

Computational Statistics

Chapter 6: Bootstrap

1. Let X_1, \dots, X_n be an iid sample from an exponential distribution with rate θ , i.e., the pdf of X_i is

$$f_\theta(x_i) = \theta \exp(-\theta x_i) \mathbb{1}_{[0, +\infty)}(x_i)$$

for $i = 1, \dots, n$. The purpose of this exercise is to estimate the median $m = F_\theta^{-1}(0.5)$ of this distribution using the bootstrap.

- (a) Generate a sample x_1, \dots, x_n of size $n = 50$, with $\theta = 0.5$. What is the true value of m ?
- (b) Write a function `bootstrap` that generates B bootstrap replicates of the observed data.
- (c) Compute the plug-in estimate \hat{m} of m and its standard error.
- (d) Compute a 95% confidence on m using the percentile method.
- (e) Compute a 95% confidence on m using the bootstrap- t method. (You will need to use a double bootstrap loop).