

Negative Cycle Separation in Wireless Network Design*

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Abstract The Wireless Network Design Problem (WND) consists in choosing values of radio-electrical parameters of transmitters of a wireless network, to maximize network coverage. We present a pure 0-1 Linear Programming formulation for the WND that may contain an exponential number of constraints. Violated inequalities of this formulation are hard to separate both theoretically and in practice. However, a relevant subset of such inequalities can be separated more efficiently in practice and can be used to strengthen classical MILP formulations for the WND. Preliminary computational experience confirms the effectiveness of our new technique both in terms of quality of solutions found and provided bounds.

1 Introduction

Wireless networks have shown a rapid growth over the past two decades and now play a key role in new generation telecommunications networks. Scarce radio resources, such as frequencies, have rapidly become congested and the need for more

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effective design methods arose. A general planning problem consists in establishing the radio-electrical parameters (e.g., power emission and frequency) of the transmitters of a wireless network so as to maximize the overall network coverage. To present our original contribution, in this paper we focus only on establishing power emissions. This is actually a basic problem in all wireless planning contexts that can be easily extended by introducing additional elements, such as frequencies [4, 5].

For our purposes, a wireless network can be described as a set of transmitters B distributing a telecommunication service to a set of receivers T . Each transmitter $b \in B$ emits a radio signal with power $p_b \in [0, P_{\max}]$. The power $p_b(t)$ that receiver t gets from transmitter b is proportional to the emitted power p_b by a factor $a_{tb} \in [0, 1]$, i.e. $p_b(t) = a_{tb} \cdot p_b$, commonly called *fading coefficient*. Among the signals received from transmitters in B , receiver t can select a *reference signal* (or *server*), which is the one carrying the service. All the other signals are interfering.

A receiver t is regarded as served by the network, specifically by server $\beta \in B$, if the ratio of the serving power to the sum of the interfering powers (*signal-to-interference ratio* or *SIR*) is above a threshold δ [11], (*SIR threshold*), whose value depends on the technology and the desired quality of service:

$$\frac{a_{t\beta} \cdot p_\beta}{\mu + \sum_{b \in B \setminus \{\beta\}} a_{tb} \cdot p_b} \geq \delta \quad (1)$$

where the system noise $\mu > 0$ is assimilated to an interfering signal with fixed (very low) power emission.

For every $t \in T$, we have one inequality of type (1) for each potential server $\beta \in B$: in particular, we denote by $SIR(t, b)$ the inequality (1) associated with receiver t and server b . Receiver t is served if at least one of these inequalities is satisfied or, equivalently, if the following disjunctive constraint is satisfied:

$$\bigvee_{\beta \in B} \left(a_{t\beta} \cdot p_\beta - \delta \cdot \sum_{b \in B \setminus \{\beta\}} a_{tb} \cdot p_b \geq \delta \cdot \mu \right) \quad (2)$$

Each linear inequality of the above disjunction is obtained by simple algebra from the SIR expression (1).

If each receiver $t \in T$ is associated to a value $r_t > 0$ that expresses revenue obtained by serving t , the *Wireless Network Design Problem (WND)* consists in setting the power emission of each transmitter $b \in B$ and the server of each receiver in $t \in T$ with the aim of maximizing the overall revenue of served receivers.

2 A pure 0-1 Linear Programming formulation for the WND

The WND is often approached by solving a suitable Mixed-Integer Linear Program (MILP): first, a binary variable x_{tb} is introduced for every $t \in T$, $b \in B$, with $x_{tb} = 1$ if and only if b serves t ; then, variables x_{tb} are used to replace each disjunction (2)

with a set of $|B|$ linear constraint, that, however, include large positive constants, the notorious *big-M coefficients* [5, 7]. The (linear) objective function aims to maximize the overall revenue from coverage, i.e. $\max \sum_{t \in T} \sum_{b \in B} r_t \cdot x_{tb}$ and requires the additional constraints:

$$\sum_{b \in B} x_{tb} \leq 1 \quad t \in T \quad (3)$$

to ensure that each receiver is associated to at most one server. A vector $x \in \{0, 1\}^{T \times B}$ satisfying (3) is a *server assignment*.

The resulting MILP presents severe drawbacks, highlighted in several works, e.g. [5, 7, 8]. First, the coefficients in the SIR inequalities may vary over a very wide range, with differences up to 10^{12} or even larger. This makes the constraint matrix very ill-conditioned and the solutions returned by solvers are often inaccurate and may contain errors. Also, the presence of big- M terms results in weak bounds thus leading to very large search trees. To tackle these problems a number of different approaches were recently proposed. For a comprehensive introduction to these related works, we refer the reader to [4, 5, 7].

In this paper, we propose an alternative pure 0-1 Linear Programming formulation for the WND, whose defining inequalities are linear constraints in the assignment variables x_{tb} . Such inequalities are thus valid for all the formulations that are derived from the previously introduced MILPs and can be included to strengthen them.

Let now $\tilde{x} \in \{0, 1\}^{T \times B}$ be a server assignment and let Σ denote the set of all the SIR inequalities $SIR(t, b)$ and the lower and upper bounds constraints $0 \leq p_b \leq P_{\max}$ on power emissions. With \tilde{x} we associate the subsystem $I(\tilde{x})$ of SIR inequalities (1) whose corresponding variables \tilde{x}_{tb} are activated, i.e:

$$I(\tilde{x}) = \{SIR(t, b) \in \Sigma : \tilde{x}_{tb} = 1\}$$

It is easy to check if $I(\tilde{x})$, extended with lower and upper bounds on the variables p_b , is feasible. If this is the case, all of the assigned testpoints can actually be served by the network, and we say that x is a *feasible server assignment*.

At this point, we can restate the WND as the problem of finding a feasible server assignment that maximizes the revenue function. To this aim, a simple characterization of all the feasible server assignments goes as follows. Denote by IS the set of subsystems $I(x)$ such that x is *not feasible*. Then $\tilde{x} \in \{0, 1\}^{T \times B}$ is a feasible server assignment if and only if \tilde{x} satisfies the following system of linear inequalities:

$$\sum_{(t,b) \in I} \tilde{x}_{tb} \leq |I| - 1 \quad \forall I \in IS \quad (4)$$

The above system is in general very large and the inequalities must be generated dynamically. Unfortunately, the separation of violated inequalities (4) is hard, both theoretically and in practice [2, 10]. Moreover, it may entail some of the numerical difficulties associate with the MILP formulations for the WND. Still, a relevant subset of these inequalities can be separated more effectively, as we describe next.

To this end, we proceed in a similar way to [8]. Namely, we generate a new system Σ' obtained from Σ by substituting each (1) with the system:

$$\frac{a_{t\beta} \cdot P_\beta}{a_{tb} \cdot p_b} \geq \delta \quad \forall b \in B \setminus \{\beta\} \quad (5)$$

where, to simplify the notation, we assume that B also contains the noise μ as a fictitious transmitter with fixed power emission. It is not difficult to see that Σ' is a relaxation of Σ and every infeasible subsystem of Σ' corresponds to an infeasible subsystem of Σ . Basically, this relaxation corresponds to considering a receiver as served if the power emission of its server suffices to contrast each interferer individually and the thermal noise. Or, alternatively, if its *best server* is “stronger” than its *strongest interferer*. In [8] the authors show that, in most cases of practical interest, this is indeed a good approximation of the original SIR constraint.

By assuming $p_b \in [\varepsilon, P_b]$, with $\varepsilon > 0$ very small, and by taking the logarithm² of both left and right hand side multiplied by 10, the system Σ' can be rewritten as:

$$q_b - q_\beta \leq w_{\beta b}^t \quad t \in T, \beta \in B, b \in B \setminus \{\beta\} \quad (6)$$

where $q_b = 10 \log_{10} p_b$ for all $b \in B$ and $w_{\beta b}^t = \lceil 10(\log_{10} a_{t\beta} - \log_{10} a_{tb} - \log_{10} \delta) \rceil$, extended with the lower and upper bounds $10 \log_{10} \varepsilon \leq q_b \leq 10 \log_{10} P_b$, for all $b \in B$.

In this way, the system Σ' is transformed into a *system of difference inequalities* (lower and upper bounds can be easily represented in this form as well), where each constraint (6) is associated with a server β and a receiver t and thus with an assignment variable $x_{t\beta}$.

Now, given a generic system of difference constraints Σ^d :

$$(i) \quad t_v - t_u \leq l_{uv}, (u, v) \in A \quad (7)$$

where $t \in \mathbb{R}^A$ and $l \in \mathbb{Z}^A$, we can consider the associated weighted directed graph $G = (V, A)$, with weight function l . Then, it is well known that every infeasible subsystem of (7) contains (the constraints corresponding to) the arcs of a negative directed cycle of G [9]. Also, denoting by $x \in \{0, 1\}^A$ the incidence vector of (the arcs corresponding to) a feasible subsystem of Σ^d , then x is the set of solutions to:

$$(i) \quad \sum_{uv \in C} x_{uv} \leq |C| - 1, C \in \mathcal{C}^- \quad (8)$$

$$x \in \{0, 1\}^A$$

where \mathcal{C}^- is the set of negative directed cycles of G .

In [6] we develop an exact approach to the separation of violated inequalities (8.i). The resulting algorithm can be used to separate the violated inequalities associated with the system (6) (including upper and lower bounds on the q variables expressed as difference inequalities) which correspond to negative directed cycles in the associated directed graph. One of these cycles C corresponds to a subset of

² This corresponds to rewriting all quantities in dB format.

constraints of (6) associated with the pairs $I_C = \{(\beta_1, t_1), \dots, (\beta_m, t_m)\} \subseteq B \times T$ (plus possibly some lower and upper bound constraints).

One can show that $\beta_1 \neq \beta_2 \neq \dots \neq \beta_m$ and $t_1 \neq t_2 \neq \dots \neq t_m$ and the valid constraint:

$$\sum_{(t,b) \in I_C} x_{tb} \leq |I_C| - 1 \quad (9)$$

may be added to the formulation. In our preliminary results, however, we limit to consider cycle inequalities with $|C| = 2$ and separate them by enumeration.

3 Preliminary Computational Results

We test the performance of our new approach to WND on a set of 15 realistic instances, developed with the Technical Strategy & Innovations Unit of British Telecom Italia (BT Italia SpA). All the instances refer to a *Fixed WiMAX Network* [1], deployable in an urban residential area and consider various scenarios with up to $|T| = 529$ receivers, $|B| = 36$ transmitters, $|F| = 3$ frequencies, $|H| = 4$ burst profiles (see Table 1). We remark that the experiments refer to a formulation that extends the basic one considered in Section 1, by including frequency channels and modulation schemes as additional decision variables. Such formulation is denoted by *(BM)* and captures specific features of so-called *Next Generation Networks* like WiMAX [1]. For a detailed description of *(BM)*, we refer the reader to [3].

For each instance, we present preliminary computational results obtained by solving the big-M formulation *(BM)* and its corresponding Power-Indexed formulation *(PI)* [5]. We consider *(BM)* and *(PI)* formulations with and without the valid inequalities (8) obtained for $|C| = 2$. Formulations strengthened through (8) are distinguished by adding *S*-, i.e. *(S-BM)* and *(S-PI)*.

Experiments are run by imposing a time limit of 1 hour and by using a machine with a 1.80 GHz Intel Core 2 Duo processor and 2 GB of RAM. Table 1 reports the performance of the four considered formulations over the set of WiMAX instances. We solve *(BM)* and *(S-BM)* by direct application of IBM ILOG Cplex 11.1 and we report i) the upper bound UB_0 obtained at node 0 of the branch-and-bound tree, ii) the value $|T^*|$ of the best solution found within the time limit and iii) the final integrality gap *gap%*. The presence of two values in some lines of the column $|T^*|$ of *(BM)* indicates that the coverage plans returned by Cplex contain errors and some receivers are actually not covered. We instead solve *(PI)* and *(S-PI)* by the incremental algorithm WPLAN described in [5] and we report i) the upper bound UB_0 obtained at node 0 when considering the basic set of power levels, and ii) the value $|T^*|$ of the best solution found by WPLAN within the time limit.

By adding the new valid inequalities (8) for $|C| = 2$, in most cases stronger bounds are obtained at node 0 and smaller integrality gaps are reached within the time limit. In particular, the benefits are particularly evident in the case of the big-M formulation: in three cases, namely I6, I12, I15, the value of the best solution is increased, even eliminating coverage errors (I6, I15).

Table 1 Comparisons between (BM) and (PI) with and without valid inequalities (8)

ID	T	B	F	H	(BM)			(S-BM)			(PI)		(S-PI)	
					UB ₀	T*	gap%	UB ₀	T*	gap%	UB ₀	T*	UB ₀	T*
I1	100	12	1	1	98.36	66 (70)	29.43	96.78	66 (70)	27.68	90.77	75	90.23	75
I2	169	12	1	1	165.47	97	59.81	163.15	97	57.39	153.12	101	152.37	101
I3	196	12	1	1	193.61	102 (105)	77.87	192.02	102 (105)	75.11	179.35	108	177.92	108
I4	225	12	1	1	219.76	92	81.13	218.36	92	79.36	202.44	92	201.54	92
I5	289	12	1	1	287.20	76 (77)	195.44	287.20	76 (77)	194.92	274.62	85	274.13	85
I6	361	12	1	1	352.01	126 (132)	154.87	350.43	140	138.76	337.22	156	336.46	156
I7	400	18	1	1	397.21	166	132.01	396.79	166	131.32	386.07	184	384.95	184
I8	400	18	3	4	400.00	356	12.36	400.00	356	12.36	396.53	372	395.80	372
I9	441	18	3	4	441.00	266 (270)	63.33	441.00	266 (270)	63.33	438.28	295	437.52	295
I10	484	27	3	4	484.00	120 (122)	296.72	484.00	120 (122)	296.72	479.10	242	478.68	242
I11	529	27	3	4	529.00	77	587	529.00	77	587	523.15	168	521.76	168
I12	400	36	1	4	398.04	72 (74)	287.30	396.93	77 (78)	264.85	389.61	102	389.14	102
I13	441	36	1	4	433.21	184	131.03	431.42	184	129.77	414.93	194	413.78	194
I14	484	36	1	4	482.78	209	108.31	481.66	209	107.56	472.44	251	471.58	251
I15	529	36	1	4	517.89	98 (105)	226.44	516.14	114	198.57	503.32	232	502.67	232

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