Introduction to belief functions
Exercises on statistical inference

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1 Exercise 1

The one-parameter Fréchet distribution with shape parameter $\alpha > 0$ has the cumulative distribution function (cdf)

$$P(X \leq x) = \exp(-x^{-\alpha}) \mathbb{1}_{(0,+\infty)}(x)$$

1. Write a program that simulates this distribution using the probability integral transform method.
2. Write functions to compute the log-likelihood and the relative likelihood, given a realization $x_1, \ldots, x_n$ of an iid sample. Plot these functions for a particular sample.
3. Let $X_1, \ldots, X_n, X_{n+1}$ be an iid random sample from the Fréchet distribution with unknown shape parameter $\alpha > 0$. Write a program that computes the belief and the plausibility of the event $X_{n+1} \in [a, b]$ for any real interval $[a, b]$, given a realization $x_1, \ldots, x_n$ of $X_1, \ldots, X_n$.

2 Exercise 2

Write a program to solve the same problems as in Questions 2 and 3 of Exercise 1, this time assuming that the sample is generated from the three-parameter Fréchet distribution with shape parameter $\alpha > 0$, location parameter $m \in \mathbb{R}$ and scale parameter $\sigma > 0$, with cdf

$$P(X \leq x) = \exp\left[-\left(\frac{x-m}{\sigma}\right)^{-\alpha}\right] \mathbb{1}_{(m,+\infty)}(x).$$

(Use a constrained nonlinear optimization function such as function `constrOptim.nl` in R package `alabama`).