Introduction to belief functions
Exercises on statistical inference

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1. Exercise on approximating Dempster's rule by Monte Carlo simulation:

   (a) Write a function that approximates by Monte Carlo simulation the combination of two consonant belief functions on $\mathbb{R}$, with contour functions of the form

   $$pl(x) = \exp\left(-\frac{h}{2}(x-m)^2\right),$$

   with mode $m \in \mathbb{R}$ and precision $h > 0$. The function should also return the degree of conflict between the two belief functions.

   (b) Apply this function to combine two belief functions with $m_1 = 0$, $h_1 = 0.3$, $m_2 = 1$, $h_2 = 2$. Plot the contour function as well as the lower and upper cds of the combined belief function.

   (c) What is the expression of the combined contour function? Verify it experimentally.

2. The one-parameter Fréchet distribution with shape parameter $\alpha > 0$ has the cumulative distribution function (cdf)

   $$P(X \leq x) = \exp(-x^{-\alpha})\mathbb{1}_{(0,+\infty)}(x)$$

   (a) Write a program that simulates this distribution using the probability integral transform method.

   (b) Write functions to compute the log-likelihood and the relative likelihood, given a realization $x_1, \ldots, x_n$ of an iid sample. Plot these functions for a particular sample.

   (c) Let $X_1, \ldots, X_n, X_{n+1}$ be an iid random sample from the Fréchet distribution with unknown shape parameter $\alpha > 0$. Write a program that computes the belief and the plausibility of the event $X_{n+1} \in [a, b]$ for any real interval $[a, b]$, given a realization $x_1, \ldots, x_n$ of $X_1, \ldots, X_n$.

3. We now assume that the sample is generated from the two-parameter Fréchet distribution with shape parameter $\alpha > 0$ and scale parameter $\sigma > 0$, with cdf

   $$P(X \leq x) = \exp\left[-\left(\frac{x}{\sigma}\right)^{-\alpha}\right]\mathbb{1}_{(0,+\infty)}(x).$$

We will us package functions `dfrechet` and `rfrechet` in package `VGAM` for, respectively, computing the density function and generating random data from this distribution.

Write a program to solve the same problems as in Questions 2 and 3 of Exercise 1. (Use a constrained nonlinear optimization function such as function `constrOptim.nl` in R package `alabama`).