Introduction to belief functions
Exercises on statistical inference

Thierry Denœux

1 Exercise 1

The one-parameter Fréchet distribution with shape parameter $\alpha > 0$ has the cumulative distribution function (cdf)

$$P(X \leq x) = \exp(-x^{-\alpha})\mathbb{1}_{(0,+\infty)}(x)$$

1. Write a program that simulates this distribution using the probability integral transform method.

2. Let $X_1, \ldots, X_n, X_{n+1}$ be an iid random sample from the Frchet distribution with unknown shape parameter $\alpha > 0$. Write a program that computes the belief and the plausibility of the event $X_{n+1} \in [a, b]$ for any real interval $[a, b]$, given a realization $x_1, \ldots, x_n$ of $X_1, \ldots, X_n$.

2 Exercise 2

Write a program to solve the same problem as in Question 2 of Exercise 1, this time assuming that the sample is generated from the two-parameter Fréchet distribution with shape parameter $\alpha > 0$ and location parameter $m \in \mathbb{R}$, with cdf

$$P(X \leq x) = \exp[-(x - m)^{-\alpha}]\mathbb{1}_{(m,+\infty)}(x).$$

(Use a constrained nonlinear optimization function such as function `constrOptim.nl` in R package `alabama`).