Robust Optimization under multi-band uncertainty

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Classical Optimization

$$\max \sum_{j \in J} c_j x_j \qquad \text{(NOMINAL PROBLEM)}$$
$$\sum_{j \in J} a_{ij} x_j \le b_i \qquad \forall i \in I$$
$$x_j \ge 0 \qquad \forall j \in J$$
$$x_j \in \mathbb{Z}_+ \qquad \forall j \in J_{\mathbb{Z}} \subseteq J$$

Theoretical assumption: all the coefficients a_{ij} are known exactly.

However, many real problems involve **uncertain** coefficients.

Consequences of neglecting uncertainty:

- optimal solutions may heavily lose in quality;
- feasible solutions may become infeasible.

Robust Optimization

Inclusion of coefficient uncertainty in the problem through stricter constraints that protect against coefficient deviations.

$$\max \sum_{j \in J} c_j x_j \qquad \text{ROBUST COUNTERPART general form}$$
$$\sum_{j \in J} \bar{a}_{ij} x_j + DEV(x, D) \le b_i \qquad \forall i \in I$$
$$x_j \ge 0 \qquad \forall j \in J$$
$$x_j \in \mathbb{Z}_+ \qquad \forall j \in J_{\mathbb{Z}} \subseteq J$$

- $\bar{a}_{ij}$ = known nominal value of the uncertain coefficient a_{ij}
- $DEV(x, D)$ = worst deviation allowed by the uncertainty set D for a solution x

Single-band uncertainty

The classical uncertainty model by Bertsimas and Sim (2004).

Main assumptions:

- single symmetric deviation band $a_{ij} \in [\bar{a}_{ij} - d_{ij}^{\max}, \bar{a}_{ij} + d_{ij}^{\max}]$
- upper bound on the number of coefficients deviating from the nominal value in each constraint (i.e., $a_{ij} \neq \bar{a}_{ij}$)

Strongpoint:

- linear and compact Robust Counterpart;

Drawback:

- the behaviour of uncertainty inside the band is completely neglected

$\Rightarrow$ POOR MODELING OF ARBITRARY NON-SYMMETRIC DEVIATION BEHAVIOUR (common case in real problems)

Multi-band uncertainty

Generalization of the Bertsimas-Sim model that breaks the single deviation band into multiple bands.

Main assumptions:

- multiple non-symmetric deviation bands $d_{ij}^k \in (d_{ij}^{k-1}, d_{ij}^k]$;
- lower and upper bounds on the number of coefficients deviating from the nominal value in each constraint.

Strongpoints:

- linear and compact Robust Counterpart;
- efficient separation of cuts imposing robustness (based on solving a min-cost flow problem;
- high modeling power of arbitrary deviation distributions (in particular deviation histograms built on historical data).

Project ROBUKOM

All the proposed developments about multi-band uncertainty are produced within ROBUKOM, a research project that aims for developing new models and algorithms for the design of robust telecommunication networks. The Project Partners of ZIB in ROBUKOM are:

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