Introduction to belief functions, Lecture 1– Exercises

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1. An urn contains 90 balls, of which 30 are white, and 60 are either black or yellow. A ball is going to be drawn from the urn. Represent the uncertainty about the outcome of this experiment using a mass function on a suitable frame. Compute the corresponding belief and plausibility functions.

2. Let $\Omega = \{a, b, c\}$ and f the following function from 2^Ω to $[0, 1]$:

A	$\emptyset$	$\{a\}$	$\{b\}$	$\{a, b\}$	$\{c\}$	$\{a, c\}$	$\{b, c\}$	$\{a, b, c\}$
$f(A)$	0	0.5	0.2	0.8	0	0.5	0.5	1

   Is f a belief function?

3. An expert has given the following contour function on $\Omega = \{a, b, c, d, e, f\}$:

ω	a	b	c	d	e	f
$pl(\omega)$	0.1	0.3	0.5	1	0.7	0.3

   Compute the corresponding mass function, assuming that it is consonant.

4. Let m be a consonant mass function on a frame Ω and let Bel and Pl be the corresponding belief and plausibility functions. Show that, for any subset A of Ω, $Bel(A) > 0 \Rightarrow Pl(A) = 1$.

5. Let m_1 and m_2 be two mass functions on $\Omega = \{a, b, c, d\}$ defined as follows

$$m_1(\{a\}) = 0.3 \quad m_1(\{a, c\}) = 0.5 \quad m_1(\{b, c, d\}) = 0.2$$

   and

$$m_2(\{b, c\}) = 0.4 \quad m_2(\{a, c, d\}) = 0.5 \quad m_2(\{d\}) = 0.1.$$

   Compute the combined mass function by Dempster's rule. What is the degree of conflict between m_1 and m_2?

6. Let $\Omega = \{a, b\}$, and let m and m' be the following mass functions on Ω,

$$m = \{a\}^\alpha \oplus \{b\}^\beta, \quad m' = \{a\}^{\alpha'} \oplus \{b\}^{\beta'},$$

   where A^w denotes the mass function m such that $m(A) = 1 - w$ and $m(\Omega) = w$.

   (a) Compute m and m'.

   (b) Compute $m \oplus m'$.