Computational Statistics
Chapter 4: Classical simulation

1. Suppose we ten observations (8,3,4,3,1,7,2,6,2,7) are observed from a Poisson distribution $\mathcal{P}(\lambda)$. A lognorm prior distribution for λ is assumed: $\log\lambda \sim \mathcal{N}(\log(4), 0.5^2)$. Denote the likelihood as $L(\lambda; \mathbf{x})$ and the prior as $\pi(\lambda)$. The MLE of λ is $\widehat{\lambda} = \overline{x}$.

   (a) Plot $q(\lambda|\mathbf{x}) = \pi(\lambda)L(\lambda; \mathbf{x})$ and $e(\lambda) = \pi(\lambda)L(\widehat{\lambda}; \mathbf{x})$ as a function of λ.

   (b) Generate a sample of size $n = 1000$ from the posterior distribution $f(\lambda|\mathbf{x})$ using the rejection sampling method. Draw a histogram of this sample and plot the prior distribution on the same graph.

   (c) Compute a 95% confidence interval on the posterior expectation $\mathbb{E}(\lambda|\mathbf{x})$.

   (d) Repeat the same operations as in the two previous questions, using now the SIR method.

   (e) Compare graphically the two samples using the functions `qqplot` and `boxplot`.