Elicitation of Expert Opinions for Constructing Belief Functions

Amel Ben Yaghlane*, Thierry Denœux† and Khaled Mellouli‡

*,‡*LARODEC, Institut Supérieur de Gestion de Tunis*
41 rue de la liberté Cité Bouchoucha 2000 Le Bardo Tunisie
**amel.ben-yaghlane@hds.utc.fr*

*,†*HeuDiaSyC, Université de Technologie de Compiègne, CNRS*
Centre de Recherches de Royallieu
B.P. 20529 F-60205 Compiègne Cedex France
†*Thierry.Denoeux@hds.utc.fr*

‡*Institut des Hautes Etudes Commerciales*
2016 Carthage Présidence Tunisie
‡*khaled.mellouli@ihec.rnu.tn*

Abstract

This paper presents a method for constructing belief functions from elicited expert opinions expressed in terms of qualitative preference relations. These preferences are transformed into constraints of an optimization problem whose resolution allows the generation of the least informative belief functions according to some uncertainty measures. Mono-objective and Multi-objective optimization techniques are used to optimize one or different uncertainty measures simultaneously.

1. Introduction

When dealing with real-world problems, uncertainty can rarely be avoided. In general, uncertainty emerges whenever information pertaining to the situation is incomplete, imprecise, contradictory or deficient in some other respect.[1]

In such situations and especially when data needed for the considered problem are not all available, a way to complement missing information is to use opinions elicitated from experts in the problem domain, i.e. individuals who have special skills in a subject area and are recognized as qualified to address the problem at hand. Expert opinions are statements, based on knowledge and experience, that experts provide in response to a given question.[2] Hence, the elicitation of expert opinions may be defined as the process of collecting and representing expert knowledge regarding the uncertainties of a problem.

For representing uncertainty, we can use appropriate frameworks such as probability theory, evidence theory or possibility theory. In this paper, we are interested in representing expert opinions in the evidence theory framework and precisely in the context of the Transferable Belief Model (TBM).[3] In the last twenty years, this theory, also known as the theory of belief functions (BFs) or Dempster-Shafer (DS) theory,[4] has attracted considerable interest as a rich and flexible framework for representing and reasoning with imperfect information. The concept of BFs subsumes those of probability and possibility measures, making the theory very general. The TBM is a recent variant of DS theory developed by Smets which is considered to be a coherent and axiomatically justified interpretation of BF theory.

For collecting expert opinions, we can proceed quantitatively or qualitatively. In a quantitative manner, we may ask the expert to provide his opinions as numbers according to the uncertainty theory that will be used to represent them. This approach supposes that the expert is familiar enough with the concepts of the theory framework to be able to correctly quantify his judgments. This is not always obvious. An alternative way is to elicit expert opinions qualitatively. This allows experts to express their opinions in a natural way, while deferring the use of numbers.

Recently, several authors have addressed the problem of eliciting qualitatively expert opinions and generating associated quantitative BFs.[5–7]

In this paper, we propose a new method for constructing BFs from elicited expert opinions. Our method consists in representing qualitatively expert opinions in terms of preference relations that will be transformed into constraints of an optimization problem. The resolution of this problem allows the generation of the least informative BFs according to some uncertainty measures (UMs). Mono-objective and Multiobjective optimization techniques are used and different optimization models are proposed and discussed.

The following section summarizes the background concepts related to the TBM, uncertainty measures, and the Least Commitment Principle (LCP). In Sec. 3, we summarize previous work addressing the problem considered in this paper. Section 4 presents the new method proposed for constructing BFs from qualitative expert opinions. The optimization models introduced in this section are illustrated by examples. Section 5 concludes the paper.

2. Background

2.1. *The transferable belief model*

The TBM is based on a two-level model: a credal level where beliefs are entertained, combined and updated, and a pignistic level where beliefs are converted to probabilities to make decisions.

2.1.1. *Credal level*

Let Ω denote a finite set called the frame of discernment. A basic belief assignment (bba) or mass function is a function $m : 2^\Omega \to [0, 1]$ verifying:

$$\sum_{A \subseteq \Omega} m(A) = 1. \tag{1}$$

$m(A)$ measures the amount of belief that is exactly committed to A. A bba m such that $m(\emptyset) = 0$ is said to be normal. Notice that this condition is relaxed in the TBM: the allocation of a positive mass to the empty set ($m(\emptyset) > 0$) is interpreted as a consequence of the open-world assumption and can be viewed as the amount of belief allocated to none of the propositions of Ω. A bba verifying this condition is said to be subnormal, or unnormalized. The subsets A of Ω such that $m(A) > 0$ are called focal elements (FEs). Let $\mathcal{F}(m) \subseteq 2^\Omega$ denote the set of FEs of m.

The belief function induced by m is a function bel: $2^\Omega \to [0, 1]$, defined as:

$$\text{bel}(A) = \sum_{\emptyset \neq B \subseteq A} m(B), \tag{2}$$

for all $A \subseteq \Omega$. bel(A) represents the amount of support given to A.

The plausibility function associated with a bba m is a function pl: $2^\Omega \to [0,1]$ defined as:

$$\mathrm{pl}(A) = \sum_{\emptyset \neq B \cap A} m(B). \tag{3}$$

pl(A) represents the total amount of potential specific support that could be given to A.

The commonality function associated with a bba m is a function q: $2^\Omega \to [0,1]$ defined as:

$$\mathrm{q}(A) = \sum_{B \supseteq A} m(B), \tag{4}$$

where $A, B \subseteq \Omega$.

2.1.2. *Pignistic level*

At this level, beliefs are used to make decisions. When a decision must be made, the beliefs held at the credal level induce a probability measure at the pignistic level. This transformation is called the pignistic transformation. Let m be a bba defined on Ω, the probability function induced by m at the pignistic level, denoted by BetP and also defined on Ω is given by:

$$\mathrm{BetP}(\omega) = \sum_{A:\omega \in A} \frac{m(A)}{|A|}, \tag{5}$$

for all $\omega \in \Omega$ and where $|A|$ is the number of elements of Ω in A. This probability function can be used in order to make decisions using expected utility theory. Its justification is based on rationality requirements detailed by Smets and Kennes.[3]

2.2. *Uncertainty measures*

Several measures have been proposed to quantify the information content or the degree of uncertainty of a piece of information.[1] In this section we will focus on some of these measures proposed within the theory of evidence. For more details see Refs. 1, 8 and 9.

Klir[1] noticed that in BFs theory two types of uncertainty can be expressed: nonspecificity or imprecision on the one hand, and discord or strife on the other hand. Nonspecificity is connected with sizes (cardinalities) of FEs while discord expresses conflicts among the various FEs. Composite measures, referred to as global or total measures of uncertainty, have also been proposed. They attempt to capture both nonspecificity and conflict.

2.2.1. *Nonspecificity measures*

Dubois and Prade[1] proposed to measure the nonspecificity of a normal bba by a function N defined as:

$$N(m) = \sum_{A \in \mathcal{F}(m)} m(A) \log_2 |A|. \qquad (6)$$

The bba m is all the more imprecise (least informative) that $N(m)$ is large. The minimum ($N(m) = 0$) is obtained when m is a Bayesian BF (FEs are singletons) and the maximum ($N(m) = \log_2 |\Omega|$) is reached when m is vacuous ($m(\Omega) = 1$). The function N is a generalization of the Hartley function ($H(A) = \log_2 |A|$ where A is a finite set).

2.2.2. *Conflict measures*

Conflict measures are considered as the generalized counterparts of the Shannon's entropy ($-\sum_{\omega \in \Omega} p(\omega) \log_2 p(\omega)$ where p is a probability measure). Yager, Hohle, and Klir and Ramer[1,8,9] defined different conflict measures that may be expressed as follows:

$$Conflict(m) = -\sum_{A \in \mathcal{F}(m)} m(A) \log_2 f(A), \qquad (7)$$

where f is, respectively, pl, bel or BetP. These conflict measures are called, respectively, Dissonance (E), Confusion (C) and Discord (D).

Notice that different conflict measures that are not a generalization of the Shannon's entropy have been proposed by Smets[8] and George and Pal.[10]

2.2.3. *Composite measures*

Different global measures have been proposed by several authors.[1,8,9] Among them, the pignistic entropy (EP), and the total uncertainty (H) defined, respectively, as:

$$EP(m) = -\sum_{\omega \in \Omega} \mathrm{BetP}(\omega) \log_2 \mathrm{BetP}(\omega), \tag{8}$$

$$H(m) = \sum_{A \in \mathcal{F}(m)} m(A) \log_2 \left(\frac{|A|}{m(A)} \right). \tag{9}$$

The interesting feature of $H(m)$ is that it has a unique maximum.

2.3. *Least commitment principle*

The LCP,[11] also referred to as the principle of maximum uncertainty,[1] plays a central role in the TBM. It formalizes the idea that one should never presuppose more beliefs than justified. Given a family of BFs compatible with a set of constraints, depending on how their "information content" is compared, the LCP indicates that the most appropriate is the *least committed*. Dubois and Prade[11] have made three proposals to order BFs according to their informational content: pl-ordering, q-ordering, and s-ordering.[11] In this paper, however, we propose to apply the LCP using UMs for comparing the informational content of BFs. As pointed out by Klir [1],the degree of uncertainty of a piece of information is intimately connected to its information content. The LCP plays a role similar to the Maximum Entropy principle in Bayesian theory.

3. Previous Works

Methods for eliciting qualitatively expert opinions and generating associated quantitative BFs have been proposed by several authors.[5–7] In the sequel, some of these works are summarized.

3.1. *Wong and Lingras' method*

Wong and Lingras[5] proposed a method for generating BFs from qualitative preference relations. To express expert opinions, they defined two binary relations $\cdot >$ and $\sim$ defined on 2^{Ω} and called, respectively, the preference relation and the indifference relation. The idea behind this method is that given a pair of propositions A and B, an expert may express which of the propositions is more likely to be true using the preference relation $\cdot >$, or may judge the two propositions equally likely to be true using the indifference relation $\sim$ defined as:

$$A \sim B \Leftrightarrow (\neg(A \cdot > B), \neg(B \cdot > A)). \tag{10}$$

The objective of Wong and Lingras' method is to represent these preference relations by a BF *bel*, such that:

$$A \cdot > B \Leftrightarrow bel(A) > bel(B), \tag{11}$$

$$A \sim B \Leftrightarrow bel(A) = bel(B), \tag{12}$$

where $A, B \in 2^{\Omega}$.

Note that this method does not require that the expert supply the preference relations between all pairs of propositions in $2^{\Omega} \times 2^{\Omega}$. In fact, it allows the generation of BFs using *incomplete* qualitative preference relations. It has been shown that the existence of such BF depends on the structure of the preference relation $\cdot >$ that should satisfies the following axioms:

(1) *Asymmetry*: $A \cdot > B \Rightarrow \neg(B \cdot > A)$.
(2) *Negative Transitivity*: $\neg(A \cdot > B)$ and $\neg(B \cdot > C) \Rightarrow \neg(A \cdot > C)$.
(3) *Dominance*: For all $A, B \in 2^{\Omega}, A \supseteq B \Rightarrow A \cdot > B$ or $A \sim B$.
(4) *Partial monotonicity*: For all $A, B, C \in 2^{\Omega}$, if $A \supset B$ and $A \cap C \neq \emptyset$, then $A \cdot > B \Rightarrow (A \cup C) \cdot > (B \cup C)$.
(5) *Nontriviality*: $\Omega \cdot > \emptyset$.

Since the preference relation $\cdot >$ is asymmetric and negatively transitive, $\cdot >$ is a *weak order*.[12] It should be noted that Axioms 1 and 2 imply that $\cdot >$ is transitive. It has also been shown[13] that the binary relation $\sim$ defined by 10 is an *equivalence relation* on 2^{Ω}, i.e. it is reflexive, symmetric and transitive. Let $S = \cdot > \cup \sim$, defined on 2^{Ω}. S is a *complete preorder*[12] since $\cdot >$ is a weak order and $\sim$ is an equivalence relation.

To generate a BF from such preference relations, Wong and Lingras proceeded in two steps: determine the FEs, and compute the bba. The first step consists in considering that all the propositions that appear in the preference relations are potential FEs. Then, some of them are eliminated according to the following condition: if $A \sim B$ for some $B \subset A$, then A is not a FE. The second step enables the generation of a bba from the preference relations through the resolution of the system of equalities and inequalities defined by Eqs. (11)

and (12) using a perceptron algorithm. It should be noted that several BFs may be solutions of this system. However, the perceptron algorithm selects arbitrary only one of them.

It has been noted[6] that this method does not address the issue of inconsistency in the pairwise comparisons. In fact, the expert may provide inconsistent preference relations ($A\cdot > B$, $B\cdot > C$, and $C\cdot > A$).

3.2. *Bryson et al.' method*

Bryson, *et al.*[6] proposed a method called "Qualitative discrimination process" (QDP) for generating belief functions from qualitative preferences. The QDP is a multi-step process. First, it involves a qualitative scoring step in which the expert assign propositions first into a *Broad* category bucket, then to a corresponding *Intermediate* bucket, and finally to a corresponding *Narrow* category bucket. The qualitative scoring is done using a table where each *Broad* category is described by a linguistic quantifier in the sense of Parsons.[6,7] Hence, it allows the expert to progressively refine the qualitative distinctions in the strength of his beliefs in the propositions. In the second step, the qualitative scoring table from step 1 is used to identify and remove non-focal propositions by determining if the expert is indifferent in his strength of belief of any propositions and their subsets in the same or lower *Narrow* category bucket. It should be noted that this step is consistent with Wong and Lingras' approach presented in the previous section. Step 3 is called "imprecise pairwise comparisons" because the expert is required to provide numeric intervals to express his beliefs on the relative truthfulness of the propositions. In step 4, the consistency of the belief information provided by the expert is checked. Then, the belief function is generated in step 5 by providing a bba interval for each FE. Finally, in step 6, the expert examines the generated BF and stops the QDP if it is acceptable, otherwise the process is repeated.

It should be noted that the QDP, in spite of being proposed as a qualitative approach for generating BFs from qualitative information, involves numeric intervals in the elicitation process between all the pairs of propositions to provide BFs. Furthermore, Yaghlane *et al.*[14] showed that the QDP may violate the axiom of transitivity of the indifference relation.

4. Constructing Belief Functions from Qualitative Preferences

In this section we propose a new method for constructing BFs from qualitative expert opinions expressed in terms of preference relations. Our method allows the generation of *optimized* BFs in the sense of one or several UMs. We first present the main ideas behind our method, then we propose the optimization techniques and models used for deriving BFs along with illustrative examples. We also point out the main differences between our method and those presented in the previous section (see also Refs. 14 and 15).

4.1. *Main ideas*

Expressing expert opinions in terms of qualitative relations as proposed by Wong and Lingras[5] seems to be very attractive. In fact, it is natural and quite easy to make pairwise comparisons between propositions of a frame of discernment modeling a certain problem. Convinced of this motivation, we also propose to use the preference and indifference relations $(\cdot >, \sim)$ defined by Wong and Lingras to express expert judgments in our method. We assume that the preference relation satisfies axioms (1)–(5) introduced in Sec. 3.1.

Given such binary relations, we propose to convert them into constraints of an optimization problem whose resolution, according to some UMs, allows the generation of the *least informative* or the *most uncertain* BFs, as prescribed by the LCP recalled in Sec. 2.3.

Consequently, the criterion or objective function we optimize is an UM of the BF to be generated and the constraints are derived from the expert preferences, as defined in equations (11) and (12) as follows:

$$A \cdot > B \Leftrightarrow bel(A) - bel(B) \geq \varepsilon, \tag{13}$$

$$A \sim B \Leftrightarrow |bel(A) - bel(B)| \leq \varepsilon, \tag{14}$$

where $\varepsilon > 0$ is considered to be the smallest gap that the expert may discern between the degrees of belief in two propositions A and B. Note that ε is a constant specified by the expert before beginning the optimization process.

A crucial step for generating BFs before solving such optimization problem is to determine BF focal elements. We propose to consider that all the propositions existing in the preference and the indifference relations expressed by the expert are potential FEs. Furthermore, we assume that Ω should always be considered as a potential FE, which seems to us to be more coherent with BF theory.

Therefore, considering the problem of generating quantitative BFs from qualitative preference relations as an optimization problem, allows us to integrate the issue of the information content in the constructed BFs in our method. It should be noted that none of the methods presented in Sec. 3 address this issue. Furthermore, our method addresses the inconsistency of the preference relations provided by the expert. In fact, if these relations are consistent, then the optimization problem is feasible. Otherwise no solutions will be found. Thus, the expert may be guided to reformulate his preferences.

4.2. *Mono-objective optimization model*

According to the ideas presented above, we propose to formulate the problem by the following mono-objective optimization model. This model allows the construction of BFs that maximize one UM.

Model 1
$\text{Max}_m \, UM(m)$
s.t.
$bel(A) - bel(B) \geq \varepsilon \qquad \forall A \cdot > B$
$bel(A) - bel(B) \leq \varepsilon \qquad \forall A \sim B$
$bel(A) - bel(B) \geq -\varepsilon \qquad \forall A \sim B$
$\sum_{A \in \mathcal{F}(m)} m(A) = 1;\ m(A) \geq 0\ \forall A \subseteq \Omega;\ m(\emptyset) = 0,$

where the first, second and third constraints are derived from Eqs. (13) and (14), representing the quantitative constraints corresponding to the qualitative preference relations.

Example 1. Let $\Omega = \{a, b\}$ be a frame of discernment and let $\{a\} \cdot > \{b\}$ be the preference relation given by an expert. To construct a BF from this preference relation, we first define the potential FEs of the BF, as proposed in Sec. 4.1. Hence, $\mathcal{F}(m_1) = \{\{a\}, \{b\}, \Omega\}$. Then, we formulate this problem according to Model 1. The UM we propose to optimize is the pignistic entropy (EP) given by Eq. (8). Assume that $\varepsilon = 0.01$.

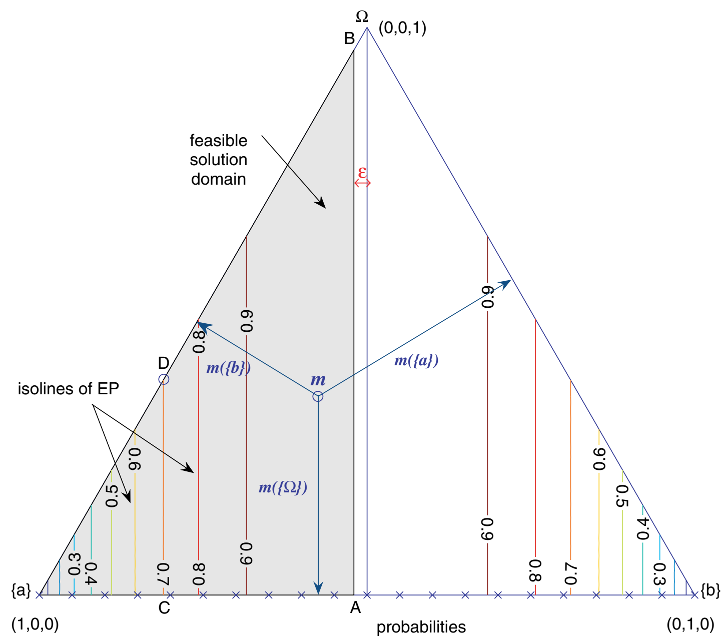

Fig. 1. Graphical representation.

Notice that several BFs are solutions of this optimization problem. These BFs are plotted in Fig. 1 (segment AB). Each BF is represented as a point in an equilateral triangle using barycentric coordinates. The lower left corner, the lower right corner, and the upper corner correspond, respectively, to $\{a\}$, $\{b\}$, and Ω. The orthogonal distance to the lower side of the triangle is thus proportional to $m(\Omega)$, while the distances to the right-hand and the left-hand sides are, respectively, proportional to $m(\{a\})$ and $m(\{b\})$. The gray region corresponds to the domain of feasible solutions. Although this model allows the construction of BFs from qualitative preference relations, we consider that having only these preferences constitute too weak information to generate BFs.

4.3. *Multiobjective optimization models*

As an alternative formulation of the BF generation problem, we propose to use multiobjective optimization techniques.[16] One of the well-known multiobjective methods is goal programming. This model allows to take into account simultaneously several objectives in a problem for choosing the most satisfactory solution within a set of feasible solutions.[17]

The idea behind the use of goal programming to formulate our problem is to be able to integrate additional information about the BFs to be generated. We can do this by asking the expert to give besides the preference relations, his certainty degree for the considered problem. Hence, we consider this additional information as a goal to be reached and formulate the problem by the following goal programming model:

Model 2
$\text{Min}_{m,\delta^+,\delta^-} \quad (\delta^+ + \delta^-)$
s.t.
$UM(m) - \delta^+ + \delta^- = G$
$bel(A) - bel(B) \geq \varepsilon \qquad \forall A \cdot > B$
$bel(A) - bel(B) \leq \varepsilon \qquad \forall A \sim B$
$bel(A) - bel(B) \geq -\varepsilon \quad \forall A \sim B$
$\sum_{A \in \mathcal{F}(m)} m(A) = 1;\ m(A) \geq 0\ \forall A \subseteq \Omega;\ m(\emptyset) = 0;\ \delta^+, \delta^- \geq 0,$

where δ^+ and δ^- indicate, respectively, positive and negative deviations of the achievement level from aspirated level.[17] This model allows us to restrict the search space to the proximity of the goal G (level of aspiration) associated with the objective (UM).

Example 2. Let us consider the setting of Example 1. Suppose that the degree of certainty of the expert is equal to 70%. The problem is formulated according to Model 2.

Notice that, again we have several solutions, as the goal G may be attained in several points. Figure 1 (segment CD) shows that any bba such that $EP(m)$ are points of the isoline of EP equal to G are solutions of Model 2.

To overcome the problem encountered with the two previous models, we propose to integrate in the objective function of Model 2, the nonspecificity measure. So, the model is transformed as follows:

Model 3
$\text{Min}_{m,\delta^+,\delta^-} \quad (\delta^+ + \delta^-) - N(m)$
s.t.
$UM(m) - \delta^+ + \delta^- = G$
$bel(A) - bel(B) \geq \varepsilon \quad \forall A \cdot > B$
$bel(A) - bel(B) \leq \varepsilon \quad \forall A \sim B$

$bel(A) - bel(B) \geq -\varepsilon \quad \forall A \sim B$
$\sum_{A \in \mathcal{F}(m)} m(A) = 1;\ m(A) \geq 0\ \forall A \subseteq \Omega;\ m(\emptyset) = 0;\ \delta^+, \delta^- \geq 0.$

Solving this model allows us to generate a tradeoff solution. The BF constructed is the *least specific* and the *least informative* BF in the neighborhood of G. It is represented by the point D in Fig. 1. Notice that this point is the intersection of the isolines of $N(m)$ which are horizontal lines, and $EP(m) = 70\%$ (segment CD).

Detect inconsistencies

We also propose a different goal programming model allowing us to check the consistency of the preference relations provided by the expert. This is done by introducing slack variables in the constraints as follows:

Model 4
Min $(\delta^+ + \delta^-) + \sum_{A \cdot > B} \eta_{AB} + \sum_{C \sim D} \varphi_{CD} + \sum_{C \sim D} \varphi'_{CD} - N(m)$
s.t.
$UM(m) - \delta^+ + \delta^- = G$
$bel(A) - bel(B) + \eta_{AB} \geq \varepsilon \quad \forall A \cdot > B$
$bel(C) - bel(D) \leq \varepsilon + \varphi_{CD} \quad \forall C \sim D$
$bel(C) - bel(D) + \varphi'_{CD} \geq \varepsilon \quad \forall C \sim D$
$\sum_{A \in \mathcal{F}(m)} m(A) = 1;\ m(A) \geq 0 \quad \forall A \subseteq \Omega;\ m(\emptyset) = 0;\ \delta^+, \delta^- \geq 0;$
$\eta_{AB} \geq 0 \quad \forall A, B \text{ s.t. } A \cdot > B;\ \varphi_{CD} \geq 0, \varphi'_{CD} \geq 0 \quad \forall C, D \text{ s.t. } C \sim D.$

Consequently, the inconsistencies are detected when the slack variables are positive.

5. Conclusion

A new method for constructing BFs from elicited expert opinions expressed in terms of qualitative preference relations has been defined. It consists in transforming the preference relations provided by the expert into constraints of an optimization problem involving one or several uncertainty measures. Mono-objective and multi-objective optimization techniques were used to solve such constrained optimization problem. The BFs generated are the least informative ones. Further work is under way to extend our method to combine multi-expert qualitative opinions.

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