Introduction to belief functions, Lecture 1– Exercises

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- 1. An urn contains 90 balls, of which 30 are white, and 60 are either black or yellow. A ball is going to be drawn from the urn. Represent the uncertainty about the outcome of this experiment using a mass function on a suitable frame. Compute the corresponding belief and plausibility functions.
- 2. Let $\Omega = \{a, b, c\}$ and f the following function from 2^{Ω} to [0, 1]:

A	Ø	$\{a\}$	$\{b\}$	$\{a, b\}$	$\{c\}$	$\{a, c\}$	$\{b, c\}$	$\{a, b, c\}$
f(A)	0	0.5	0.2	0.8	0	0.5	0.5	1

Is f a belief function?

3. An expert has given the following contour function on $\Omega = \{a, b, c, d, e, f\}$:

ω	a	b	c	d	e	f
$pl(\omega)$	0.1	0.3	0.5	1	0.7	0.3

Compute the corresponding mass function, assuming that it is consonant.

- 4. Let *m* be a consonant mass function on a frame Ω and let *Bel* and *Pl* be the corresponding belief and plausibility functions. Show that, for any subset *A* of Ω , $Bel(A) > 0 \Rightarrow Pl(A) = 1$.
- 5. Let m_1 and m_2 be two mass functions on $\Omega = \{a, b, c, d\}$ defined as follows

$$m_1(\{a\}) = 0.3$$
 $m_1(\{a,c\}) = 0.5$ $m_1(\{b,c,d\}) = 0.2$

and

$$m_2(\{b,c\}) = 0.4$$
 $m_2(\{a,c,d\}) = 0.5$ $m_2(\{d\}) = 0.1$

Compute the combined mass function by Dempster's rule. What is the degree of conflict between m_1 and m_2 ?

6. Let $\Omega = \{a, b\}$, and let m and m' be the following mass functions on Ω ,

$$m = \{a\}^{\alpha} \oplus \{b\}^{\beta}, \quad m' = \{a\}^{\alpha'} \oplus \{b\}^{\beta'},$$

where A^w denotes the mass function m such that m(A) = 1 - w and $m(\Omega) = w$.

- (a) Compute m and m'.
- (b) Compute $m \oplus m'$.