

As most wireless planning problems, the WiMAX Network Planning Problem poses complex modelling and computational challenges. Preliminary experiments show that running times are quite sensitive to the number of testpoints and antennas. Moreover, they exhibit a strong dependency of the running times on the number of available frequencies. This is because the MILP model shows a large symmetry due to the equivalence of variables associated with different frequencies. The development of symmetry breaking methodologies is thus a fundamental issue of future algorithmic research.

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