Risk Assessment in Drinking Water Production Using Belief Functions

Sabrina Démotier[1,2], Thierry Denœux[2], and Walter Schön[2]

[1] Ondeo Services, Information Technology Division,
38, rue du Président Wilson,
78230 Le Pecq, France
sabrina.demotier@ondeo.com

[2] Université de Technologie de Compiègne, UMR CNRS 6599 Heudiasyc,
BP 20529, 60205 Compiègne, France
{tdenoeux, wschon}@hds.utc.fr

Abstract. This paper presents an original method for risk assessment in water treatment, based on belief functions. The risk of producing non-compliant drinking water (i.e., such that one of the quality parameter exceeds the regulation standards), is estimated taking into account the quality parameters of raw water and the process line of the treatment plant (technology, different failure modes and corresponding failure rates). Uncertainty on available data (treatment steps efficiency, failure rates, times to repair and raw water quality) is modeled using belief functions that are combined to compute a degree of confidence that the produced water will meet quality standards. The methodology recovers the classical results (obtained by fault tree analysis) as a limit case when uncertainties on input data are modeled by probabilities, and still provides informative results when only weaker forms of knowledge are available.

1 Problem Description

The production and distribution of good quality water to consumers is a fundamental issue, given the medical and financial consequences that could result from the delivery of insufficient quality water. It is therefore necessary to set up a treatment process adapted to the raw water to be treated, and to estimate the residual risk of producing water that does not comply with the regulation (in most cases the problem is detected by real time monitoring and the production is stopped, but this induces heavy financial penalties).

In the classical approach, this risk assessment process is performed by determining the probability of the undesirable event "production of non-compliant water", taking into account the quality of the resource to be treated (i.e., the estimated probability to find a given concentration of an undesirable component), characteristics of the treatment unit (efficiency of the treatment steps), as well as different failure modes that can occur in the process line (failure rates and repair times of each mode). Such a process was developed and described in detail in previous papers [5, 1].

One major difficulty in applying risk assessment methods in the environmental engineering domain is that basic data are not perfectly known and are often determined by expert judgement with a high level of uncertainty. In this paper, it is proposed to model the uncertainty on raw water quality, process line efficiency and state of the treatment plant (nominal or failure mode) in the belief function framework. Each source of information will be modeled by a belief function and combined to obtain an assessment of the plausibility to produce non compliant water.

In the limit case where the basic belief functions are probabilities, the proposed methodology recovers the results obtained by the classical approach: in that case, the belief function obtained for the variable representing the non compliance of produced water is also a probability measure. In the general case, however, the belief function obtained is no more a probability and it is possible to compute the belief, the plausibility and the pignistic probability to produce non compliant water. These results can be used to estimate a level of confidence to meet contractual requirements with a given treatment plant technology, and therefore to help treatment plant designers to choose the optimal architecture, given an objective level of residual risk.

The rest of the paper is organized as follows. The classical methodology is first recalled in Section 2. Our approach is then described in Section 3, and compared to the classical approach in Section 4. Simulations are presented in Section 5, and Section 6 concludes the paper.

2 Classical Approach

The following is just a brief reminder of the major concepts described in [5, 1]. The current regulation on potable water takes into accont 62 quality parameters of various types (turbidity, colour, concentration of mineral or organic components, physicochemical properties...). To meet these requirements, the treatment process must be adapted, on the one hand, to the general quality of water, and on the other hand to exceptional pollution peaks. To take into account this resource variability, a treatment plant is composed of the succession of various treatment processes (preoxydation, clarification, polishing, disinfection...).

For each quality parameter, the efficiency of each treatment step is represented by a transfer function, giving the output concentration C_{out} as a function of the input concentration C_{in} for the considered parameter. In most cases, this transfer function is linear and can be expressed using a single parameter α, called the abatement rate or reduction factor: $C_{out} = (1 - \alpha)C_{in}$. It is also possible to account for nonlinear transfer functions by defining different abatement rates according to the input concentration, but we will not consider that case in this paper.

By combining these local transfer functions (established for each treatment step), it is possible to define a global transfer function, which represents the global efficiency of the treatment line for a given parameter. This global transfer function must be determined for the nominal mode of the treatment plant (abatement rate α_0) and also for all possible failure modes. The "Failure Modes Effects and Criticality Analysis" (FMECA) methodology allows to determine, for each of the n possible failure modes, the corresponding degraded abatement rate α_i, the failure rate λ_i and the repair time T_i. The probability to be in failure mode i is then

$$p_i = \lambda_i T_i, \tag{1}$$

and the probability of being in the nominal state is given by

$$p_0 = 1 - \sum_{i=1}^{n} p_i. \tag{2}$$

Based on previously defined transfer functions, it is possible to define acceptable water quality thresholds for raw water in each operating mode (nominal and degraded) of the treatment plant, by inverting all global transfer functions and by applying these inverse functions to the normalized threshold N imposed for produced water. This step is illustrated in Figure 1: $n+1$ thresholds concerning raw water are obtained as: $\eta_i = N/(1-\alpha_i)$, $(0 \leq i \leq n)$, and $n+2$ possible raw water states are defined as: $e_i = [\eta_i, \eta_{i-1}]$, $(0 \leq i \leq n+1)$, with $\eta_{n+1} = 0$ and $\eta_{-1} = \infty$. Two possible states for the produced water are also defined: s_0 (corresponding to non compliance with the norm: $C_{out} > N$) and s_1 ($C_{out} \leq N$).

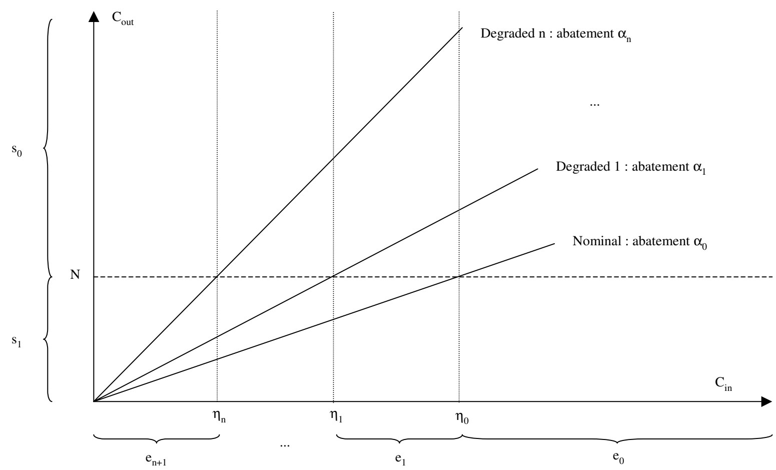

Fig. 1. Transfer function (output concentration of an undesirable water characteristics, as a function of the input concentration) in nominal mode and for n degraded mode of the treatment plant. In this linear approximation, the transfer function for mode i only depends on an abatement rate α_i, such as $C_{out} = (1 - \alpha_i)C_{in}$.

The last step of classical methodology is performed via Fault Tree Analysis (FTA), as illustrated in Figure 2. For the considered parameter, the top level

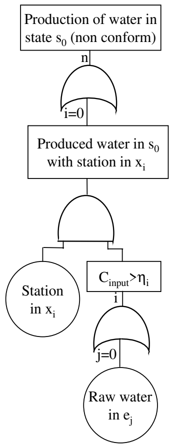

Fig. 2. Fault tree analysis of event "Production of water in state s_0" (non conform to the norm) .

event of the fault tree is "Produced water in state s_0 (non compliant with norm N)". The first level of the fault tree is a decomposition between the different possible modes (nominal and failure modes) of the treatment plant by the top level OR gate. The plant being in a given mode i will not accept input concentrations exceeding η_i for the considered parameter: second level AND gate. The raw water states whose concentration is more than η_i are states $e_j (0 \leq j \leq i)$: third level OR gate. The minimal cutset is easily obtained and the probability of non compliance (unavailability concerning the considered parameter) is given by:

$$p(s_0) = \sum_{i=0}^{n} \sum_{j=0}^{i} p_i q_j \ , \tag{3}$$

where q_j is the probability that the raw water is in state e_j. The global unavailability is simply obtained by adding the contribution of each quality parameter.

3 Belief Function Approach

3.1 Rationale

The classical solution described above assumes the availability of precise and complete prior knowledge of transfer functions, failure rates and repair times, as well as enough historical data to estimate the distribution of water quality parameters. However, such knowledge and data are usually not available, in particular in the case of call for bids, where a proposal must be submitted based on partial information. Moreover, transfer functions, repair time and failure rates can only be obtained by tests laboratories, expert knowledge or feedback from operational sites, which generally does not allow to obtain reliable estimates for a specific site. That is why an approach integrating these various uncertainties was developed. The belief function framework was chosen because of its flexibility for representing weak forms of knowledge [7], and because it generalizes Probability Theory, allowing to recover the classical results when all required data are available.

3.2 Notations and Background

The interpretation of belief functions adopted in this paper is that of Smets' Transferable Belief Model (TBM) [9]. In this model, a belief function is understood as representing an agent's state of belief, without resorting to an underlying probability model. Only the essential definitions and specific notations will be given here. A detailed exposition of the TBM may be found in [9].

A basic belief assignment (bba) on domain (or frame of discernment) X is noted m^X (for convenience, we use the same notation X for a variable and its domain). It is defined as a function from the powerset 2^X of X to $[0,1]$ verifying $\sum_{A\subseteq X} m^X(A) = 1$. The corresponding belief and plausibility functions are defined, respectively, as:

$$bel^X(A) = \sum_{\emptyset\neq B\subseteq A} m^X(B), \tag{4}$$

$$pl^X(A) = \sum_{B\cap A\neq\emptyset} m^X(B). \tag{5}$$

Given a bba $m^{X\times Y}$ defined on the Cartesian product of two domains X and Y, the marginal bba $m^{X\times Y\downarrow X}$ on X is defined, for all $A \subseteq X$, as

$$m^{X\times Y\downarrow X}(A) = \sum_{\{B\subseteq X\times Y \mid \mathrm{Proj}(B\downarrow X)=A\}} m^{X\times Y}(B)\ , \tag{6}$$

where $\mathrm{Proj}(B \downarrow X)$ denotes the projection of B onto X, defined as

$$\mathrm{Proj}(B \downarrow X) = \{x \in X \mid \exists y \in Y, (x, y) \in B\}\ . \tag{7}$$

Conversely, let m^X be a bba on X. Its *vacuous extension* on $X\times Y$ is defined as:

$$m^{X\uparrow X\times Y}(B) = \begin{cases} m^X(A) \text{ if } B = A \times Y \text{ for some } A \subseteq X, \\ 0 \qquad\quad \text{otherwise.} \end{cases} \tag{8}$$

Another useful notion is that of *ballooning extension* [8]. Let $m^X[y]$ denote the conditional bba on X, given that $Y = y$. The ballooning extension of $m^X[y]$ on $X \times Y$ is the least committed bba, whose conditioning on y yields $m^X[y]$ (see [8] for detailed justification). It is obtained as:

$$m^X[y]^{\Uparrow X\times Y}(B) = \begin{cases} m^X(A) \text{ if } B = (A \times \{y\}) \cup (X \times (Y \setminus \{y\})) \text{ for some } A \subseteq X, \\ 0 \qquad\quad \text{otherwise.} \end{cases} \tag{9}$$

Let us now consider two bba's m_1^X and m_2^X induced by two distinct sources of information. If both sources are known to be reliable, they can be combined using the (unnormalized) Dempster's rule of combination, leading to a new bba $m_{1 \textcircled{\cap} 2}^X = m_1^X \textcircled{\cap} m_2^X$, defined as:

$$m_{1 \textcircled{\cap} 2}^X(A) = \sum_{B \cap C = A} m_1^X(B) m_2^X(C). \tag{10}$$

Finally, the TBM is based on a two level mental modals: the *credal level* where beliefs are entertained and represented by belief functions, and the *pignistic level* where decisions are made. The *pignistic transformation* maps a bba m^X to a probability measure $BetP^X$ on X, defined as:

$$BetP^X(A) = \sum_{B \subseteq X} \frac{m^X(B)}{1 - m^X(\emptyset)} \frac{|A \cap B|}{|B|}, \quad \forall A \subseteq X. \tag{11}$$

3.3 Application

Available data. Let us go back to the problem presented in Section 2, but let us now assume that, for each failure mode x_i $(i = 1, \ldots, n)$, the abatement rate α_i, the failure rate λ_i and the repair time T_i are not known precisely. Let $\widetilde{\alpha}_i = (\alpha_i^-, \alpha_i^0, \alpha_i^+)$ be a triangular fuzzy number defining a flexible constraint on α_i, and let $[\lambda_i^-, \lambda_i^+]$ and $[T_i^-, T_i^+]$ denote interal-valued assessments of λ_i and T_i, respectively. Furthermore, the probability distribution (q_j) of input concentrations is no longer assumed to be known. Instead, we more realistically assume that a finite sample $C_{in,1}, \ldots, C_{in,K}$ has been observed.

Discretization of input and output spaces. To translate this information in the TBM framework, we first have to define the underlying variables expressed on suitable finite domains. In the classical approach, the output concentration was discretized in two categories: water either meets or does not meet a fixed quality limit. However, this approach is too restrictive and results in a loss of information. To refine this discretization, we now define $\ell+1$ thresholds σ_k, $k = 0, \ldots, \ell$, which induce $\ell + 2$ possible states s_k, $k = 0, \ldots, \ell + 1$ for the output water (see Figure 3). For a given mode x_i of the treatment plant, the output threshold σ_k defines an input threshold $\theta_{k,i}$ (the input concentration must be less than $\theta_{k,i}$ for the output concentration to be less than σ_k when the treatment plant is in mode x_i). In order to recover the classical limit, one of the output thresholds must be the norm ($N = \sigma_k$ for some k) and the input thresholds must at least contain the $n + 1$ values $\theta_{k,i}$ obtained with that k and all functioning modes i of the treatment plant. However, the discretization can take into account more values than this minimal set. We note η_j $(0 \leq j \leq m)$ the input thresholds arranged in decreasing order and e_j $(0 \leq j \leq m + 1)$ the corresponding input states. Note that we define $\ell + 1$ thresholds for each of the $n + 1$ modes, so that $m + 1 \leq (n + 1)(l + 1)$ (the upper bound may not be strict because some of the thresholds may be equal).

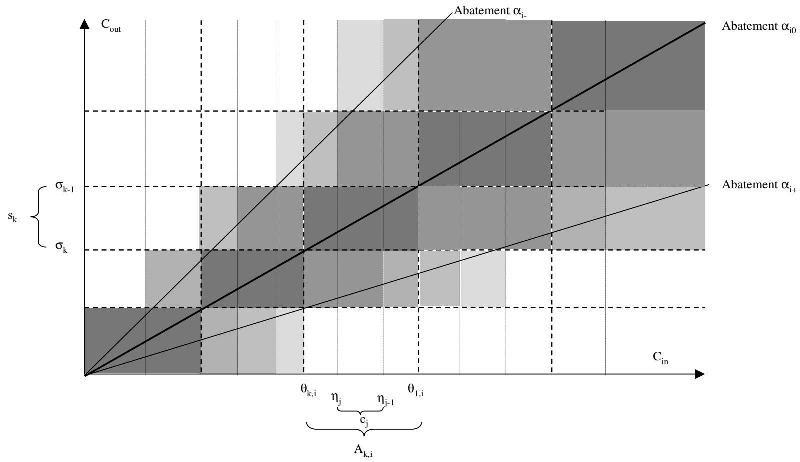

Fig. 3. Discretization of input and output concentrations, and fuzzy transfer function.

Frames of discernment. We thus have three underlying variables: the discretized input concentration taking values in $E = \{e_0, \ldots, e_{m+1}\}$, the discretized output concentration taking values in $S = \{s_0, \ldots, s_{\ell+1}\}$, and the plant state in $X = \{x_0, \ldots, x_n\}$. We now have to translate the available pieces of information into belief functions on the joint space $X \times E \times S$, combine this evidence, and marginalize on S to obtain our belief concerning the output concentration values.

Representation of transfer functions. The fuzzy abatement rate $\widetilde{\alpha}_i$ for operating mode i may be seen as defining a fuzzy relation between input and output concentrations. This fuzzy relation may be expressed as a possibility distribution π_i on variables C_{in} and C_{out} defined as a function of the ration $\rho = C_{out}/C_{in}$ as:

$$\pi_i(C_{in}, C_{out}) = \begin{cases} 0 & \text{if } \rho \leq \alpha_i^- \text{ or } \rho \geq \alpha_i^+ \\ \dfrac{\rho - \alpha_i^-}{\alpha_i^0 - \alpha_i^-} & \text{if } \alpha_i^- < \rho \leq \alpha_i^0 \\ \dfrac{\alpha_i^+ - \rho}{\alpha_i^+ - \alpha_i^0} & \text{if } \alpha_i^0 < \rho < \alpha_i^+ \end{cases} \tag{12}$$

After discretization of input and output concentrations, π_i induces possibility distribution $\pi_i^{E\times S}$ on the product space $E \times S$ (see Figure 3), defined as:

$$\pi_i^{E\times S}(e_j, s_k) = \sup_{C_{in}\in e_j, C_{out}\in s_k} \pi_i(C_{in}, C_{out}) \tag{13}$$

Such a possibility distribution is known to be equivalent to a consonant bba $m^{E\times S}[x_i]$ (see [2]).

Belief on X. In the classical case, knowledge of failure rates λ_i and repair times T_i induced a propability function on X using (1) and (2). Since λ_i and T_i are now only known to lie in given intervals, we have a family $\mathcal{P}^X$ of probability distributions on X, defined by the contraints $p_i^- \leq p_i \leq p_i^+$ $(i = 0, \ldots, n)$, with $p_i^- = \lambda_i^- T_i^-$, $p_i^+ = \lambda_i^+ T_i^+$ $(i = 1, \ldots, n)$, $p_0^- = 1 - \sum_{i=1}^n \lambda_i^+ T_i^+$ and $p_0^+ = 1 - \sum_{i=1}^n \lambda_i^- T_i^-$. The lower and upper probability of an event $A \subseteq X$ are given by:

$$P^-(A) = \max\left(\sum_{x_i \in A} p_i^-, 1 - \sum_{x_i \notin A} p_i^+\right) \tag{14}$$

$$P^+(A) = \min\left(\sum_{x_i \in A} p_i^+, 1 - \sum_{x_i \notin A} p_i^-\right) \tag{15}$$

These lower and upper probabilities do not, in general, verify the axioms of belief and plausibility measures. However, we may represent this information in the belief function framework by the most specific bba m^X (according, e.g., to the nonspecificity uncertainty measure [4]), whose set of compatible probability functions includes $\mathcal{P}^X$. This may be obtained by solving the following linear program:

$$\min_{m^X} \sum_{\emptyset \neq A \subseteq X} m^X(A) \log |A|$$

under the constraints:

$$bel^X(A) \leq P^-(A) \leq P^+(A) \leq pl^X(A), \quad \forall A \subseteq X.$$

Belief on E. The available information on E is of a different nature: it consists of a finite sample $C_{in,1}, \ldots, C_{in,K}$ of sample values. A simple approach to build a belief function on E might be to consider the histogram, i.e. to define $m^E(e_j)$ as the relative frequency of observations falling in class e_j. This approach, however, is not satisfactory in the small sample case because it does not take into account the sample size. Ideally, the inferred belief function should reflect the amount of available information, and hence the sample size. One way to achieve this goal is to generate B bootstrap replicates of the data [3]. Let $p_b(e_j)$ be the relative frequency of class e_j in bootstrap sample b, and define $p^-(e_j)$ and $p^+(e_j)$ as, say, the 1st and 9th deciles of the distribution $p_b(e_j), b = 1, \ldots, B$. We then obtain lower and upper probabilities on E, which define a family $\mathcal{P}^E$ of probability distributions. As before, we may translate this information in the belief function format by considering the most specific bba m^E, whose set of compatible probability distributions includes $\mathcal{P}^E$. However, as E may be much larger than X, the complexity of this solution might become too high. A simpler approach is to restrict the number of focal elements of m^E. For instance, if m^E is constrained to be quasi-Bayesian (i.e., to have only singletons and E as focal elements), the solution can be shown to be:

$$m^E(\{e_j\}) = p^{*-}(e_j) \quad j = 0, \ldots, m+1 \tag{16}$$

$$m^E(E) = \max_j (p^{*+}(e_j) - p^{*-}(e_j)) \tag{17}$$

$$m^E(A) = 0 \quad \forall A \subseteq E, A \neq E, |A| \neq 1 \tag{18}$$

with

$$p^{*-}(e_j) = \max\left(p^-(e_j), 1 - \sum_{j' \neq j}^{m+1} p^+(e_{j'})\right) \quad j = 0, \ldots, m+1 \tag{19}$$

$$p^{*+}(e_j) = \min\left(p^+(e_j), 1 - \sum_{j' \neq j}^{m+1} p^-(e_{j'})\right) \quad j = 0, \ldots, m+1 \tag{20}$$

Combination and marginalization. The final step is to combine all the available evidence, and marginalize on S. For that purpose, all belief functions must first be extended to the product space $X \times E \times S$ using the ballooning extension for $m^{E\times S}[x_i]$ $(i = 0, \ldots, n)$, and using the vacuous extension for m^X and m^E. The resulting belief functions are combined using Dempster's rule, and the result is marginalized on S. Formally, the final bba on S is thus defined as:

$$\left(\left(\textcircled{\cap}_{i=0}^{n} m^{E\times S}[x_i]^{\Uparrow X\times E\times S}\right) \textcircled{\cap} m^{X\uparrow X\times E\times S} \textcircled{\cap} m^{E\uparrow X\times E\times S}\right)^{\downarrow S}. \tag{21}$$

Note that these operations may be performed very efficiently using local computation algorithms such as the one described in [6].

4 Comparison with the Classical Solution

In this section, we will show that our method yields the same results as the classical method where all necessary data are precisely known. When α_i is known, $m^{E\times S}[x_i]$ has a unique focal element: we have

$$m^{E\times S}[x_i](B_i) = 1, \tag{22}$$

where B_i is defined as $B_i = \bigcup_{k=0}^{\ell+1} A_{k,i} \times \{s_k\}$, with

$$A_{k,i} = \{e_j | e_j \subseteq [\theta_{k,i}, \theta_{k-1,i}]\}$$

The ballooning extension of $m^{E\times S}[x_i]$ yields:

$$m^{E\times S\Uparrow X\times E\times S}[x_i](\{x_i\} \times B_i \cup \{\overline{x_i}\} \times E \times S) = 1. \tag{23}$$

It can be shown by induction that

$$\bigcap_{i=0}^{n}(\{x_i\} \times B_i \cup \{\overline{x_i}\} \times E \times S) = \bigcup_{i=0}^{n}(\{x_i\} \times B_i). \tag{24}$$

Hence, if we note $m_1^{X\times E\times S} = \textcircled{\cap}_{i=0}^{n} m^{E\times S}[x_i]^{\Uparrow X\times E\times S}$, we obtain:

$$m_1^{X\times E\times S}\left(\bigcup_{i=0}^{n}\{x_i\} \times B_i\right) = 1. \tag{25}$$

In a second step, we have to express our beliefs concerning the raw water and plant states. In the classical case, m^X and m^S are probability functions defined by $m^X(\{x_i\}) = p_i$ $(i = 0, \ldots, n)$ and $m^S(\{e_j\}) = q_j$ $(j = 0, \ldots, m+1)$. The vacuous extension of these bba's on the joint space yields:

$$m^{X\uparrow X\times E\times S}(\{x_i\} \times E \times S) = p_i \quad i = 0, \ldots, n \tag{26}$$

$$m^{E\uparrow X\times E\times S}(X \times \{e_j\} \times S) = q_j \quad j = 0, \ldots, m+1. \tag{27}$$

Let $m_2^{X\times E\times S}$ denote the combination of $m_1^{X\times E\times S}$ with m^X. We have

$$m_2^{X\times E\times S}(\{x_i\} \times B_i) = p_i \quad i = 0, \ldots, n. \tag{28}$$

Finally, let $m_3^{X\times E\times S}$ denote the combination of $m_2^{X\times E\times S}$ with m^E. By construction, there is only one k such that $e_j \in A_{k,i}$, considering a fixed state i of the plant. Consequently, the resulting focal sets are all of the form:

$$(\{x_i\} \times \bigcup_{k=0}^{\ell+1}(A_{k,i} \times \{s_k\})) \cap (X \times \{e_j\} \times S) = \{x_i\} \times \{e_j\} \times \{s_k\} \tag{29}$$

with $e_j \in A_{k,i}$. Hence, $m_3^{X\times E\times S}$ is a probability function.

In order to conclude this demonstration and to show the equality between our approach and the classical one, we will now study the particular case developed in the classical method presentation, which consists in considering only one threshold for treated water, which must be the norm N. We then only have two states for treated water s_0 and s_1 and, consequently, two states for raw water $A_{0,i}$ and $A_{1,i}$. In this case, we have

$$m_3^{X\times E\times S}(D_{i,j}) = p_i q_j \tag{30}$$

with

$$D_{i,j} = \begin{cases} \{x_i\} \times \{e_j\} \times \{s_0\} & \text{if } j \leq i \\ \{x_i\} \times \{e_j\} \times \{s_1\} & \text{if } j > i. \end{cases} \tag{31}$$

We then finally obtain, after marginalization:

$$m^{X\times E\times S\downarrow S}(\{s_0\}) = \sum_{i=0}^{n} \sum_{j=0}^{i} p_i q_j, \tag{32}$$

which completes the proof.

5 Simulations

Figure 4 shows simulation results, for one normal mode x_0, one failure mode x_1, and seven output concentration thresholds $s_k, k = 0, \ldots, 6$. The three graphs correspond to three states of knowledge:

1. In the first case (upper left), the abatement rates, failure rate and latency time are known:
$$\alpha_0 = 0.8 \quad \alpha_1 = 0.4$$
$$\lambda_1 = 2 \times 10^{-3}\ \mathrm{h}^{-1} \quad T_1 = 4 \times 24\ \mathrm{h}.$$
2. In the second case (upper right), the abatement rates are still known, but the failure rate and latency time are only bounded by:
$$\lambda_1^- = 10^{-3}\ \mathrm{h}^{-1}, \quad \lambda_1^+ = 2 \times 10^{-3}\ \mathrm{h}^{-1}$$
$$T_1^- \in 4 \times 24\ \mathrm{h}, \quad T_1^+ = 8 \times 24\ \mathrm{h}.$$
3. In the third case (lower graph), the same knowledge of λ_1 and T_1 is assumed, and the abatement rates are constrained by fuzzy numbers:
$$\widetilde{\alpha}_0 = (0.79, 0.8, 0.81), \quad \widetilde{\alpha}_1 = (0.39, 0.4, 0.41).$$

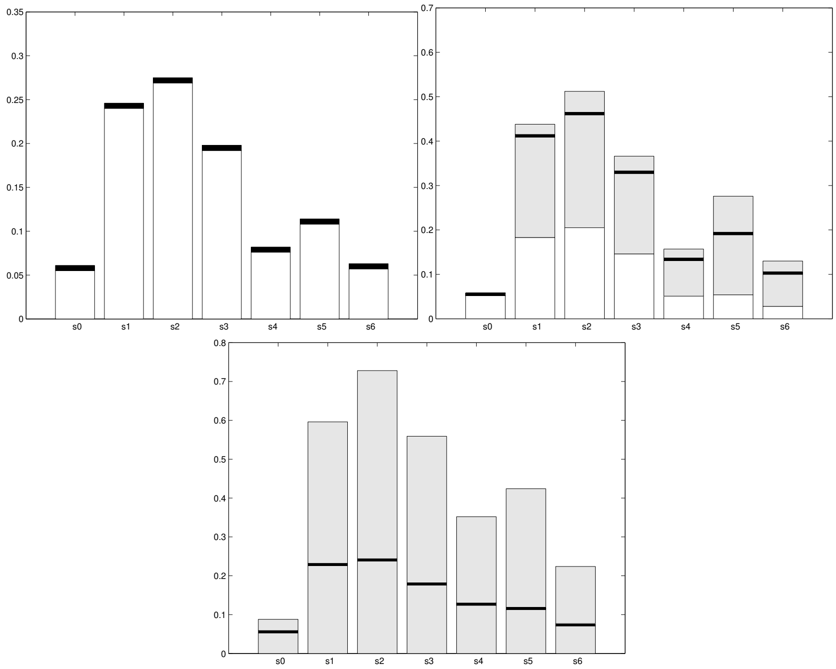

Fig. 4. Results obtained in case 1 (upper left), case 2 (upper right) and case 3 (down). The white bars correspond to the belief given to each singleton of S. The black lines indicate the pignistic probabilities, and the full bars (white and grey parts) show the plausibities.

As expected, the induced belief measure on S is a probability measure when all necessary data are available. In that case, the solution is identical to that of the classical fault tree approach. In contrast, results with the TBM approach degrade gracefully when the imprecision in input data increases, the pignistic probability getting closer to the uniform distribution.

6 Conclusion

In this paper, an original method allowing to take into account data uncertainties in risk assessment for drinking water production process has been described. In order to evaluate compliant water unavailability, this method takes into account resource quality, characteristics of treatment plant, and different operating modes of the treatment plant. Belief functions are used to describe expert knowledge of treatment steps efficiency, failure rates, times to repair and raw water quality. By combination of these belief functions, it is possible to define the belief to produce compliant water. In the case where all data are precisely known, this approach was shown to be equivalent to the classical fault tree analysis.

Preliminary validation showed that this approach fits well the experts needs, by allowing to simulate various process line scenarios to propose the best technical option, based on available partial information.

References

1. Démotier, S., Odeh, K., Schön, W., Charles, P., Footohi, F., Allioux, J-F: Risk assessment for drinking water production process. In proceedings of European Conference on System Dependability and Safety, pp: 544-550. Lyon, France, 2002.
2. Dubois, D., Prade, H.: Possibility theory, An approach to computerized processing of uncertainty. Ed. Plenum, 59-66, 1988.
3. Efron, B., Tibshirani, R.: An introduction to the bootstrap. Chapman and Hall, New York, 199).
4. Klir, G. J., Wierman, M. J.: Uncertainty-Based Information. Elements of Generalized Information Theory. Springer-Verlag, New-York, 1998.
5. Lainé, J-M., Démotier, S., Odeh, K., Schön, W., Charles, P.: Risk assessment for drinking water production. Assessing the potential risk due to the presence of cryptosporidium oocysts in water. In proceedings of IWA. Berlin, Germany, 2001.
6. Shenoy, P. P.: Binary joint trees for computing marginals in the Shenoy-Shafer architecture. International Journal of Approximate Reasoning, Vol. 17, 239-263, 1997.
7. Smets, P.: The transferable belief model for expert judgments and reliability problems. Reliability Engineering and Systems Safety, 59-66, 1992.
8. Smets, P.: Belief functions: the disjunctive rule of combination and the generalized bayesian theorem. International Journal of Approximate Reasoning, Vol. 9, 1-35, 1993.
9. Smets, P. : The transferable belief model for quantified belief representation. In Smets, P. (ed.), Handbook of defeasible reasoning and uncertainty managment systems Vol.1., pp: 267-301, Kluwer, Doordrecht, 1998.