Introduction to belief functions, Lecture 2– Exercise

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1. An oil company must decide whether or not to drill for oil. They are uncertain whether the hole will be dry (D), have a trickle of oil (T), or be a gusher (G). Drilling a hole costs \$70,000. The payoffs for hitting a gusher, a trickle or a dry hole are \$270,000, \$120,000, and \$0, respectively.

   (a) Which act do we select using the Laplace, maximax, maximin criteria?

   Solution: We have the following payoff matrix (in 1000\$):

	D	T	G
drill (f_1)	-70	50	200
not drill (f_2)	0	0	0

   Using the Laplace criterion, the average utility of drilling is

   $$\frac{1}{3}(-70 + 50 + 200) = 60$$

   and the utility of not drilling is 0, so $f_1 \succcurlyeq f_2$.

   Using the maximax criterion, the maximum utilities of drilling and not drilling are, respectively, 200 and 0, so again $f_1 \succcurlyeq f_2$.

   Using the maximin criterion, the minimum utilities of drilling and not drilling are, respectively, -70 and 0, so $f_2 \succcurlyeq f_1$.

   (b) Discuss the decision based on the Hurwicz criterion, for different values of the pessimism index.

   Solution: Let α denote the degree of pessimism. Act f_1 is preferred to f_2 iff

   $$-70\alpha + 200(1-\alpha) \geq 0 \Leftrightarrow \alpha \leq \frac{20}{27} \approx 0.74.$$

   (c) Based on seismic soundings, we have obtained the following mass function on $\Omega = \{D, T, G\}$:

   $$m(\{D\}) = 0.1, \quad m(\{T, D\}) = 0.4, \quad m(\{G, T\}) = 0.2, \quad m(\Omega) = 0.3$$

   Compute the lower and upper expected utilities for each of the two acts, as well as the pignistic expected utilities.

   Solution: If f_2 is chosen, the loss is surely zero, so the three expectations are equal to zero.

   For f_1, the induced mass function on $\mathcal{C} = \{-70, 50, 200\}$ is

   $$m_1(\{-70\}) = 0.1, \quad m_1(\{-70, 50\}) = 0.4, \quad m_1(\{50, 200\}) = 0.2,$$

   $$m_1(\{-70, 50, 200\}) = 0.3.$$

   The lower and upper expected utilities are, respectively,

   $$\underline{\mathbb{E}}_{m_1}(u) = -70 \times 0.1 - 70 \times 0.4 + 50 \times 0.2 - 70 \times 0.3 = -46$$

   and

   $$\overline{\mathbb{E}}_{m_1}(u) = -70 \times 0.1 + 50 \times 0.4 + 200 \times 0.2 + 200 \times 0.3 = 113.$$

The pignistic probability distribution corresponding to m is

$$p_1(-70) = 0.1 + 0.4/2 + 0.3/3 = 0.4$$

$$p_1(50) = 0.4/2 + 0.2/2 + 0.3/3 = 0.4$$

$$p_1(200) = 0.2/2 + 0.3/3 = 0.2.$$

The pignistic expected utility for act f_1 is, thus,

$$\mathbb{E}_{p_1}(u) = -70 \times 0.4 + 50 \times 0.4 + 200 \times 0.2 = 32.$$

(d) Discuss the decisions made using the generalized Hurwicz criterion, as a function of the pessimism index.

Solution: Let α denote the degree of pessimism. Act f_1 is preferred to f_2 iff

$$-46\alpha + 113(1-\alpha) \geq 0 \Leftrightarrow \alpha \leq \frac{113}{157} \approx 0.72.$$