# Introduction to belief functions, Lecture 1- Exercises 

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1. An urn contains 90 balls, of which 30 are white, and 60 are either black or yellow. A ball is going to be drawn from the urn. Represent the uncertainty about the outcome of this experiment using a mass function on a suitable frame. Compute the corresponding belief and plausibility functions.
2. Let $\Omega=\{a, b, c\}$ and $f$ the following function from $2^{\Omega}$ to $[0,1]$ :

| $A$ | $\emptyset$ | $\{a\}$ | $\{b\}$ | $\{a, b\}$ | $\{c\}$ | $\{a, c\}$ | $\{b, c\}$ | $\{a, b, c\}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $f(A)$ | 0 | 0.5 | 0.2 | 0.8 | 0 | 0.5 | 0.5 | 1 |

Is $f$ a belief function?
3. An expert has given the following contour function on $\Omega=\{a, b, c, d, e, f\}$ :

| $\omega$ | $a$ | $b$ | $c$ | $d$ | $e$ | $f$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $p l(\omega)$ | 0.1 | 0.3 | 0.5 | 1 | 0.7 | 0.3 |

Compute the corresponding mass function, assuming that it is consonant.
4. Let $m$ be a consonant mass function on a frame $\Omega$ and let $B e l$ and $P l$ be the corresponding belief and plausibility functions. Show that, for any subset $A$ of $\Omega, \operatorname{Bel}(A)>0 \Rightarrow \operatorname{Pl}(A)=1$.
5. Let $m_{1}$ and $m_{2}$ be two mass functions on $\Omega=\{a, b, c, d\}$ defined as follows

$$
m_{1}(\{a\})=0.3 \quad m_{1}(\{a, c\})=0.5 \quad m_{1}(\{b, c, d\})=0.2
$$

and

$$
m_{2}(\{b, c\})=0.4 \quad m_{2}(\{a, c, d\})=0.5 \quad m_{2}(\{d\})=0.1
$$

Compute the combined mass function by Dempster's rule. What is the degree of conflict between $m_{1}$ and $m_{2}$ ?
6. Let $\Omega=\{a, b\}$, and let $m$ and $m^{\prime}$ be the following mass functions on $\Omega$,

$$
m=\{a\}^{\alpha} \oplus\{b\}^{\beta}, \quad m^{\prime}=\{a\}^{\alpha^{\prime}} \oplus\{b\}^{\beta^{\prime}}
$$

where $A^{w}$ denotes the mass function $m$ such that $m(A)=1-w$ and $m(\Omega)=w$.
(a) Compute $m$ and $m^{\prime}$.
(b) Compute $m \oplus m^{\prime}$.

