

Introduction to belief functions

Exercises on statistical inference

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1 Exercise 1

The one-parameter Fréchet distribution with shape parameter $\alpha > 0$ has the cumulative distribution function (cdf)

$$P(X \leq x) = \exp(-x^{-\alpha}) \mathbb{1}_{(0, +\infty)}(x)$$

1. Write a program that simulates this distribution using the probability integral transform method.
2. Let X_1, \dots, X_n, X_{n+1} be an iid random sample from the Frchet distribution with unknown shape parameter $\alpha > 0$. Write a program that computes the belief and the plausibility of the event $X_{n+1} \in [a, b]$ for any real interval $[a, b]$, given a realization x_1, \dots, x_n of X_1, \dots, X_n .

2 Exercise 2

Write a program to solve the same problem as in Question 2 of Exercise 1, this time assuming that the sample is generated from the three-parameter Fréchet distribution with shape parameter $\alpha > 0$, location parameter $m \in \mathbb{R}$ and scale parameter $\sigma > 0$, with cdf

$$P(X \leq x) = \exp \left[- \left(\frac{x - m}{\sigma} \right)^{-\alpha} \right] \mathbb{1}_{(m, +\infty)}(x).$$

(Use a constrained nonlinear optimization function such as function `constrOptim.nl` in R package `alabama`).