

Extended Component Importance Measures Considering Aleatory and Epistemic Uncertainties

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Abstract—We introduce extended component importance measures (Birnbaum importance, RAW, RRW, and Criticality importance) considering aleatory and epistemic uncertainties. The Dempster-Shafer theory, which is considered to be a less restricted extension of probability theory, is proposed as a framework for taking into account both aleatory and epistemic uncertainties. The epistemic uncertainty defined in this paper is the total lack of knowledge of the component state. The objective is to translate this epistemic uncertainty to the epistemic uncertainty of system state, and to the epistemic uncertainty of importance measures of components. Affine arithmetic allows us to provide much tighter bounds in the computing process of interval bounds of importance measures, avoiding the error explosion problem. The efficiency of the proposed measures is demonstrated using a bridge system with different types of reliability data (aleatory uncertainty, epistemic uncertainty, and experts' judgments). The influence of the epistemic uncertainty on the components' rankings is described. Finally, a case study of a fire-detector system located in a production room is provided. A comparison between the proposed measures and the probabilistic importance measures using two-stage Monte Carlo simulations is also made.

Index Terms—Affine arithmetic, Dempster-Shafer theory, epistemic uncertainty, experts' judgments, importance measures, pignistic reliability.

ACRONYMS

TBM	Transferable Belief Model
D-S	Dempster-Shafer
RAW	Risk Achievement Worth
RRW	Risk Reduction Worth
CR	Criticality importance
BBA	Basic Belief Assignment

NOTATION

Bel	Belief measure
Pl	Plausibility measure
m^X	BBA on the frame of discernment X

F_i	Failure state of component i
W_i	Working state of component i
R_S	System's reliability
$BetR_S$	Pignistic system's reliability
Q_S	System's unreliability
$BetQ_S$	Pignistic system's unreliability
I^B	Birnbaum importance measure
I_{Bet}^B	Birnbaum pignistic importance measure
I^{RAW}	RAW importance measure
I_{Bet}^{RAW}	RAW pignistic importance measure
I^{RRW}	RRW importance measure
I_{Bet}^{RRW}	RRW pignistic importance measure
I^{CR}	CR importance measure
I_{Bet}^{CR}	CR pignistic importance measure
k	Conflict factor between BBAs
d	Distance between BBAs
S	Similarity function
$S_i(t)$	Survival function of the i th component
n	Number of components
β_i	Shape parameter in $S_i(t)$
η_i	Scale parameter in $S_i(t)$
$\underline{\lambda}$	Lower bound of failure rate
$\bar{\lambda}$	Upper bound of failure rate
\underline{R}_s	Lower bound of system's reliability
\bar{R}_s	Upper bound of system's reliability

I. INTRODUCTION

AN important problem in reliability theory is to identify components within the system that more significantly influence the system's behavior with respect to reliability or availability. As we cannot improve all components at one time to improve a system's reliability, priority should be given to components that are more important. In that way, reliability managers can prioritize where investments should be made to guarantee the maximum increase of reliability considering the whole system.

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Historically, Birnbaum [1] was the first to quantify the contributions of components' reliability to binary coherent systems' reliability. A feature of Birnbaum importance measure is that it does not depend on the given component's reliability. Therefore, two components may have a similar measure value, although their current levels of reliability could differ substantially. The Criticality importance measure (CR) is another widely used measure [2]. This measure is a natural extension of the Birnbaum measure which includes the component's unreliability. Two other measures that are widely used for ranking the components' importance are the Risk Achievement Worth (RAW), and the Risk Reduction Worth (RRW) [3]. Andrews and Beeson [4] extended the Birnbaum importance measure to promote the importance measures of components in non-coherent systems. In [72], Kuo and Zuo provided an overview of the applications of importance measures in the areas of reliability, network, risk, mathematical programming, and optimization. However, these traditional importance measures were only defined for crisp values of components' reliability data. This is why several uncertainty importance measures were introduced. Borgonovo [5], and Modarres and Aggarwal [6] studied the influence of uncertainty in importance measures results. Borgonovo [7] proposed also a common categorization of uncertainty measures based on variance and moment indicators. The first family of measures is based on the correlation between input variables and the output. The second category of measures is based on the variance of the probability distribution. The third category of measure is the moment-independent sensitivity indicators. However, these measures assume a choice of probability density functions which can make the results imprecise or even erroneous if there is not enough reliability data. Recently, Borgonovo and Smith [8] proposed an epistemic RAW, and studied its properties for series and parallel systems. In fuzzy set theory, fuzzy importance measures based on structural importance were introduced by Furuta and Shiraishi [9]. Liang and Wang [10] developed a fuzzy measure based on the use of triangular fuzzy numbers with maximal and minimal sets. Guimarees *et al.* [11] presented a fuzzy measure using the Euclidean distance between two fuzzy sets. Sallak *et al.* [12] proposed a fuzzy uncertainty measure based on the use of fuzzy triangular numbers to reduce uncertainty in the evaluation of Safety Integrity Levels (SILs). We should note that, in the case of no prior information about components reliabilities, we can use the structural importance which does not take into account the reliabilities of the system components because the importance of a component is measured by the number of times it appears in the reliability models [13], [14]. To our knowledge, there is no work treating the use of importance measures in D-S theory.

In this paper, importance measures such as Birnbaum, RAW, RRW, and CR are extended to make them compatible with D-S theory for taking into account aleatory and epistemic uncertainties. The use of D-S theory is due to the fact that, during the last years, the reliability assessment community recognized that the distinction between different types of uncertainties plays an important role in reliability evaluation [15], [16]. Uncertainty can be considered essentially of two types: aleatory uncertainty which arises from natural stochasticity or environmental vari-

ation across space or through time, and epistemic uncertainty which arises from incompleteness of knowledge or data [17]. The distinction is useful because epistemic uncertainty can be reduced by acquiring knowledge on the studied system. This distinction is similar to the distinction used, in quality, between system variation and measurement error. Furthermore, it has been proved that uncertainties in reliability and risk assessments are mainly epistemic [18].

In the case of a large amount of reliability data, the classical probabilistic approach was widely used to manage uncertainties in risk and reliability assessments [19]. This approach was based on the definition given by Laplace of the probability of an event as the ratio of the number of cases favorable to it to the number of all possible cases when all cases are equally possible [20]. Also widely used was the frequentist probabilistic approach introduced by Venn [21], which defined the event probability as the limit of its relative frequency in a large number of trials. However, in the case of components that fail only rarely (nuclear systems, chemical processes, railway systems, etc.) or components that have not been operated long enough to generate a sufficient quantity of reliability data, expert judgment is an option. Several methods were proposed to manage uncertainties in expert judgment such as the Bayesian approach, interval approach, evidence theory, possibility theory, etc. In the framework of probabilities, De Finetti [22] introduced the subjective probabilities of an event to indicate the degree to which the expert believes it. Kaplan and Garrick [23] introduced the concept of probability of frequency to expand their definition of risk. Pate-Cornell [24] considered six levels of sophistication in the analysis of the uncertainties according to the circumstances. She also proposed an uncertainty analysis to obtain a family of risk curves in the presence of several levels of uncertainty. The Bayesian approach proposed the use of subjective probabilities to represent expert judgment. The probability distributions representing the aleatory uncertainties are first proposed. The epistemic uncertainties about the parameter values of the distributions are then represented by prior subjective probability distributions [23]. The equation of Bayes is used to compute the new epistemic uncertainties in terms of the posterior distributions in the case of new reliability data. Finally, the predictive distributions of the quantities of interest are derived by using the total probability law. The predictive distributions are subjective, but they also take into account the aleatory uncertainties represented by the prior probability models [25]. However, there are some criticisms about the Bayesian approach and representing epistemic uncertainties using probabilities. These criticisms were exposed particularly by Walley [26], and Caselton and Luo [27]. When there is sparse information about the value of a parameter α , the choice of probability distribution may not be appropriate. For example, there is a difference between saying that all that is known about the parameter α is that its value is located somewhere in an interval $[x, y]$ and saying that a uniform distribution on $[x, y]$ characterizes degrees of belief with respect to where the value of this parameter is located in the interval $[x, y]$ [19], [28]. Furthermore, in a situation of ignorance, a Bayesian approach must equally allocate subjective probabilities over the frame of discernment. Thus there is no distinction between uncertainty and ignorance.

A number of alternative theories based on different notions of uncertainty were proposed. The evidence theory also known as the Dempster-Shafer or belief functions theory is a generalization of the Bayesian theory of subjective probability. Whereas the Bayesian theory requires probabilities for each question of interest, belief functions allow us to base degrees of belief for one question on probabilities for a related question [29]. To illustrate the idea of obtaining degrees of belief for one question from subjective probabilities for another, we propose an example in risk assessment inspired from the example of limb given by Shafer. Suppose we have subjective probabilities for the infallibility of a risk expert A. The probability that A is infallible is 0.75, and the probability that A is not infallible is 0.25. The risk expert A reports to us that a component i is failed. This information, which must be true if A is infallible, is not necessarily false if A is not infallible. The risk expert testimony justifies a 0.75 degree of belief that the component i is failed, but only a 0 degree of belief (not a 0.25 degree of belief) that the component i is not failed. This value does not mean that we are sure that the component i is failed, as a zero probability would. It means that the risk expert's testimony gives us no reason to believe that the component i is failed. The 0.75 and the 0 constitute a belief function. Thus there is no requirement that belief not committed to a given proposition should be committed to its negation. The second point of evidence theory is that belief measures of uncertainty may be assigned to overlapping sets and subsets of hypotheses, events or propositions, as well as to individual hypothesis. D-S theory, which can be considered as an alternative approach to represent uncertainties, has gained an increasing amount of attention both from the theoretical and the applied point of view [30]–[33]. In a finite discrete space, D-S theory is a generalization of probability theory where probabilities are assigned to sets instead of mutually exclusive singletons. This theory is still a young field compared to other theories, and its main application is data fusion.

II. D-S AND RELIABILITY ASSESSMENT

D-S is a theory of uncertainty that was first developed by Dempster [34], and extended by Shafer [35]. The idea of using D-S theory in reliability assessment was introduced by Dempster and Kong [36]. D-S theory, which is considered to be a less restricted extension of probability theory, is proposed in this paper as a framework for taking into account both aleatory and epistemic uncertainties.

A. Basic Concepts

The frame of discernment X is the set of all hypotheses for which the information sources can provide evidence. A Basic Belief Assignment (BBA) on a frame of discernment X is a function $m^X : 2^X \rightarrow [0, 1]$ which maps beliefs masses on subsets of events as

$$\sum_{A \in 2^X} m^X(A) = 1 \quad (1)$$

A focal element is a subset X of the frame of discernment, such that $m(X) > 0$. If all the focal elements are singletons (i.e. one-element subsets of the frame of discernment), then we speak about a Bayesian belief function. The two important measures

of uncertainty provided by D-S theory are called the belief function, and the plausibility function. They are defined respectively by

$$\begin{aligned} Bel(A) &= \sum_{B \subseteq A} m^X(B), \\ Pl(A) &= \sum_{B \cap A \neq \emptyset} m^X(B) \quad \forall A \subseteq X, \forall B \subseteq X \quad (2) \end{aligned}$$

The interval $[Bel(A), Pl(A)]$ represents the uncertainty of A .

The formulas of marginal BBA $m^{\Omega_x \Omega_y \downarrow \Omega_x}$ defined on a frame of discernment Ω_x , and the vacuous extension defined on the Cartesian product $\Omega_x \Omega_y$, are given in the Appendix.

The two most known rules of combination are the conjunctive \odot and the disjunctive \oplus rules [35]. The Dempster rule is a widely used rule calculated from the conjunctive combination of two BBAs followed by normalization. The formulas of these three rules are given in the Appendix. Depending on the reliability of experts and the conflict between them, several other combination rules were defined in D-S theory (Yager rule [37], Dubois and Prade rule [38], Cautious rule [39], etc.). For more details, see [34], [35], [39], [40].

B. Reliability Assessment

In our reliability model, the frame of discernment X_i of a component i is defined by $X_i = \{F_i, W_i\}$, where F_i , and W_i denote respectively the failure, and the working states of the component i . An expert α should use an interval $[Bel(W_A), Pl(W_A)]$ to represent its interval belief that the component A is in the working state at time t . From this result, we can say that the values of $Bel(\{W_A\})$ and $Pl(\{W_A\})$ are going to bound the correct value to be in the working state which represents the aleatory uncertainty concerning the functioning state of the component A . Moreover, the epistemic uncertainty is equal to $Pl(\{W_A\}) - Bel(\{W_A\})$, which represents the length of the interval $[Bel(W_A), Pl(W_A)]$. It represents the part of the belief expert allocated to the lack of knowledge concerning the state of the component A . Thus, we can say that aleatory and epistemic uncertainties are treated separately in our approach. The BBAs about component A are computed as

$$\begin{aligned} m_\alpha^{X_A}(\{W_A\}) &= Bel(\{W_A\}) \\ m_\alpha^{X_A}(\{F_A\}) &= 1 - Pl(\{W_A\}) \\ m_\alpha^{X_A}(\Omega_A) &= Pl(\{W_A\}) - Bel(\{F_A\}) \end{aligned}$$

Thus, unlike the probabilistic relationship between a set and its complement, the belief assigned to a set does not uniquely determine the belief assigned to its complement. Similarly, the plausibility assigned to a set does not uniquely determine the plausibility assigned to its complement [28].

To evaluate the whole reliability of a system S composed of n components $\{1, 2, \dots, n\}$ in a given configuration, we use the BBA defined as

$$\begin{aligned} m^{X_S} &= \left(m^{X_1 \uparrow X_1 \times X_2 \dots \times X_n \times X_S} \oplus m^{X_2 \uparrow X_1 \times X_2 \dots \times X_n \times X_S} \oplus \right. \\ &\quad \left. \dots \oplus m^{X_n \uparrow X_1 \times X_2 \dots \times X_n \times X_S} \oplus m_{Config}^{X_1 \times X_2 \dots \times X_S} \right) \downarrow^{X_S} \quad (3) \end{aligned}$$

TABLE I
DEMPSTER COMBINATION OF BBAS CONCERNING COMPONENT 1

	$\{0_1\}$	$\{1_1\}$	$\{0_1, 1_1\}$
Expert 1	0.1	0.3	0.6
Expert 2	0	0.4	0.6
Expert 1 \oplus Expert 2	0.0625	0.5625	0.3750

The BBA $m_{Conf}^{X_1 \times X_2 \times \dots \times X_n \times X_S}$ represents the relation between the system S and its components (structure function). Then, belief and plausibility measures are obtained from m^{X_S} . The reliability of the system R_S is obtained by

$$R_S = [\underline{R}_S, \overline{R}_S] = [Bel(\{W_S\}), Pl(\{W_S\})] \quad (4)$$

For example, let us consider a system S with two components 1 and 2 in parallel. For simplicity, we consider that the system S and its components have two states: 0_i , and 1_i . Let us suppose that Expert 1 asserts an interval belief $[0.3, 0.9]$ that component 1 is working at time t . The value 0.3 represents the part of the expert's belief that supports that the component 1 is in a working state, whereas the value 0.9 is obtained by adding the value 0.6 to the lower value 0.3, which represents the epistemic uncertainty of the expert about the state of component 1. Expert 2 asserts an interval belief $[0.4, 1]$ that component 1 is working at time t . Dempster's rule of combination applied to the BBAs from Experts 1 and 2 gives the new BBAs shown in Table I. Also, because BBAs have been combined, the level of conflict k can be gauged. In this case, $k = 0.04$ indicating a minor conflict in the evidence from the two experts. The same reasoning is done to obtain the BBA of component 2. Then, we use the vacuous extension to extend m^{X_1} and m^{X_2} to the product space $X_1 \times X_2 \times X_S$, and we combine the obtained BBAs using Dempster rule. The resulting BBAs are combined with $m_{Conf}^{X_1 \times X_2 \times X_S}$. For parallel configuration, we have $m_{Parallel}^{L_1 \times L_2 \times L_S}(\{(1_1, 1_2, 1_S), (0_1, 0_2, 0_S), (0_1, 1_2, 1_S), (1_1, 0_2, 1_S)\})$. To obtain the BBAs of the system S , the final result is marginalized on L_S . Belief and plausibility functions are then computed to obtain upper and lower bounds of the system's reliability.

C. Construction of BBAs

Within the framework of DS theory, the construction of BBAs remains an important problem which can considerably influence final results. Nevertheless, there are very few works concerning the construction of BBAs from reliability data [33], [41]. The problem of elicitation of experts' judgments has long been addressed in the probability theory framework related to reliability and risk assessments. In [42], Ouchi reviews the literature on the use of expert opinion in probabilistic risk analysis. However, these elicitation techniques are not suited to other uncertainty theories (possibility theory, D-S theory, etc.). This next section is mainly dealing with the elicitation process of experts' judgments in D-S theory. The integration of experts' reliability will be presented. The dependence of BBAs on the number of components' failures will also be discussed.

1) *Expert Elicitation Using Reliability Parameters*: Let us consider a component i with a constant failure rate λ . The reliability expert 1 tells us that $\lambda \in [\underline{\lambda}, \overline{\lambda}]$ [41]. Variable t rep-

resents the amount of time that the components must function for the system to succeed, and w is the lifetime of the component. Because, the failure rate is constant, w has an exponential distribution with scale parameter $1/\lambda$. The component will fail when $w \leq t$ or $\lambda \geq 1/t$. If $1/t \leq \underline{\lambda}$, the component will certainly fail, thus, the proposition totally agrees with the event F_i , so we represent it by $Bel_1(\{F_i\}) = 1 - e^{-\underline{\lambda}t}$. Similarly, if $1/t \leq \overline{\lambda}$, the component may fail; thus, the proposition totally or partially agrees with the event F_i , so we represent it by $Pl_1(\{F_i\}) = 1 - e^{-\overline{\lambda}t}$. Finally, the obtained BBAs are

$$\begin{aligned} m_1^{X_i}(\{W_i\}) &= e^{-\overline{\lambda}t} \\ m_1^{X_i}(\{F_i\}) &= 1 - e^{-\underline{\lambda}t} \\ m_1^{X_i}(\{W_i, F_i\}) &= e^{-\underline{\lambda}t} - e^{-\overline{\lambda}t} \end{aligned}$$

2) *Discounting*: It sometimes occurs that an expert (or a source of information) induces a BBA m^Ω , but there is some doubt regarding the reliability of that expert. It may be useful to discount this BBA m^Ω by some factor $\delta \in [0, 1]$. When the BBA is discounted, the remaining assignment is applied to the combination of all options in the frame of discernment Ω . This discounting operation was introduced by Shafer [35]. The details of this operation are given in the Appendix.

3) *Dependence of BBAs on the Number of Components' Failures*: This subsection describes how to build belief function models for situations with unknown parameters of the probability distribution.

Kozine and Filimonov [43] asked how to model the s -dependence of belief functions on the number of occurrences that have happened, and how to embed this s -dependence on the number of occurrences in the BBA m . Dempster proposed a model for an unknown parameter of binomial distribution [44]. Let us consider the observations (X_1, \dots, X_n) which come from a binomial process with an unknown failure rate λ . X_i is equal to 1 if the component failed on the i th demand. Consider a_i to be a series of s -independent pivotal random values which have a uniform distribution on the interval $[0, 1]$, and consider to hold the equation

$$X_i = 1 \Leftrightarrow a_i \leq \lambda \quad (5)$$

This equation presents a logical restriction on the possible values of the triples (X_i, a_i, λ) . When the a_i are marginalized out, the result is a binomial model over the X_i .

To find the margin over λ , first arrange the pivotal variables in order, $a_0 \leq a_1 \leq \dots \leq a_{n+1}$ where $a_0 = 0$, and $a_{n+1} = 1$. Let X be the total number of failures observed in the n demand. It must be true that $\underline{a} = a_X \leq \lambda \leq \overline{a} = a_{X+1}$. The order statistics from a uniform distribution follow a beta distribution. Then the joint belief function over λ is the mass density function

$$\begin{aligned} m(\underline{a}, \overline{a}) &= \frac{n!}{(k-1)!(n-k-1)!} \\ &\quad \times \underline{a}^{k-1} (1-\overline{a})^{n-k-1} \quad 0 < k < n; \\ m(0, \overline{a}) &= n(1-\overline{a})^{n-1} \quad k = 0; \\ m(\underline{a}, 1) &= n\underline{a}^{n-1} \quad k = n; \end{aligned} \quad (6)$$

For any set B , we can get the respective Bel, and Pl functions such that $\lambda \in B$ by

$$\begin{aligned}
Bel(B) &= \int \int_{[a,b] \subseteq B} m(a,b) da db \\
Pl(B) &= \int \int_{[a,b] \cap B \neq \emptyset} m(a,b) da db
\end{aligned} \quad (7)$$

Under consideration that B is an interval, we obtain

$$\begin{aligned}
Bel(B) &= \frac{k}{n+1} \\
Pl(B) &= \frac{k+1}{n+1}
\end{aligned} \quad (8)$$

This method was also applied to the case of Poisson processes, where the uniform pivotal variables a_i become γ waiting time distributions. Almond has applied Dempster's work in some reliability problems [41]. For more details, see [41], [44].

D. Computational Complexity

The computational complexity of reasoning within the D-S theory of evidence is one of the main points of criticism this formalism has to face. In the proposed reliability model, the computational cost of the model grows exponentially with the size of the studied system. Three operations are used: extension, projection, and combination. Dempster's rule of combination requires significantly large computational cost with the increasing of the focal elements' cardinality. In the case of binary systems, for each component, there are two elements in the frame of discernment (working, and failure; $n = 2$ is the cardinality of the frame of discernment). Thus, the BBA has $n_f = 2^n - 1 = 3$ focal elements (the empty-set here is excluded). Furthermore, the combination of two BBAs requires the computation of up to $2^{n_f+1} - 2$ operations (intersections). Then the total number of combination operations for nc components (we add also the BBA of the system configuration) is $N_{comb} = (2^{n_f+1} - 2) * C_{nc+1}^2$. The total number of extensions is $N_{ext} = (nc + 1) * (2^{n_f+1} - 2)$. The total number of projection is $N_{ext} = N_{comb} * n_f$. Finally, the total number of operation is $N_{Op} = N_{comb} + N_{ext} + N_{proj}$. Thus, the DS approach can be applied on large binary coherent systems, and the maximum number of system' components to be analyzed in reasonable time is about 100 components.

However, the DS approach can be more time consuming when used to study the reliability of multi-state systems. To optimize the D-S operations, and to save time and space, some computation algorithms were given in [41], [45]. The idea of these algorithms lies in doing local calculations in smaller product spaces so as to reduce the exponential growth of the D-S. On the other hand, Smets [29] introduced a different approach that propagates conditional belief functions over a directed acyclical network called the belief network. The advantage of this approach is that the edges of the graphs are weighted by conditional belief functions and not by joint belief functions over a product space; thus, it's a more optimal method in terms of computational cost. The approach is less general, but the loss of generality does not affect the reliability analysis. Smets [46] introduced matrix calculus using the

Möbius transformation to the D-S. This approach simplifies greatly the development of software, and optimizes the computational time. Reducing the number of focal elements of the BBAs under consideration while retaining the essence of the information they represent is also another way to overcome the computational complexity.

III. INTERVAL AND AFFINE ARITHMETICS

A. Interval Arithmetic (IA)

To define importance measures based on upper and lower values of reliability defined in (4), we need to use interval arithmetic (IA) operations. IA was first proposed by Moore [47]. An interval number $[\underline{x}, \bar{x}]$ is the set of real numbers x such that $\underline{x} \leq x \leq \bar{x}$ where \underline{x} , and \bar{x} are elements of the set of real numbers \mathbb{R} with $\underline{x} \leq \bar{x}$. Addition, subtraction, multiplication, and division of intervals are defined in the Appendix. However, when a given variable occurs more than once in an interval computation, it is treated as a different variable in each occurrence. This effect is called the dependency problem. Therefore, standard interval arithmetic provides guaranteed bounds, but they are often too conservative [48]. Several techniques have been developed to avoid the dependence problems, and hence reduce the over-estimation: quadratic approximation and Gauss transformation, deconvolution, generalized interval arithmetic, modal interval, central forms, and Affine Arithmetic.

B. Affine Arithmetic (AA)

Affine arithmetic (AA) was introduced in 1993 by Comba and Stolfi [49], and developed by De Figueiredo and Stolfi in [50]. AA is an extension of IA obtained by considering affine forms in place of intervals. This method is similar to standard IA, but, in addition, it keeps track of s -correlations between quantities during the computations. This extra information allows us to provide much tighter s -bounds in the computing process, avoiding the error explosion problem. In AA, each quantity x is represented by an affine form \hat{x} , which is a first-degree polynomial:

$$\hat{x} = x_0 + x_1 \epsilon_1 + \dots + x_n \epsilon_n \quad (9)$$

Each noise symbol ϵ_i is a symbolic real variable whose value is only known to lie in the interval $U = [-1, +1]$, and is s -independent from the other noise symbols. The coefficients x_i are known real values. The coefficient x_0 is called the central value of \hat{x} . The coefficients x_1, \dots, x_n are called the partial deviations associated with the noise symbols $\epsilon_1, \dots, \epsilon_n$ in x . Conversions between affine forms and intervals are performed as follows

- Affine Form \rightarrow Interval:

$$\hat{x} = x_0 + \sum_{i=1}^n x_i \epsilon_i \rightarrow x = \left[x_0 - \sum_{i=1}^n |x_i|, x_0 + \sum_{i=1}^n |x_i| \right] \quad (10)$$

- Interval \rightarrow Affine Form:

$$x = [\underline{x}, \bar{x}] \rightarrow \hat{x} = \frac{\underline{x} + \bar{x}}{2} + \frac{\bar{x} - \underline{x}}{2} \epsilon \quad (11)$$

where ϵ is a new variable having a value that is only known to lie in the range $[-1, +1] \dots$

Extending non-affine operations requires that we use good affine approximation to the exact result, and append an extra term to bound the error of this approximation. Different approximation methods can be used for the affine forms of several functions: optimal (Chebyshev), min-range, and interval approximation. All these techniques assume that the function is at least bounded on the argument interval. The Chebyshev approximation provides the best approximation result. However, its computation is prohibitive expensive [51].

IV. IMPORTANCE MEASURES USING THE D-S THEORY

The aim of this section is to introduce importance measures using D-S theory. This work will be done by introducing new definitions of usual probabilistic importance measures. These new measures will be first used to rank the importance of components in the presence of only aleatory uncertainty, both aleatory and epistemic uncertainties, and in the case when many experts' judgments must be taken into account. Then, the measures will be used to reflect to what degree the epistemic uncertainties on the components' reliability influence the components' rankings. In this work, the following key assumptions are taken into account.

- System and components are allowed to take only two possible states: either working, or failed.
- Component failures are s -independent. Failure of one component does not impact the failures of the other components.
- The structure function is coherent. That is, improvement of component states cannot damage the system.
- The components are not repairable.

A. Interval Definitions

In 1969, Birnbaum first introduced the concept of a components' reliability importance [1]. This measure was defined as the probability that a component i is critical to system failure, *i.e.* when component i fails it causes the system to move from a working to a failed state. The Birnbaum importance measure of a component i can be interpreted as the rate at which the system's reliability improves as the reliability of component i is improved. Analytically, Birnbaum's importance interval measure of a component i can be defined using D-S theory by

$$\begin{aligned} [I^B(i)] &= [Bel(\{W_S\}|\{W_i\}), Pl(\{W_S\}|\{W_i\})] \\ &\quad - [Bel(\{W_S\}|\{F_i\}), Pl(\{W_S\}|\{F_i\})] \\ &= [\underline{R}_{S|W_i}, \overline{R}_{S|W_i}] - [\underline{R}_{S|F_i}, \overline{R}_{S|F_i}] \end{aligned}$$

Where $Bel(\{W_S\}|\{W_i\})$, and $Pl(\{W_S\}|\{W_i\})$ denote respectively the belief, and plausibility measures that the system is functioning when it is known that component i is in a working state. Whereas $Bel(\{W_S\}|\{F_i\})$, and $Pl(\{W_S\}|\{F_i\})$ denote respectively the belief, and plausibility measures that the system is functioning when component i is in a failed state.

To evaluate $[I^B(i)]$ using AA, we first need to define the affine form $\hat{I}^B(i)$ of the quantity $I^B(i)$. This result will be defined by writing the affine forms of the quantities $R_{S|W_i}$ and $R_{S|F_i}$, and then evaluating the affine form of their difference.

The affine forms of the quantities $R_{S|W_i}$, and $R_{S|F_i}$ are respectively: $\hat{R}_{S|W_i} = R_{S|W_i,0} + R_{S|W_i,1}\epsilon_1$, and $\hat{R}_{S|F_i} =$

$R_{S|F_i,0} + R_{S|F_i,2}\epsilon_2$ where $R_{S|W_i,0} = (\underline{R}_{S|W_i} + \overline{R}_{S|W_i})/2$, $R_{S|W_i,1} = (\overline{R}_{S|W_i} - \underline{R}_{S|W_i})/2$, $R_{S|F_i,0} = (\underline{R}_{S|F_i} + \overline{R}_{S|F_i})/2$, and $R_{S|F_i,2} = (\overline{R}_{S|F_i} - \underline{R}_{S|F_i})/2$. It follows that the affine form of the quantity $I^B(i)$ is defined by

$$\begin{aligned} \hat{I}^B(i) &= \hat{R}_{S|W_i} - \hat{R}_{S|F_i} \\ &= I_0^B(i) + I_1^B(i)\epsilon_1 + I_2^B(i)\epsilon_2 \end{aligned} \quad (12)$$

where

$$\begin{aligned} I_0^B(i) &= R_{S|W_i,0} - R_{S|F_i,0} \\ I_1^B(i) &= R_{S|W_i,1} \\ I_2^B(i) &= R_{S|F_i,2} \end{aligned}$$

The Birnbaum importance measures obtained using AA and IA methods are expressed in Table II. Note that in the case of Birnbaum measures, AA and IA give the same results because we use only subtraction operations. To compute for example the Birnbaum measure of component k , we first assume that $m(\{W_k\}) = 1$. Then all the masses are combined so as to obtain at the end the system's reliability. First, each BBA on Ω_k must be extended to the product space $\Omega_1\Omega_2 \cdots \Omega_n\Omega_S$. Then, all of the components' reliability masses and the system's configuration mass m_{Config}^Ω are combined using the combination rule. Finally, the obtained mass is projected to the frame of discernment of the system's states Ω_s . This process is represented by Fig. 1, and repeated for the hypothesis $m(\{W_k\}) = 0$. Then, using AA operations, we compute the difference between the system's reliability in the two cases to obtain the importance measure of component k .

The RAW is defined as the ratio of the conditional system's unreliability if component i is failed over the system's unreliability. This measure quantifies the maximum possible percentage increase in system unreliability generated by a particular component. Using D-S theory, the RAW of a component i can be defined by

$$\begin{aligned} [I^{RAW}(i)] &= \frac{[Bel(\{F_S\}|\{F_i\}), Pl(\{F_S\}|\{F_i\})]}{[Bel(\{F_S\}), Pl(\{F_S\})]} \\ &= \frac{[\underline{Q}_{S|F_i}, \overline{Q}_{S|F_i}]}{[\underline{Q}_S, \overline{Q}_S]} \end{aligned}$$

where $Bel(\{F_S\}|\{F_i\})$, and $Pl(\{F_S\}|\{F_i\})$ respectively denote the belief, and plausibility measures that the system is in a failed state when it is known that component i is in a failed state.

The RRW is the ratio of the system's unreliability over the conditional system's unreliability if component i is replaced by a perfect component [16], [52], [53]. Using D-S theory, the RRW of a component i is defined by

$$\begin{aligned} [I^{RRW}(i)] &= \frac{[Bel(\{F_S\}), Pl(\{F_S\})]}{[Bel(\{F_S\}|\{W_i\}), Pl(\{F_S\}|\{W_i\})]} \\ &= \frac{[\underline{Q}_S, \overline{Q}_S]}{[\underline{Q}_{S|W_i}, \overline{Q}_{S|W_i}]} \end{aligned}$$

Whereas the Birnbaum importance provided the probability that a given component would be responsible for the failure at time t , CR (also called the Criticality measure of Lambert) is another

TABLE II
INTERVAL AND PIGNISTIC IMPORTANCE MEASURES

	Interval form using AI	Interval form using AA	Pignistic form
Birnbaum	$[\underline{R}_{S W_i}, \overline{R}_{S W_i}] - [\underline{R}_{S F_i}, \overline{R}_{S F_i}]$	$[I_0^B(i) - h_i^B, I_0^B(i) + h_i^B]$ $h_i^B = I_1^B(i) + I_2^B(i) $ $I_0^B(i) = R_{S W_i,0} - R_{S F_i,0}$ $I_1^B(i) = R_{S W_i,1}$ $I_2^B(i) = R_{S F_i,2}$	$ Bet(R_S \{W_i\}) - Bet(R_S \{F_i\}) $
RAW	$\frac{[\underline{Q}_{S F_i}, \overline{Q}_{S F_i}]}{[\underline{Q}_S, \overline{Q}_S]}$	$[I_0^{RAW}(i) - h_i^{RAW}, I_0^{RAW}(i) + h_i^{RAW}]$ $h_i^{RAW} = I_1^{RAW}(i) + I_2^{RAW}(i) $ $+ I_3^{RAW}(i) + I_4^{RAW}(i) $ $I_0^{RAW}(i) = \alpha Q_{S F_i,0} Q_{S,0} + \beta Q_{S F_i,0}$ $I_1^{RAW}(i) = Q_{S F_i,1}(\alpha Q_{S,0} + \beta)$ $I_2^{RAW}(i) = \alpha Q_{S,2} Q_{S F_i,0}$ $I_3^{RAW}(i) = \delta Q_{S F_i,0}$ $I_4^{RAW}(i) = \alpha Q_{S,2} Q_{S F_i,1} + \delta Q_{S F_i,1}$ $\alpha = -1/\underline{Q}_S \overline{Q}_S$ $\delta = (\beta_1 - \beta_2)/2 $ $\beta_1 = 1/\underline{Q}_S - \alpha \overline{Q}_S$ $\beta_2 = 1/x' - \alpha x'$ $x' = \sqrt{-1/\alpha}$ $\beta = (\beta_1 + \beta_2)/2$	$\frac{Bet(Q_S \{F_i\})}{BetQ_S}$
RRW	$\frac{[\underline{Q}_S, \overline{Q}_S]}{[\underline{Q}_{S W_i}, \overline{Q}_{S W_i}]}$	$[I_0^{RRW}(i) - h_i^{RRW}, I_0^{RRW}(i) + h_i^{RRW}]$ $h_i^{RRW} = I_1^{RRW}(i) + I_2^{RRW}(i) $ $+ I_3^{RRW}(i) + I_4^{RRW}(i) $ $I_0^{RRW}(i) = \alpha Q_{S,0} Q_{S W_i,0} + \beta Q_{S,0}$ $I_1^{RRW}(i) = Q_{S,0}(\alpha Q_{S W_i,0} + \beta)$ $I_2^{RRW}(i) = \alpha Q_{S W_i,2} Q_{S,0}$ $I_3^{RRW}(i) = \delta Q_{S,0}$ $I_4^{RRW}(i) = \alpha Q_{S W_i,2} Q_{S,1} + \delta Q_{S,1}$	$\frac{BetQ_S}{Bet(Q_S \{W_i\})}$
CR	$[I^B(i)] \frac{[\underline{Q}_i, \overline{Q}_i]}{[\underline{Q}_S, \overline{Q}_S]}$	$[I'_0(i)I_0^B(i) - h_i^{CR}, I'_0(i)I_0^B(i) + h_i^{CR}]$ $h_i^{CR} = I'_0(i) \sum_{j=5}^6 I_j^B(i) + I_0^B(i) \sum_{j=1}^4 I'_j(i) $ $+ (\sum_{j=1}^4 I'_j(i))(\sum_{j=1}^2 I_j^B(i))$ $I'_0(i) = \alpha Q_{i,0} Q_{S,0} + \beta Q_{i,0}$ $I'_1(i) = Q_{i,0}(\alpha Q_{S,0} + \beta)$ $I'_2(i) = \alpha Q_{S,2} Q_{i,0}$ $I'_3(i) = \delta Q_{i,0}$ $I'_4(i) = \alpha Q_{S,2} Q_{i,1} + \delta Q_{i,1}$	$I^B(i) \frac{BetQ_i}{BetQ_S}$

well-known measure used to determine the probability that the given component was responsible for system failure before time t [2]. Thus, the CR measure is suitable for prioritizing maintenance actions. As compared to Birnbaum's measure, the components' reliability is integrated into the measure. Using the TBM, the CR measure can be defined by

$$\begin{aligned} [I^{CR}(i)] &= [I^B(i)] \frac{[Bel(\{F_i\}), Pl(\{F_i\})]}{[Bel(\{F_S\}), Pl(\{F_S\})]} \\ &= [I^B(i)] \frac{[\underline{Q}_i, \overline{Q}_i]}{[\underline{Q}_S, \overline{Q}_S]} \end{aligned}$$

Similar to the way the interval form using AA of Birnbaum measures was obtained, the interval forms of $I^{RAW}(i)$, $I^{RRW}(i)$, and $I^{CR}(i)$ are computed and represented in Table II.

B. Pignistic Definitions

In Table II, we give interval values of Birnbaum importance measures. However, these interval values are not sufficient to rank components. To rank components, we have to convert these interval values to a probability measure. Such a transformation is called a probabilistic transformation. We define the probabilistic transformation as a mapping $f : m \rightarrow P$, where P denotes the probability distribution, and m the BBA function [54]. A probabilistic transformation f is potentially consistent in three ways.

- *ulb*-consistent (upper and lower bound consistent): if $Bel(A) \leq f(A) \leq Pl(A)$ for any $A \subseteq X$.
- *p*-consistent (or probabilistically consistent): if $f(m) = m$ for any Bayesian BBA m .

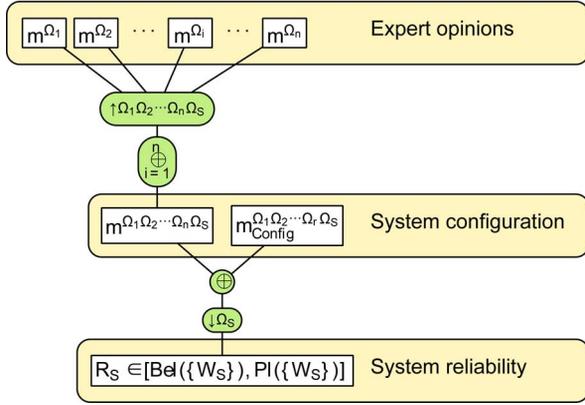


Fig. 1. Algorithm to compute the reliability of systems.

- α -consistent: if $f(\alpha m_1 + (1 - \alpha)m_2) = \alpha f(m_1) + (1 - \alpha)f(m_2)$ for any BBAs m_1 and m_2 .

The most known probabilistic transformation is the pignistic transformation $BetP$. It was introduced by Smets and Kennes [55], and corresponds to the generalized insufficient reason principle: a BBA assigned to the union of n atomic sets is split equally among these n sets. It is defined for any set $B \subseteq X$ and $B \neq \emptyset$ by

$$BetP(B) = \sum_{A \neq \emptyset | A \subseteq X} \frac{|A \cap B|}{|A|} \frac{m(A)}{1 - m(\emptyset)} \quad (13)$$

$|A|$ denotes the cardinality of $A \subseteq X$. We should note that, in the case of binary systems and closed world hypothesis (*i.e.* $m(\emptyset) = 0$), the pignistic system's reliability $BetR_S$ is given by $BetR_S = m^{X_S}(\{W_S\}) + m^{X_S}(\{W_S, F_S\})/2$.

The formula of the normalized plausibility probabilistic transformation PlP and the normalized belief probabilistic transformation $BelP$ are given in the Appendix.

Other probabilistic transformations were also defined using different kinds of mappings, either proportional to the plausibility, to the normalized plausibility, to all plausibilities, to the belief or a hybrid mapping. For more details, see [56].

Then, based on the pignistic transformation, we propose to define our pignistic Birnbaum measure as

$$I_{Bet}^B(i) = |Bet(R_S|\{W_i\}) - Bet(R_S|\{F_i\})| \quad (14)$$

where $Bet(R_S|\{W_i\})$ is the system's pignistic reliability when component i is functioning, and $Bet(R_S|\{F_i\})$ is the system's pignistic reliability when component i is in a failed state. In the case of coherent systems, we can write $I_{Bet}^B(i) = Bet(R_S|\{W_i\}) - Bet(R_S|\{F_i\})$. We also note that the pignistic Birnbaum measure $I_{Bet}^B(i)$ is the midpoint of the interval $[I^B(i)] = [I_0^B(i) - h_i^B, I_0^B(i) + h_i^B]$. The other pignistic measures are given in Table II. Furthermore, this pignistic Birnbaum measure can be defined using some other distances instead of subtraction operation. In [57], Tessem aimed to assess the quality of Bayesian approximation algorithms of belief functions by proposing an error measure between two pignistic values of BBAs m and m' based on the Minkowski family, and defined as q . In our reliability model, the parameter

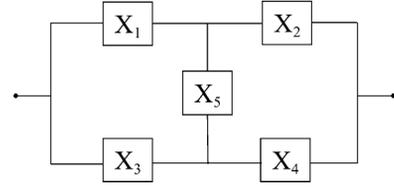


Fig. 2. Reliability block diagram of a bridge system.

θ_i belongs to $\{F_S, W_S, \{F_S, W_S\}\}$. $BetP$ is the system's pignistic reliability when component i is functioning, and $BetP'$ is the system's pignistic reliability when component i is in a failed state. When θ_i is equal to W_S , the Tessem distance is equivalent to our proposed measure. We can also use the extension of the Euclidean measure between two BBAs from the probability theory to the D-S theory proposed by Cuzzolin [58] as $d(m, m') = \sqrt{\sum_{A \subseteq X_S} (m(A) - m'(A))^2}$. Josselme *et al.* [59] proposed a distance based on the similarity function, and defined by $d(m, m') = \sqrt{d_{11}/2 - d_{12} + d_{22}/2}$ where $d_{ij} = \sum_{A \subseteq X} \sum_{B \subseteq X} m_i(A)m_j(B)S(A, B)$, and $S(A, B)$ is the similarity function given by $S(A, B) = |A \cap B|/|A \cup B|$. For more details about distances defined in D-S theory, see [58]–[61].

According to Dubois *et al.* [62], the pignistic transformation is mathematically equivalent to changing each piece of partial ignorance E_i into a uniform probability P_i on E_i , representing a Bayesian uninformed prior, and $BetP$ is then the convex mixture of the P_i with weights m_i . Hence, the pignistic transformation approach to decision with belief functions is consistent with the higher-order probability approach, viewing the weights m_i as meta-probabilities. It is thus a generalized indifference Laplacean principle. Furthermore, the pignistic transformation is the only probabilistic transformation in the belief framework which has the three properties of consistency [54], [63]. Moreover, if we consider the set of probability distributions $\mathcal{P} = \{P | P \geq Pl\}$ where P is a probability distribution, and Pl is the plausibility function induced by the BBA m_i , then it was proved [64], [65] that the pignistic probability coincides with the center of gravity of \mathcal{P} . That's why we advise users to use the pignistic transformation in the absence of a total order of components' importance measures, or to get new information to have a direct total order on component ranking.

V. NUMERICAL EXAMPLE

To examine the general applicability of the importance measure definitions, a system with a bridge configuration is studied (cf. Fig. 2). The bridge system is composed of five components X_1, X_2, X_3, X_4 , and X_5 . Depending on the cases, the components have reliability data presented in Tables III–V, and VII. Four different cases are considered. The first case corresponds to the case when there is no epistemic uncertainty about reliability data of the components. The second one corresponds to the presence of both aleatory and epistemic uncertainties. The third one consists of treating many experts' judgments. The last case corresponds to reliability data with variable epistemic uncertainty.

TABLE III
 CASE I—BBAS OF COMPONENTS

Components	$m^{X_i}(\{F_i\})$	$m^{X_i}(\{W_i\})$	$m^{X_i}(\{F_i, W_i\})$
X_1	0.21	0.79	0
X_2	0.26	0.74	0
X_3	0.22	0.78	0
X_4	0.31	0.69	0
X_5	0.28	0.72	0

 TABLE IV
 CASE II—BBAS OF COMPONENTS

Components	$m^{X_i}(\{F_i\})$	$m^{X_i}(\{W_i\})$	$m^{X_i}(\{F_i, W_i\})$
X_1	0.18	0.72	0.10
X_2	0.23	0.71	0.06
X_3	0.22	0.68	0.10
X_4	0.20	0.66	0.14
X_5	0.27	0.62	0.11

 TABLE V
 CASE III—BBAS OF COMPONENTS $X_1, X_2, X_3, X_4,$ AND X_5

Components	Expert I		Expert II		Expert III	
	$m^{X_i}(\{F_i\})$	$m^{X_i}(\{W_i\})$	$m^{X_i}(\{F_i\})$	$m^{X_i}(\{W_i\})$	$m^{X_i}(\{F_i\})$	$m^{X_i}(\{W_i\})$
X_1	0.10	0.80	0.15	0.82	0.15	0.75
X_2	0.20	0.75	0.20	0.77	0.22	0.68
X_3	0.18	0.79	0.10	0.85	0.30	0.60
X_4	0.20	0.70	0.22	0.68	0.25	0.65
X_5	0.16	0.75	0.16	0.77	0.18	0.72

A. Case I: Reliability Data With Only Aleatory Uncertainty

We consider only aleatory uncertainties ($m^{X_i}(\{F_i, W_i\}) = 0$) for components $X_1, X_2, X_3, X_4,$ and X_5 , as shown in Table III. Importance measures are computed by using equations presented in Table II. The obtained importance measures (cf. Fig. 3) are precise values because there is no epistemic uncertainty related to components' reliability data, and in this case the belief measures are equals to the plausibility measures. Additionally, we can see that RRW and CR measures give the same rankings for all components ($I^{RRW}(X_5) < I^{RRW}(X_3) < I^{RRW}(X_1) < I^{RRW}(X_2) < I^{RRW}(X_4)$ and $I^{CR}(X_5) < I^{CR}(X_3) < I^{CR}(X_1) < I^{CR}(X_2) < I^{CR}(X_4)$). Birnbaum and RAW measures give the same most important component X_2 . Therefore, we can only consider the two measures RAW and RRW because their definitions are easier to understand. If we have the potential to upgrade components, the RAW measure indicates that the component X_2 is the most important in the system. Whereas if no upgrades or improvements are possible, the RRW measure indicates that the deterioration of component X_4 is the most critical in the system, and thus inspections must be first done on X_4 .

B. Case II: Reliability Data With Both Aleatory and Epistemic Uncertainties

Let us consider epistemic uncertainties ($m^{X_i}(\{F_i, W_i\}) > 0$) for components $X_1, X_2, X_3, X_4,$ and X_5 as shown in Table IV. Using the equations presented in Table II, we compute the importance measures of the system's components. The obtained importance measures (cf. Fig. 4) are interval

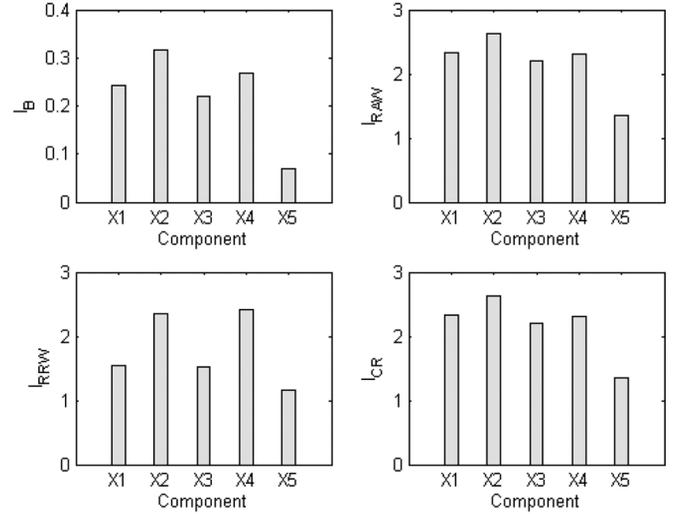


Fig. 3. Case I—Importance measures of components.

values because, in the presence of epistemic uncertainties, the belief measures are not equal to the plausibility measures. However, within these interval values of importance measures, it is very difficult to rank components' importance. This is why we compute the pignistic values of importance measures. As we can see in Fig. 5, within the pignistic values, it is easy to rank components' importance. For example, according to RRW, we have $I^{RRW}(X_5) < I^{RRW}(X_1) < I^{RRW}(X_3) < I^{RRW}(X_4) < I^{RRW}(X_2)$.

Another interesting idea is to compare the results obtained by the distances defined in Section IV-B and our proposed distance in the computation of the Birnbaum importance measures. According to the results presented in Fig. 6, we obtain the same ranking of components in all distances ($I^B(X_5) < I^B(X_3) < I^B(X_4) < I^B(X_1) < I^B(X_2)$). Particularly, the importance measure values obtained by our proposed distance are identical to those obtained using Tessem distance, and are very close to those obtained using Jousselme distance. This example shows that, in this case study, the distance choice does not influence the components' rankings.

Additionally, we propose also to compare the results obtained for probability transformations presented in Table XI. As we can see in Fig. 7, the pignistic and the plausibility transformations give the same ranking: $I_{pignistic}^B(X_5) < I_{pignistic}^B(X_3) < I_{pignistic}^B(X_4) < I_{pignistic}^B(X_1) < I_{pignistic}^B(X_2)$ and $I_{plausibility}^B(X_5) < I_{plausibility}^B(X_3) < I_{plausibility}^B(X_4) < I_{plausibility}^B(X_1) < I_{plausibility}^B(X_2)$. The belief transformation gives the same ranking except for components X_1 and X_2 : $I_{belief}^B(X_5) < I_{belief}^B(X_3) < I_{belief}^B(X_4) < I_{belief}^B(X_2) < I_{belief}^B(X_1)$.

C. Case III: Reliability Data Based on Several Experts' Judgments

Let us consider three experts' judgments about components $X_1, X_2, X_3, X_4,$ and X_5 , as shown in Table V. To aggregate the experts' judgments, we use three combination rules: Dempster rule, Cautious rule, and Yager rule. Table VI shows the results of the reliability data related to the use of each rule between

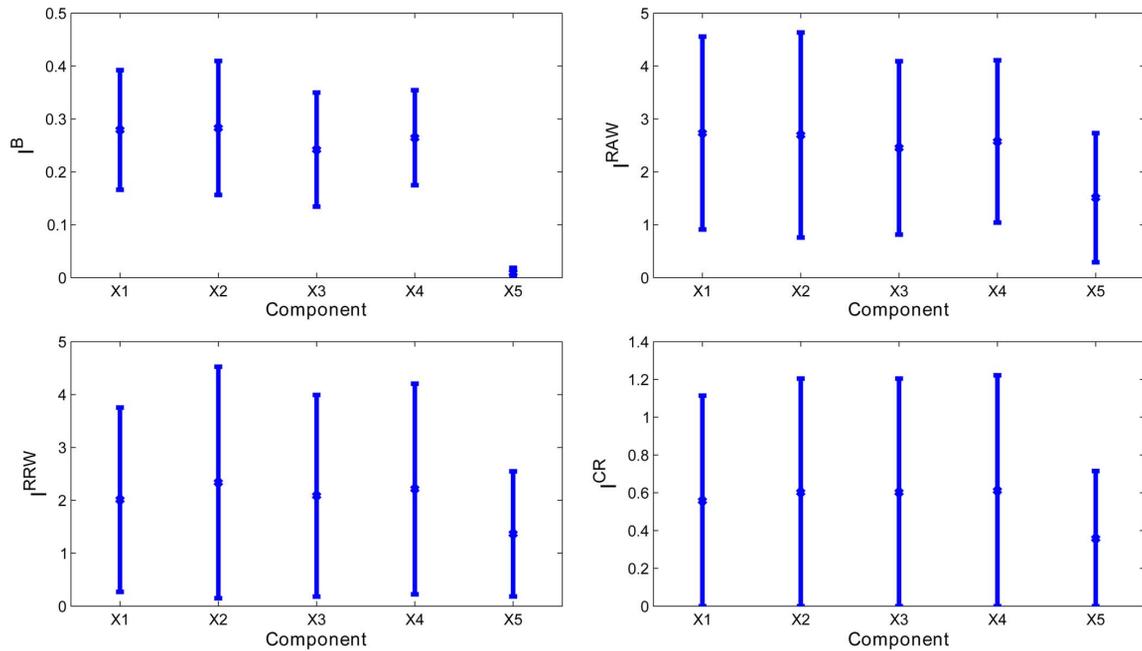


Fig. 4. Case II—Pignistic importance measures of components.

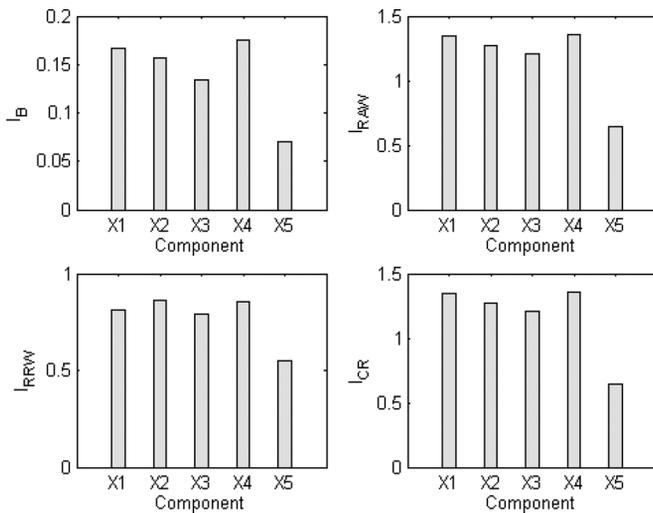


Fig. 5. Case II—Importance measures of components.

BBAs. Note that the conflict factor k between BBAs is not high ($k = \sum_{A \cap B = \emptyset, \forall A, B \subseteq X} m_i^X(A) m_j^X(B) < 0.6$). Thus, the Dempster rule does not lead to obvious contradictions in combination results. The Dempster rule considers that the three experts are equally reliable and s -independent. The Cautious rule considers that the experts are not s -independent. Instead of normalizing out the conflict, as in the Dempster rule, the Yager rule is based on the attribution of the conflict to the frame of discernment X . Then, we use the equations presented in Table II to compute and rank the components' importance. According to results presented in Fig. 8, we obtain the same ranking of components X_3 and X_5 in Dempster, Cautious, and Yager rules ($\text{Rank}(X_3) = 4$, and $\text{Rank}(X_5) = 5$). Whereas, the ranks of other components change depending on the combination rule.

For example, according to the RRW measure, the rank of component X_2 is 2 when using the Dempster rule, and 1 when using the Cautious or Yager rules. This result is why it is very important to choose the combination rule which has the hypothesis (s -independence, conflict, etc.) corresponding to our case study. For example, if the experts are not s -independent, we have to choose the Cautious rule. Whereas, if there is a high conflict between experts, we have to choose the Yager rule.

D. Case IV: Reliability Data With Variable Epistemic Uncertainty

We aim to study the influence of the epistemic uncertainty on the components' rankings. Therefore, we consider that there is no epistemic uncertainty with only one component in each case a, b, c, d , and e (cf. Table VII). For example, Case c corresponds to $m^{X_3}(\{F_3, W_3\}) = 0$ (there is no epistemic uncertainty about reliability data of component X_3). For simplicity, we choose to evaluate only Birnbaum pignistic measures. The same analysis can be done for other importance measures. Fig. 9 shows that components X_1, X_2, X_3 , and X_4 change ranks depending on the cases. For example, component X_4 changes rank between case a ($\text{Rank}(X_4) = 3$), b ($\text{Rank}(X_4) = 1$), and d ($\text{Rank}(X_4) = 2$). Therefore, an element which is the most important with respect to a value of epistemic uncertainty can be less important for a different value of epistemic uncertainty. This case clearly shows that, in some systems' configurations, the change of an epistemic uncertainty's amount gives the greatest impact on the components' rankings.

VI. CASE STUDY : FIRE-DETECTOR SYSTEM

The function of a fire detection system is to detect fire at the earliest practicable moment, and to give signals and indications so that appropriate action can be taken [66]. To increase the overall detectors' reliability, these systems must be tested periodically to detect dormant failures, *i.e.* to check that they will

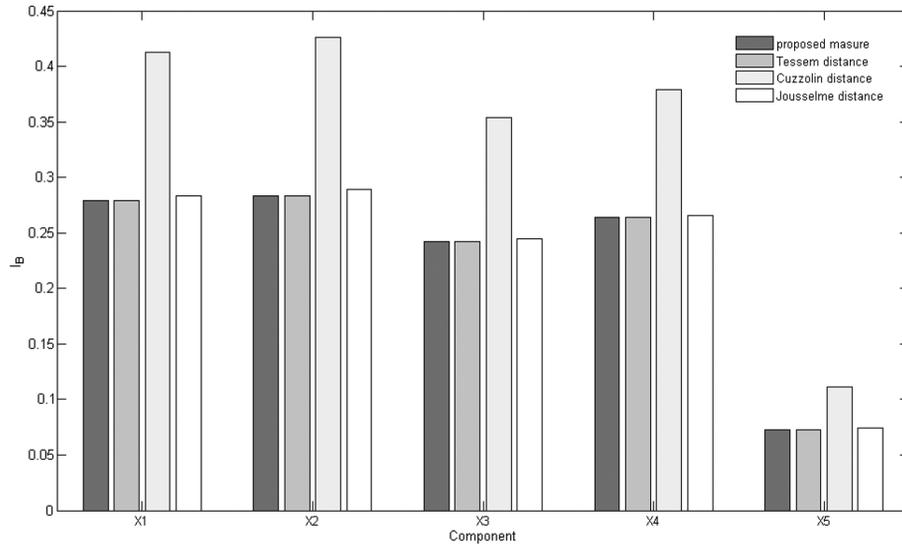


Fig. 6. Case II—Birnbaum pignistic measures based on several distances.

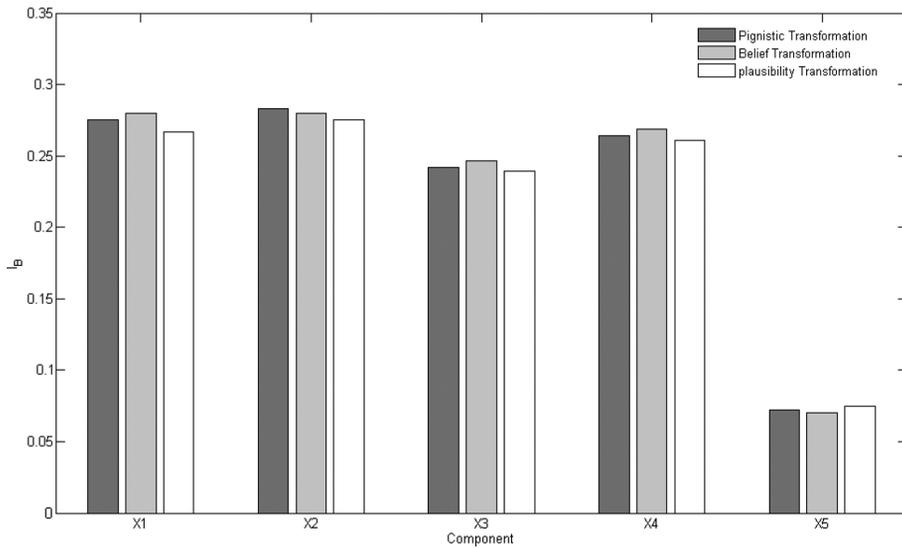


Fig. 7. Case II—Birnbaum measures based on several probability transformations.

respond if there is an actual demand. European standard EN 54 specifies requirements, test methods, and performance criteria against which the effectiveness and reliability of the component parts of fire detection can be assessed [66]. This reason is why industrialists need methods to rank fire detector components for tests and maintenance. This ranking can be carried out by identifying components which have the greatest impact on the overall system's reliability.

The system considered here is a fire detector located in a production room taken from Hoyland and Rausand [53]. It falls into two sections, heat and smoke detection sections with a manual alarm button. The configuration of the fire detector is represented in Fig. 10. In the heat detection part, there is a one circuit with four identical fuse plugs FP1, FP2, FP3, and FP4. If the temperature is more than 72 °C, the fuse plugs force the air out of the circuit. This circuit is connected to a pressure switch (PS). The PS is in a working state if at least one of the fuse plugs is in a working state. Then, it transmits a signal to a start relay (SR)

to activate an alarm, and to cause a shutdown of the system. The smoke-detection part is composed of three smoke detectors SD1, SD2, and SD3. These smoke-detectors are connected to a voting unit (VU) in a two out of three voting configuration. The DC source must be in a working state for the successful transmission of a signal from the heat-detector or smoke-detector. In the case of manual activation of this part, an operator (OP) must be always present. If the operator observes a fire, he has to turn on the manual-switch (MS) to relieve pressure in the circuit of the heat-detection part. This activates the PS, which in turn gives an electric signal to SR. The system has 13 *s*-independent Weibull components with survival functions (two-parameter Weibull distribution) given by

$$S_i(t) = \exp\left(-\frac{t}{\beta_i}\right)^{\eta_i} \quad (i = 1, \dots, 13)$$

The values of β_i and η_i given by experts are shown in Table VIII. This case study was used by Chaudhuri *et al.* [67]

TABLE VI
CASE III—THE OBTAINED BBAS BASED ON SEVERAL COMBINATION RULES

Components	Dempster rule		Cautious rule		Yager rule	
	$m^{X_i}(\{F_i\})$	$m^{X_i}(\{W_i\})$	$m^{X_i}(\{F_i\})$	$m^{X_i}(\{W_i\})$	$m^{X_i}(\{F_i\})$	$m^{X_i}(\{W_i\})$
X_1	0.0159	0.9841	0.0459	0.8214	0.0022	0.4920
X_2	0.0273	0.9727	0.1990	0.7690	0.0088	0.3927
X_3	0.0206	0.9794	0.1809	0.7915	0.0054	0.4029
X_4	0.0630	0.9370	0.2378	0.6667	0.0110	0.3094
X_5	0.0392	0.9608	0.1599	0.7704	0.0046	0.4158

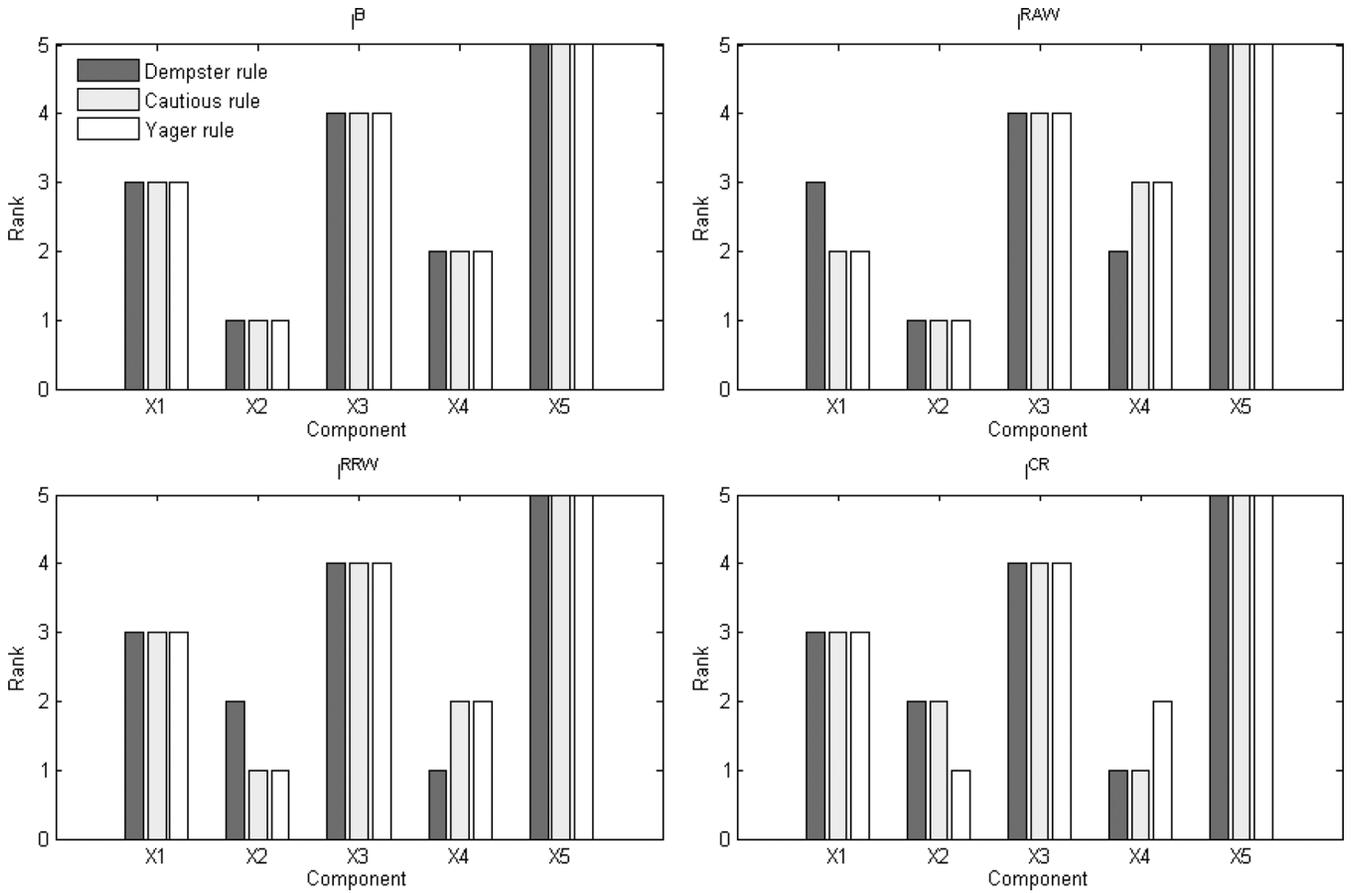


Fig. 8. Case III—Components' ranking.

TABLE VII
CASE IV—BBAS OF COMPONENTS X_1, X_2, X_3, X_4 AND X_5

Comp.	Case a		Case b		Case c		Case d		Case e	
	$m^{X_i}(\{F_i\})$	$m^{X_i}(\{W_i\})$								
X_1	0.13	0.87	0.05	0.79	0.05	0.79	0.05	0.79	0.05	0.79
X_2	0.10	0.74	0.18	0.82	0.10	0.74	0.10	0.74	0.10	0.74
X_3	0.07	0.78	0.07	0.78	0.15	0.85	0.07	0.78	0.07	0.78
X_4	0.15	0.69	0.15	0.69	0.15	0.69	0.23	0.77	0.15	0.69
X_5	0.13	0.72	0.13	0.72	0.13	0.72	0.13	0.72	0.8	0.2

to compute the Birnbaum importance of components based on a new representation of the structure function in the presence of precise values of β_i . In this study, we are in the presence of both aleatory uncertainties (Weibull components with survival

functions) and epistemic uncertainties (interval values of β_i). We aim to accomplish two goals.

- 1) Compare the components' rankings obtained in presence of epistemic uncertainty with those obtained when using

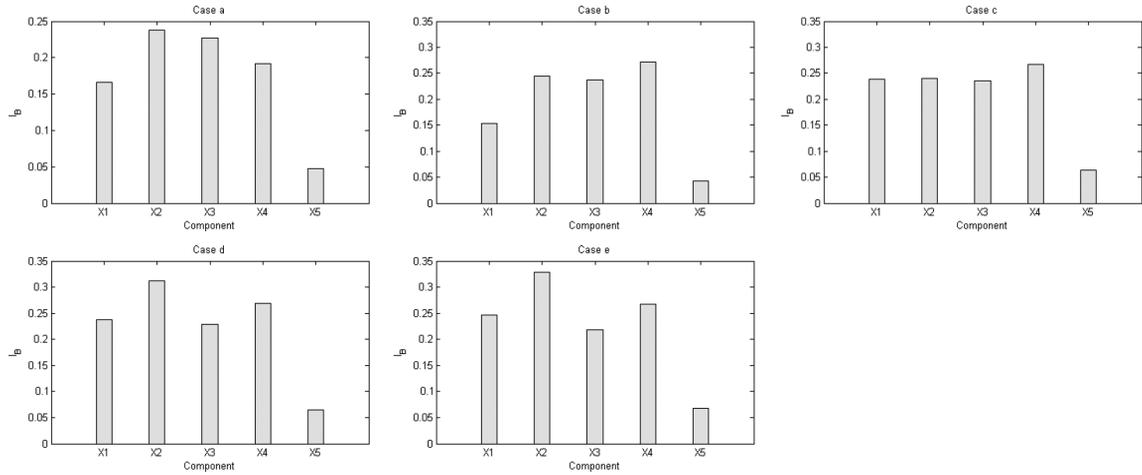


Fig. 9. Case IV—Birbaum importance measure results.

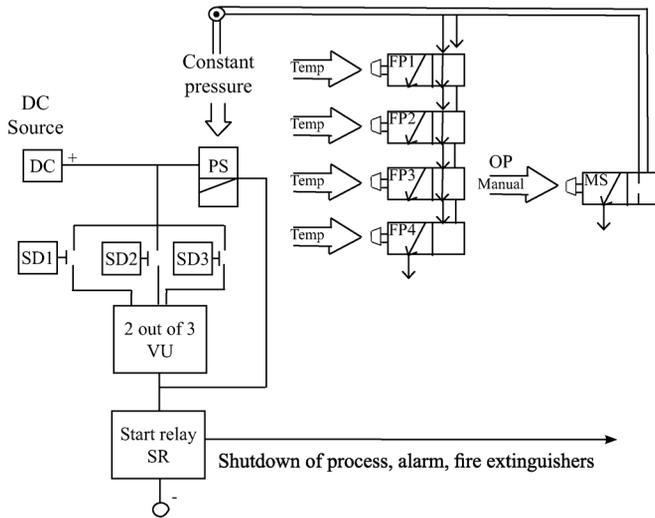


Fig. 10. The configuration of the fire-detector system [53].

 TABLE VIII
 COMPONENTS INFORMATION FOR THE FIRE-DETECTOR SYSTEM

Components number	Abbreviation	Description	Interval bounds of β (epistemic uncertainty)	η
1	DC	Current source	[0.9, 1.85]	1
2	SR	Start relay	[1.4, 1.75]	1
3	PS	Power switch	[1.2, 1.8]	1
4	OP	Operator	[1.2, 1.8]	1
5	MS	Manual switch	[1.7, 1.9]	1
6	VU	Voting	[1.7, 1.9]	1
7-10	FP1-4	Melt plugs	[2, 2.4]	1
11-13	SD1-3	Smoke detectors	[1.1, 2.15]	1

precise values of β_i ($\beta_i = \beta_{imin} + \beta_{imax}/2$) (Comparison I).

- 2) Compare the components' rankings obtained by the D-S approach with those obtained by the two-stages nested Monte-Carlo simulations approach (Comparison II).

A. Comparison I

The measures $I_{average}^B$, $I_{average}^{RAW}$, $I_{average}^{RRW}$, and $I_{average}^{CR}$ denote respectively the Birbaum, RAW, RRW, and CR impor-

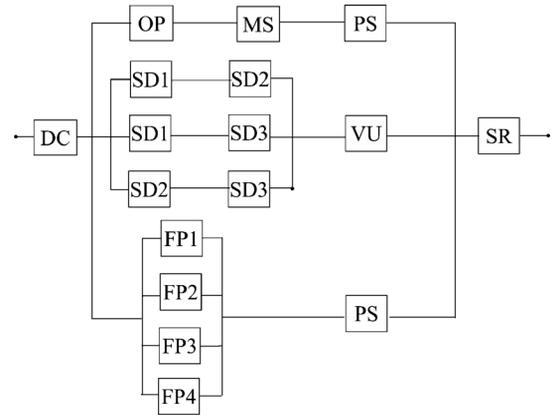


Fig. 11. Reliability block diagram of the fire-detector system.

tance measures obtained when using average values of β_i . The reliability block diagram of the system is given in Fig. 11. There are 8 minimal path sets:

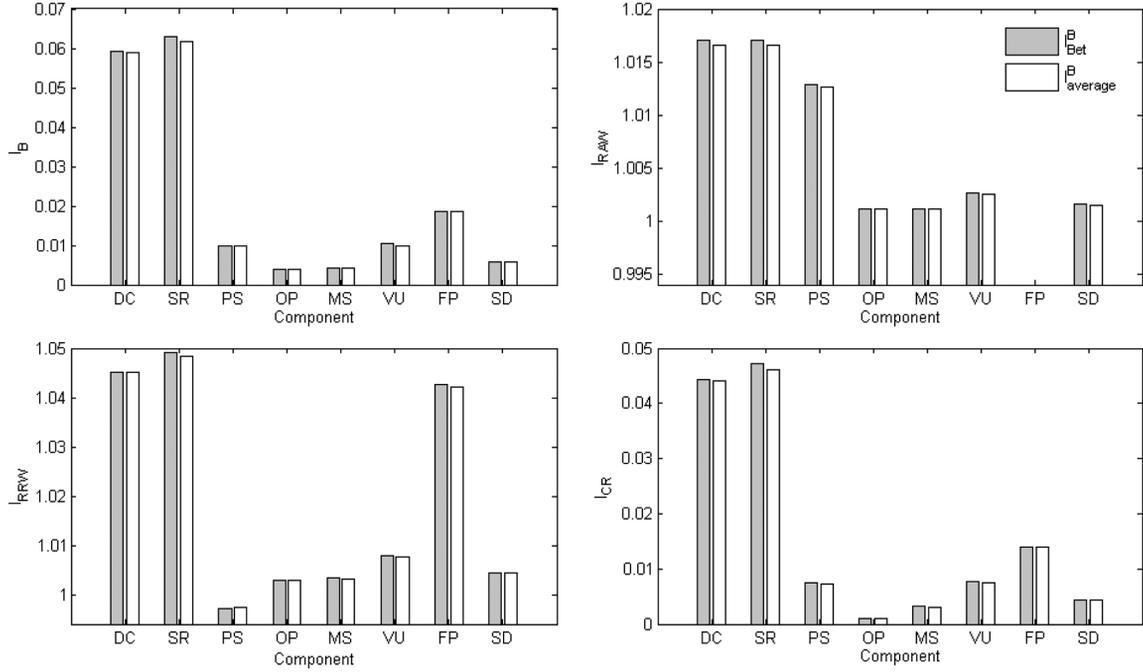
$$\{\{1, 4, 5, 3, 2\}, \{1, 11, 12, 6, 2\}, \{1, 11, 13, 6, 2\}, \{1, 12, 13, 6, 2\}, \{1, 7, 3, 2\}, \{1, 8, 3, 2\}, \{1, 9, 3, 2\}, \{1, 10, 3, 2\}\}.$$

The BBAs of components at time t are obtained as

$$m(\{W_i\}) = \exp^{-t/\beta_{imin}^\eta},$$

$$m(\{F_i\}) = 1 - \exp^{-t/\beta_{imax}^\eta} \quad (i = 1, \dots, 13)$$

Using the minimal paths method, we compute the BBA m_{Config} , which represents the system's configuration, and then we evaluate the reliability of the system using our method given in Section II-B. Finally, the importance measures are computed over time using the equations presented in Table II. Fig. 12 reports the importance measures results at time $t = 1.6$. Looking at the importance measures obtained at time $t = 1.6$, one may observe that, in the RRW importance measure, DC is more important than FP when using precise values of β_i ($I_{average}^{RRW}(DC) > I_{average}^{RRW}(FP)$), contrary to what would be happen if the components' ranking is based on the use of uncertainty values of β_i ($I_{Bet}^{RRW}(DC) < I_{Bet}^{RRW}(FP)$).

Fig. 12. Case study—Importance measures of components at $t = 1.6$.

Thus, reliability engineers can consider that the deterioration of component DC is more critical than FP, and inspections will be done first on DC instead of FP. The same remarks hold for components VU and FP according to the Birnbaum measure, and engineers can consider that the reliability of fire detector system is mostly perturbed by changes in the states of FP instead of VU. We can do the same analysis using CR measures. The drawback of the use of average values of β_i lies in the loss of epistemic uncertainty information, whereas D-S theory takes into account this information. This case study clearly shows the importance of taking into account epistemic uncertainty in the evaluation of importance measures, and the efficiency of D-S importance measures to address this problem.

B. Comparison II

We propose also to compare these results with importance measures obtained using a probabilistic approach based on Monte-Carlo simulations. The probabilistic approach is based on a two-stage nested Monte Carlo simulation: sampling of values of the probability distribution representing the epistemic uncertainty about the parameter β (outer loop), and nested sampling of values of the aleatory variables representing the failure probabilities (inner loop) of components. These aleatory uncertainties were quantified by a Weibull distribution. The parameter β of these random laws has been considered as subject to epistemic uncertainty. Its state of knowledge was expressed by a uniform and normal probability distributions. In probability theory, when a probability distribution function for an uncertain variable is not available, the uniform distribution function is often used, justified by Laplace's Principle of Insufficient Reason [26]. This principle can be interpreted to mean that all simple events for which a probability distribution function is unknown have equal probabilities. The software MATLAB has been used for generating minimal cut sets, and

TABLE IX
CASE STUDY—INTERVAL VALUES OF BIRNBAUM MEASURE: COMPARISON WITH MC SIMULATIONS

Components	MC simulations approach		D-S approach
	Normal distribution	Uniform distribution	
	99 percentile	99 percentile	
DC	[0,0.0048]	[0,0.0063]	[0,0.0098]
OP	[0,0.0007]	[0,0.0012]	[0,0.0022]
MS	[0,0.0007]	[0,0.0011]	[0,0.0021]
PS	[0.0001,0.0056]	[0,0.0078]	[0,0.0121]
SR	[0.0005,0.0083]	[0,0.0119]	[0,0.0196]
SD1-3	[0,0.00076]	[0,0.00115]	[0,0.0022]
VU	[0,0.0008]	[0,0.0012]	[0,0.0023]
FP1-4	[0,0.0017]	[0,0.0026]	[0,0.0039]

for reliability estimation. The resulting number of model runs to be performed is $n * m$, with n , and m being the sample sizes of the epistemic outer, and the aleatory inner simulation loops respectively. The sampling of epistemic and aleatory uncertainties simultaneously with a sample size $n = 10,000$ give the results presented in Table IX for a 99 percentile.

From the results of the present analysis (cf. Table IX), it is seen that both approaches can obtain interval bounds of importance measures, and provide decision-makers useful information about the importance of each components. The interval obtained from D-S theory is wider than that from the probabilistic approach. This is to be expected because the D-S approach only uses the information of the confidence bounds on the unknown parameter β , while in the probabilistic approach prior information (uniform and normal distributions) is assumed and incorporated into the analysis. However, by assuming a probability distribution for epistemic uncertainty, we are introducing once more uncertainty in the probabilistic approach. The experts state their uncertainty assessment of the value of β in terms of a range of possible values, but this does not justify the allocation of a specific distribution (uniform or normal).

TABLE X
CASE STUDY—PIGNISTIC VALUES OF BIRNBAUM MEASURE: COMPARISON WITH MC SIMULATIONS

Components	MC simulations approach		D-S approach
	Normal distribution	Uniform distribution	
	99 percentile	99 percentile	
DC	0.0024	0.0032	0.0049
OP	0.0003	0.0006	0.0011
MS	0.0003	0.0006	0.0010
PS	0.0028	0.0038	0.0060
SR	0.0043	0.0058	0.0098
SD1-3	0.0003	0.0005	0.0011
VU	0.0004	0.0006	0.0011
FP1-4	0.0008	0.0013	0.0019

On the other hand, in this case study, we offer two solutions to the reliability experts. The first solution is to obtain interval values of importance measures as represented in Table IX. In the case of $I_g^B = [a, b]$ and $I_h^B = [c, d]$ with $a > c$ and $b > d$, the component g is more important than component h . But in the case that $a > c$ and $b < d$, we cannot make a decision. In this case, we have to use pignistic values of importance measures. This is why we give pignistic importance measures of components in Table X.

VII. SUMMARY AND CONCLUSIONS

It is an important task to identify the components in a system which have the greatest impact on a system's reliability. In this paper, we introduced extended measures of component importance in the presence of both aleatory and epistemic uncertainties. These measures were based on a D-S reliability analysis approach, and affine arithmetic. A classical system was studied, and four cases representing different types of reliability data were analyzed. We finally proposed a comparison with hybrid probabilistic approaches. However, the computational complexity of the operations (extension, combination, and projection) used in D-S theory of evidence is one of the main problems that we have to solve. Particularly, D-S theory can be more time consuming when used to study the reliability of multi-state systems. Future research will focus on extending the D-S approach to other importance measures that also account for interactions such as joint reliability [68], [69], and total order reliability importance measures [70], [71].

APPENDIX

A. Formula of Marginalization

Consider a BBA $m^{\Omega_x \Omega_y}$ defined on the Cartesian product $\Omega_x \Omega_y$. The marginal BBA $m^{\Omega_x \Omega_y \downarrow \Omega_x}$ on Ω_x is defined by

$$m^{\Omega_x \Omega_y \downarrow \Omega_x}(A) = \sum_{B \subseteq \Omega_x \Omega_y / Proj(B \downarrow \Omega_x) = A} m^{\Omega_x \Omega_y}(B) \quad \forall A \subseteq \Omega_x \quad (15)$$

where $Proj(B \downarrow \Omega_x) = \{x \in \Omega_x / \exists y \in \Omega_y, (x, y) \in B\}$.

B. Formula of Vacuous Extension

Consider a BBA m^{Ω_x} defined on Ω_x . Its vacuous extension on $\Omega_x \Omega_y$ is defined by:

TABLE XI
CONSISTENCIES OF SOME PROBABILISTIC TRANSFORMATIONS

	α -consistency	p -consistency	ulb -consistency
<i>BelP</i>	Yes	Yes	Yes
<i>BelP</i>	No	Yes	No
<i>PlP</i>	No	Yes	No

$$m^{\Omega_x \uparrow \Omega_x \Omega_y}(B) = \begin{cases} m^{\Omega_x}(A) & \text{if } B = A \times \Omega_y \\ 0 & \text{otherwise.} \end{cases} \quad \forall A \subseteq \Omega_x \quad (16)$$

C. Formulas of Combination Rules

The Conjunctive \odot and Disjunctive \oplus rules are defined respectively by

$$m_{i \odot j}^{\Omega}(H) = \sum_{A \cap B = H, \forall A, B \subseteq \Omega} m_i^{\Omega}(A) m_j^{\Omega}(B), \forall H \subseteq \Omega \quad (17)$$

$$m_{i \oplus j}^{\Omega}(H) = \sum_{A \cup B = H, \forall A, B \subseteq \Omega} m_i^{\Omega}(A) m_j^{\Omega}(B), \forall H \subseteq \Omega \quad (18)$$

Dempster's rule is given by

$$m_{i \oplus j}^X(H) = \frac{\sum_{A \cap B = H, \forall A, B \subseteq X} m_i^X(A) m_j^X(B)}{1 - \sum_{A \cap B = \emptyset, \forall A, B \subseteq X} m_i^X(A) m_j^X(B)} \quad (19)$$

D. AI Operations

In AI, addition, subtraction, multiplication, and division of intervals are defined respectively as

$$\begin{aligned} x + y &= [\underline{x} + \underline{y}, \bar{x} + \bar{y}] \\ x - y &= [\underline{x} - \bar{y}, \bar{x} - \underline{y}] \\ x * y &= [\min(\underline{x}\underline{y}, \underline{x}\bar{y}, \bar{x}\underline{y}, \bar{x}\bar{y}), \max(\underline{x}\underline{y}, \underline{x}\bar{y}, \bar{x}\underline{y}, \bar{x}\bar{y})] \\ \frac{x}{y} &= [\underline{x}, \bar{x}] * \left[\frac{1}{\bar{y}}, \frac{1}{\underline{y}} \right] \quad \text{if } 0 \notin [\underline{y}, \bar{y}] \end{aligned}$$

E. Normalized Plausibility Probabilistic Transformation PlP

The formula of the normalized plausibility probabilistic transformation *PlP* is given by

$$PlP(B) = \frac{Pl(B)}{\sum_{A \in X} Pl(A)} = \frac{\sum_{C | C \cap B \neq \emptyset} m(C)}{\sum_{A \in X} \sum_{C | C \cap A \neq \emptyset} m(C)}$$

F. Normalized Belief Probabilistic Transformation BelP

The formula of the normalized belief probabilistic transformation *PlP* is given by

$$BelP(B) = \frac{m(B)}{\sum_{A \in X} Bel(A)} = \frac{m(B)}{\sum_{A \in X} \sum_{C \neq \emptyset | C \subseteq A} m(C)} \quad (20)$$

G. Consistencies of Some Probabilistic Transformations

The consistencies of some probabilistic transformations are given in Table XI.

H. Discounting Approach

Suppose we have to discount a BBA m^Ω by some factor $\delta \in [0, 1]$. When the BBA is discounted, the remaining assignment is applied to the combination of all options in the frame of discernment Ω . The number $1 - \delta$ is the degree of reliability attributable to the expert (Shafer [35] calls this a degree of trust), which leads to a BBA m_δ^Ω defined as

$$\begin{aligned} m_\delta^\Omega(A) &= (1 - \delta) \cdot m^\Omega(A) \quad \forall A \subseteq \Omega, A \neq \Omega \\ m_\delta^\Omega(\Omega) &= \delta + (1 - \delta) \cdot m^\Omega(\Omega) \end{aligned} \quad (21)$$

A discount rate $\delta = 1$ means that the expert can certainly not be trusted; the resulting BBA is then vacuous. On the contrary, a null discount rate leaves m^Ω unchanged, corresponding to the situation where the expert is known to be fully reliable. The discounting operation of a BBA m^Ω is also equivalent to the disjunctive combination of m^Ω with the BBA m_0^Ω defined by

$$m_0^\Omega(A) = \begin{cases} 1 - \delta & \text{if } A = \emptyset \\ \delta & \text{if } A = \Omega \\ 0 & \text{otherwise} \end{cases} \quad (22)$$

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