Theory of belief functions: Application to machine learning and statistical inderence Decision Analysis – Exercises

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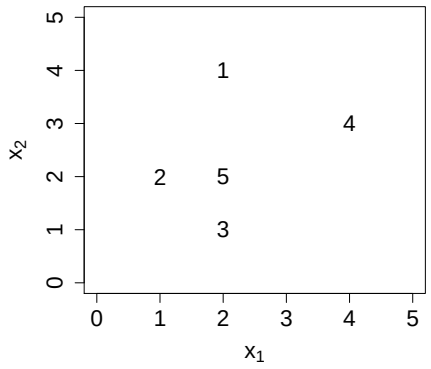
1. An oil company must decide whether or not to drill for oil. They are uncertain whether the hole will be dry (D), have a trickle of oil (T), or be a gusher (G). Drilling a hole costs \$70,000. The payoffs for hitting a gusher, a trickle or a dry hole are \$270,000, \$120,000, and \$0, respectively.
   (a) Which act do we select using the Laplace, maximax, maximin criteria?
   (b) Discuss the decision based on the Hurwicz criterion, for different values of the pessimism index.
   (c) Based on seismic soundings, we have obtained the following mass function on $\Omega = \{D, T, G\}$:
   $$m(\{D\}) = 0.1, \quad m(\{T, D\}) = 0.4, \quad m(\{G, T\}) = 0.2, \quad m(\Omega) = 0.3$$
   Compute the lower and upper expected utilities for each of the two acts, as well as the pignistic expected utilities.
   (d) Discuss the decisions made using the generalized Hurwicz criterion, as a function of the pessimism index.
2. We consider a classification problem with three classes $\Omega = \{\omega_1, \omega_2, \omega_3\}$ and two attributes. The following figure shows the feature vectors for five objects.



We have partial information about the class labels of objects 1 to 4, and we want to classify object 5 using the evidential K-nearest neighbor rule with $K = 3$ and function φ defined as follows:

$$\varphi(d) = \frac{1}{1+d}.$$

Denoting by y_i the class of object i, we have the following *partial class labels*:

$$y_1 \in \{\omega_1, \omega_2\}, \quad y_2 = \omega_2, \quad y_3 \in \{\omega_2, \omega_3\}, \quad y_4 \in \{\omega_1, \omega_3\}.$$

This means that, for instance, we only know that object 1 belongs either to class ω_1 or ω_2; we know that object 2 belongs to ω_2 for sure, etc.

(a) We wish to classify object 5. Compute the corresponding mass function.

(b) We consider four acts: f_0, f_1 , f_2 and f_3, where f_0 means rejection, and f_k assignment of object 5 to class ω_k. We have the following loss matrix:

	ω_1	ω_2	ω_3
f_0	0.6	0.6	0.6
f_1	0	1	1
f_2	1	0	2
f_3	1	0.5	0

Compute the lower and upper risks for each of the four acts. Which decision do we make for object 5, using the pessimistic and optimistic decision rules?

(c) Which decision do we make for object 5, using the pignistic decision rule?