Map matching algorithm using belief function theory

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***Abstract*—Map matching algorithms are used to integrate an initial estimated position with digital road network data for computing the vehicle position on a road map. In this paper, a map matching algorithm based on belief function theory is proposed. This method provides an accurate estimation of vehicle position relative to a digital road map using belief function theory and interval analysis. The core idea of the proposed algorithm is to handle only interval knowledge acquired from sensors and to use the multiple hypothesis technique. This technique proves to be relevant to treat junction roads situations or parallel roads. The results on simulated and real data show the usefulness of the proposed method.**

Keywords: Map matching, belief function theory, multi-sensor fusion, interval analysis, multiple hypothesis technique.

I. Introduction

Many research and industrial applications require accurate localization and/or tracking of moving vehicles. Satellite-based navigation systems like global position system (GPS) are playing an important role in vehicle localization due to the 24-hour, all weather, and free-of-charge availability. However, GPS alone is not always the ideal solution. GPS was originally designed with an inherent error of at least 10m for non-military applications. Also, GPS suffers from line of sight (LOS) issues that make it less effective in urban canyons. One possible solution for correcting GPS error is the integration of GPS data with other sensor data, e.g, dead reckoning (DR) using a data fusion algorithm. Often, classical data fusion algorithms using stochastic filters like the extended Kalman filter (EKF) are strongly affected by some types of measurement errors like bias and drift, or even by partial or total conflict between the sources of information [2] [3] [6]. Moreover, an accurate state space model and an accurate statistical model of measurement noise should be used, which is not an easy task in real applications. Sometimes, it may be more convenient to use deterministic approaches by handling only interval knowledge acquired from multiple sources. The interval framework has been shown by several authors to be a good methodology to deal with non-white and biased measurements [1] [4] [15] [16]. The importance of interval theory comes from the fact that this theory is used as a tool for so-called validated computations, i.e, computations with guaranteed accuracy taking into account all possible sources of error, from imprecise data to rounding errors during computer calculations.

Furthermore, the integrated GPS/DR system using a data fusion algorithm fails to provide the actual vehicle position on a given road segment. The availability of an accurate digital road network makes it possible to find the vehicle position in a road segment. This technique is called map matching (MM). A formal definition of MM can be found in [5] [12] [24]. A number of different algorithms have been proposed for map matching in different applications. Most of the existing map matching algorithms and their limitations are reviewed and described in [10] [24] and references therein. The multiple hypothesis technique (MHT) keeps track of several positions of the vehicle simultaneously and selects eventually which candidate is the best. In [17], an MHT for on-line map matching with embarked GPS and dead reckoning device has been proposed. In the MHT technique, probability theory is used for identifying candidate roads and for selecting the best one.

In this paper, we propose a map matching algorithm that manages multiple hypothese using belief function theory. Recently, belief theory, also known as Dempster-Shafer, or Evidence theory, has emerged as an important tool to manage and handle uncertainty and imprecision or even lack of information [25]. The use of belief theory steadily spreads out, mostly because it is used to cope with large amounts of uncertainties that are inherent of natural environments. The main idea of the proposed method is to handle only interval knowledge acquired from sensors and to select a set of candidate roads in a rectangular region. The best candidate road is eventually chosen using a decision rule of belief function theory.

This paper is organized as follows. An overview of existing map matching algorithms is given in Section II. In Sections III and IV we present the background on interval analysis and belief functions theory, respectively. A map matching method based on belief function theory, is then introduced in Section V. In Section VI, we show the results of the application of the proposed method to dynamic vehicle localization. Finally, in Section VII, we conclude and discuss the main contributions of the paper.

II. Map matching algorithms

Map matching algorithms can be classified into two categories. Algorithms in the first category consider only the geometric relationships between the estimated position of the vehicle and a digital map [22] [24]. Algorithms in the second category consider also the topology of the road network and historical data regarding the estimation position of the vehicle [12].

In the map matching algorithms of the first type, several criteria are used for choosing the best candidate road on the map, such as: distance of point to curve, distance of curve to curve and angle of curve to curve. Because these algorithms use only geometric information, they are quite unstable. Indeed, these algorithms are appropriate when one pursues simplicity rather than accuracy [18].

In the second category of algorithms, the result of MM at time step $k-1$ is used for selecting candidate roads at time step k using the topology of the road network as a constraint. Some algorithms of the second category used belief function theory for choosing the best candidate road and for computing the vehicle position [8] [9]. The main contribution of these algorithms is the use of a road selection method based on multi-criteria fusion under belief function theory. Often, proximity or angular criteria are used in order to define a belief function.

However, in the second category the determination of the vehicle position is usually not robust. Indeed, if the result of MM algorithm was wrong at time step $k-1$, then the result at time step k is likely to be wrong. Recently, some works used the multi-hypothesis technique in map matching algorithms in order to solve the above problem [17]. In this paper, we present a new map matching algorithm belonging to the second category, which handles interval knowledge about available information on sensors and uses belief function theory in order to manage some situations of map matching problems including multiple hypothesis scenarios.

III. Interval analysis

In this section we briefly introduce some notions of interval analysis. A real interval, denoted $[x]$, is defined as a closed and connected subset of $\mathbb{R}$: $[x] = [\underline{x}, \overline{x}] = \{x \in \mathbb{R} / \underline{x} \leq x \leq \overline{x}\}$, where $\underline{x}$ and $\overline{x}$ are the lower and upper bound of $[x]$. A box $[\mathbf{x}]$ of $\mathbb{R}^{n_x}$ is defined as a Cartesian product of n_x intervals: $[\mathbf{x}] = [x_1] \times [x_2] \cdots \times [x_{n_x}] = \times_{i=1}^{n_x} [x_i]$. All set-theoretic relations, e.g., $\subset, \supset, \cap, \ldots$ and the four elementary arithmetic operations $\{+, -, *, /\}$ are extended to the interval context. In general, the image of a box $[\mathbf{x}] \in \mathbb{R}^n$ by a function $\mathbf{f}$ is not a box. An inclusion function $[\mathbf{f}]$ defined as:

$$\forall [\mathbf{x}] \in \mathbb{R}^n, \mathbf{f}([\mathbf{x}]) \subset [\mathbf{f}]([\mathbf{x}]), \tag{1}$$

computes a box containing $\mathbf{f}([\mathbf{x}])$. This function should be calculated such that the box enclosing $\mathbf{f}([\mathbf{x}])$ is optimal. Different algorithms exist in order to reduce the size of boxes enclosing $\mathbf{f}([\mathbf{x}])$. For the fusion problem considered here, we have chosen to use constraint propagation techniques [13] [14], because of the great redundancy of data and equations.

A. The Constraints Satisfaction Problem (CSP)

Consider a system of m relations f_j linking variables x_i of a vector $\mathbf{x}$ of $\mathbb{R}^{n_x}$ by equations of the forms :

$$f_j(x_1, \ldots, x_{n_x}) = 0, \quad j = 1 \ldots m. \tag{2}$$

This equation can be written as a constraint satisfaction problem CSP $\mathcal{H}$:

$$\mathcal{H} : (\mathbf{f}(\mathbf{x}) = \mathbf{0}, \mathbf{x} \in [\mathbf{x}]_0) \tag{3}$$

where $\mathbf{x} = (x_1, x_2, \cdots, x_{n_x})^T$, $\mathbf{f} = (f_1, f_2, \cdots, f_m)^T$ and $[\mathbf{x}]_0$ is the prior domain of $\mathbf{x}$. A Constraint Satisfaction Problem (CSP) $\mathcal{H}$ is a problem that gathers a vector of variables $\mathbf{x}$ from an initial domain $[\mathbf{x}]_0$ and a set of constraints $\mathbf{f}$ linking the variables x_i of $\mathbf{x}$. The CSP consists in finding the values of $\mathbf{x}$ which satisfy the equality constraints (3). The solution set of the CSP will be defined as:

$$\mathbf{S} = \{\mathbf{x} \in [\mathbf{x}]_0 \mid \mathbf{f}\{\mathbf{x}\} = \mathbf{0}\}. \tag{4}$$

Note that $\mathbf{S}$ is not necessary a box. Under the interval framework, solving the CSP will be translated on *finding the minimal box* $[\mathbf{x}'] \subset [\mathbf{x}]_0$ such that $\mathbf{S} \subset [\mathbf{x}']$.

1) Waltz Contractors: A contractor is an operator applied to the domain of the CSP $\mathcal{H}$. It is used to solve the CSP, and thus to eventually provide *a minimal box* $[\mathbf{x}'] \subset [\mathbf{x}]_0$ such that $\mathbf{S} \subset [\mathbf{x}']$. The Waltz contractor is based on the propagation of primitive constraints for real variables. A primitive constraint is a constraint involving a single operator (such as $+, -, *$ or $\backslash$) or a single function (such as $\cos, \sin$ or $\sinh$). This contractor makes it possible to contract the domains of the CSP $\mathcal{H}$: $(\mathbf{f}(\mathbf{x}) = \mathbf{0}, \mathbf{x} \in [\mathbf{x}]_0)$ by taking into account any one of the m relations (constraints) in isolation, without any a priori order. The use of this method appears to be specially efficient when one has a redundancy of data and equations. Note that this method is independent of the non-linearities and provides a locally consistent contractor [13].

The principle of The Waltz contractor can be illustrated via the following example. Let us consider the constraint $z = x.\exp(y)$. At first, this constraint is decomposed into primitive constraints:

$$\begin{cases} a = & \exp(y) \\ z = & x.a \end{cases} \tag{5}$$

where a is an auxiliary variable initialized by $[a] = [0, +\infty[$. Each equation of (5) has been obtained by isolating one of the variables in the initial constraint. By using the inclusion functions $[\exp]$ and $[(\exp)^{-1}] = [\ln]$, and the initial domain of variables: $[z] = [0, 3]$, $[x] = [1, 7]$ and $[y] = [0, 1]$, the forward backward propagation algorithm (FBP) works as follows:

Set $[z] = [0, 3]$, $[x] = [1, 7]$ and $[y] = [0, 1]$,
repeat
Forward step:
F_1: $[a] = [a] \cap [\exp]([y]) = [1, e]$
F_2: $[z] = [z] \cap [x].[a] = [1, 3]$.
Backward step:
B_3: $[x] = [x] \cap ([z]/[a]) = [1, 3]$

B_4: $[a] = [a] \cap ([z]/[x]) = [1, e]$
B_5: $[y] = [y] \cap [\ln]([a]) = [0, 1]$
until The contractor becomes inefficient, e.g, there is no contractions.

IV. BELIEF FUNCTION THEORY

In this section, we introduce the main concepts of Belief function theory. Let Ω denote a finite set of mutually exclusive and exhaustive hypotheses, called the frame of discernment. A *belief structure* (BS) is a mass function m from 2^Ω to $[0, 1]$, verifying: $\sum_{A \subseteq \Omega} m(A) = 1$. Every subset A of Ω such that $m(A) > 0$ is called a *focal element* of m. A BS m such that $m(\emptyset) = 0$ is said to be normal. The *belief function* induced by m is the function *bel*: $2^\Omega \longmapsto [0, 1]$ verifying:

$$bel(A) = \sum_{\emptyset \neq B \subseteq A} m(B) \text{ for all } A \subseteq \Omega. \tag{6}$$

A categorical belief function is a belief function that satisfies: $m(A) = 1$ for some $A \subset \Omega$, $A \neq \Omega$ and $m(B) = 0$, $\forall B \subseteq \Omega$, and $B \neq A$. The belief function on Ω which haw $m(\Omega) = 1$ is the vacuous belief function.

The *plausibility function*, denoted pl, quantifies the maximum amount of potential specific support that could be given to $A \subseteq \Omega$. It is obtained by adding all the masses given to focal elements B that verify $B \cap A \neq \emptyset$:

$$pl(A) = \sum_{B \cap A \neq \emptyset} m(B) = bel(\Omega) - bel(\bar{A}). \tag{7}$$

Two different, and independent BSs m_1 and m_2 defined on the same frame of discernment Ω can be combined by the *conjunctive rule* [25] defined as:

$$\forall A \subseteq \Omega, m_{12}(A) = m_1 \cap m_2(A) = \sum_{B \cap C = A} m_1(B) m_2(C). \tag{8}$$

The conjunctive combination followed by a normalization step is known as *Dempster's rule* of combination [25]. It is denoted by $\oplus$.

Let Ω and Θ be two frames of discernment. Let $m^{\Omega \times \Theta}$ be a BS defined on the cartesian product $\Omega \times \Theta$. *The marginal* BS, denoted $m^{\Omega \times \Theta \downarrow \Omega}$, is defined , for all $A \subseteq \Omega$, as:

$$m^{\Omega \times \Theta \downarrow \Omega}(A) = \sum_{\{B \subseteq \Omega \times \Theta / proj(B \downarrow \Omega) = A\}} m^{\Omega \times \Theta}(B), \tag{9}$$

where $proj(B \downarrow \Omega)$ is the projection of B on Ω, defined as: $proj(B \downarrow \Omega) = \{w_1 \in \Omega / \exists w_2 \in \Theta; (w_1, w_2) \in B\}$. Conversely, let m^Ω be a BS defined on Ω. Its *Vacuous extension* on $\Omega \times \Theta$ is defined by:

$$m^{\Omega \uparrow \Omega \times \Theta}(B) = \begin{cases} m^\Omega(A) & if\ B = A \times \Theta \\ 0 & \text{Otherwise} \end{cases} \tag{10}$$

When a given hypothesis $h \subseteq \Omega$ is ascertained, The BS on Θ can be changed to reflect the new state of the knowledge. The conditioning operation consist on combining conjunctively the prior BS with a categorical BS supporting the hypothesis h, $m_h^\Omega(h) = 1$. Let $m^\Theta[h]$ be the BS on Θ conditioning to h, it is given by:

$$m^\Theta[h] = (m^{\Theta \times \Omega} \otimes m_h^{\Omega \uparrow \Theta \times \Omega}) \downarrow \Theta \tag{11}$$

The pignistic transformation uses the mass function to assign a pignistic probability to the subsets. Smets pignistic transformation distributes the basic belief assignments $m(A_J)$ equally among on each singleton element of $A_i \subseteq A_J$, with $A_i \in \Omega$ and $A_J \subseteq 2^\Omega$ [20]. Then, the pignistic probability is defined as:

$$Betp(A_i) = \sum_{A_i \in A_M} (\frac{1}{|A_M|})(\frac{m(A_M)}{1 - m(\emptyset)}), \tag{12}$$

where A_M is a focal element of Ω and $|A_M|$ is the width of the interval based focal element A_M.

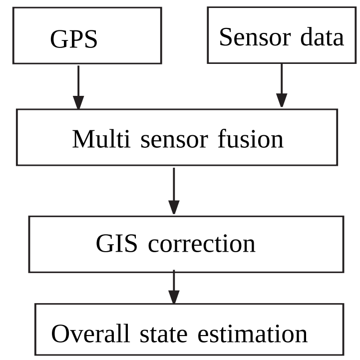

Figure 1. The basic steps of the BMM algorithm.

V. BELIEF MAP MATCHING METHOD

A. Introduction

In many application areas it is necessary to estimate the state of a dynamic system, e.g, vehicle position, using a sequence of noisy sensor measurements. Recent works use interval framework in order to deal with measurements affected by non-white noise or by significant bias, see for example [1] and references therein. This gives raise to the so-called *bounded errors methods* for state estimation, where the resulting state can be represented by a box which is equivalent to a Cartesian product of intervals.

In this section we present a belief-function based Map Matching method (BMM) using simultaneously, beside *bounded errors* estimation methods, geometrical and topological frameworks in order to attribute a basic belief assignment to a set of candidate roads (CR) extracted from an existing local map, thereby providing a more accurate estimation of the position.

Under the interval framework, and using a state space model, sensor data will be integrated with GPS measurements via a multi-sensor fusion algorithm in order to compute a state estimation position. From the resulting position, a set of CR will be computed by using a two dimensional geographical information system (GIS-2D). By handling interval knowledge

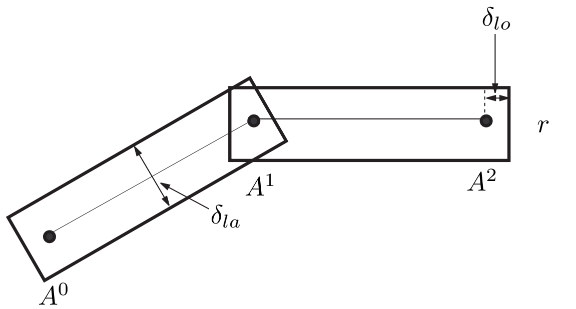

Figure 2. Rectangular roads constructed on $r = (A^0, A^1, A^2)$.

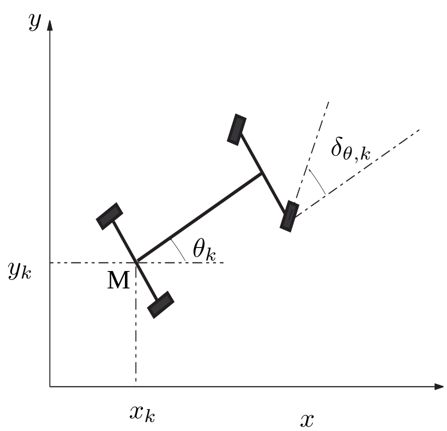

Figure 3. Definition of the frames.

of the state and rectangular representations of the roads, the BMM method will be straightforwardly formulated. Figure 1 shows the basic steps of the BMM method. We will present in section V-B the geometry of the available rectangular road map. In section V-C we will introduce the state space model to be used. The scenarios of the BMM method will be shown in section V-D.

B. Road map representation

There are several ways to represent digital spatial road network data. The planar model used in this paper is currently one of the most accepted models because of its high efficiency and low complexity [19]. As suggested by the planar model, a road r is represented by a finite sequence of points $(A^0, A^1, \cdots, A^{nA})$, where $\{A^i\}_{i=1}^{nA} \in \mathbb{R}^2$. The points A^0 and A^{nA} are the end points of road r. These points are referred to nodes while $(A^1, A^2, \cdots, A^{nA-1})$ are referred to vertices or shape points. In the BMM method, rectangular roads will be constructed from GIS data as shown in Figure 2. As it can be seen, a road r is characterized by two nodes (A^0, A^2) and a shape point A^1. The associated rectangular roads can be constructed using lateral and longitudinal imprecisions δ_{la} and δ_{lo} respectively, which are an a priori information depending on the confidence of the available map.

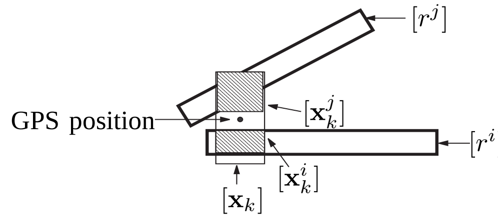

Figure 4. State update using $[\mathbf{x}_k]$ and two rectangular roads $[r^i]$ and $[r^j]$.

C. Dynamic state space model

Consider a car-like vehicle with front-wheel drive. The vehicle position is represented by the Cartesian coordinates (x_k, y_k) of the point M attached to the center of the rear axle as shown in Figure 3. The heading angle is denoted θ_k. The state $\mathbf{x}_k = (x_k, y_k, \theta_k)^T$ is calculated at each time step k thanks to the following discrete representation:

$$\begin{cases} x_{k+1} = & x_k + \delta_{S,k} \cos(\theta_k + \frac{\delta_{\theta,k}}{2}) \\ y_{k+1} = & y_k + \delta_{S,k} \sin(\theta_k + \frac{\delta_{\theta,k}}{2}) \\ \theta_{k+1} = & \theta_k + \delta_{\theta,k} \end{cases} \quad (13)$$

where $\delta_{S,k}$ is the elementary linear displacement and $\delta_{\theta,k}$ is the measure of the elementary rotation given by an ABS sensor and a gyrometer, respectively. The observation of the position at time step k, which is $\mathbf{z}_k = (x_{GPS}, y_{GPS})$, is given by a Global Position System (GPS). The *longitude, latitude* estimated point of the GPS is converted to a Cartesian local frame and the error boundary of the position is obtained thanks to the weaves *GST NMEA* [11]. Using interval framework, one will be able to handle interval quantities and to give a box representation of the state position as done in [11]. This will be a part of the BMM method. Note that, one can build a box around $\delta_{S,k}$ and $\delta_{\theta,k}$ using σ_s and σ_θ which are estimated thanks to specific static tests: $[\delta_{S,k}] = [\delta_{S,k} - 3 \cdot \sigma_s, \delta_{S,k} + 3 \cdot \sigma_s]$ and $[\delta_{\theta,k}] = [\delta_{\theta,k} - 3 \cdot \sigma_\theta, \delta_{\theta,k} + 3 \cdot \sigma_\theta]$. In the same manner, one can build a box around GPS measurement $\mathbf{z}_k$: $[\mathbf{z}_k] = ([x_{k,GPS}], [y_{k,GPS}])^T$ where $[x_{k,GPS}] = [x_{k,GPS} - 3 \cdot \sigma_x, x_{k,GPS} + 3 \cdot \sigma_x]$ and $[y_{k+1,GPS}] = [y_{k,GPS} - 3 \cdot \sigma_y, y_{k+1,GPS} + 3 \cdot \sigma_y]$.

D. Sketch of the BMM method

1) Road map mass functions: Given a state box $[\mathbf{x}_k]$ and a rectangular road map, a set R_k of CR can be selected where any rectangular road [r] in R_k should verify: $[\mathbf{x}_k] \cap [r] \neq \phi$. The associated mass function at time step k, m^{R_k}, can be computed using topology and similarity criteria.

From the map topology, a mass function $m_1^{R_k}$ which represents our belief given to the hypothesis that the vehicle is on each road included in R_k can be computed.

From the similarity between the rectangular roads in R_k and the state box $[\mathbf{x}_k]$, a mass function $m_2^{R_k}$ on R_k can be calculated. This similarity is characterized by the width of the intersection between $[\mathbf{x}_k]$ and the rectangular roads, and for geometrical convenience is calculated as the width of the minimal box englobing this intersection. Let $L^i = \frac{|[\mathbf{x}_k^i]|}{|[\mathbf{x}_k]|}$, where

1) Initialization

   a) Set $k = 0$ and from the GPS measurement create a state box $[\mathbf{x}_k]$.
   b) From each road r^i including in the local map, construct the associated rectangular road $[r^i]$.
   c) From all rectangular roads $[r^i]$ and $[\mathbf{x}_k]$, construct a set R_k of CR by using similarity criterion. Let n_{R_k} be the number of roads including in R_k.
   d) State update: By using $[\mathbf{x}_k^i] = [\mathbf{x}_k] \cap [r^i]$, $i = 1, \cdots n_{R_k}$.
   e) Constructed a categorical mass function $m_1^{R_k}$ on R_k.
   f) Construct a mass function $m_2^{R_k}$ using (14) and (15) on R_k.
   g) Compute m^{R_k} using (16)

2) Set $R_{k+1} = R_k$ and $m_1^{R_{k+1}} = m^{R_k}$.
3) For $i = 1 : n_{R_k}$
4) Prediction:
   - Input boxes: $[\delta_{S,k}] = [\delta_{S,k} - 3 \cdot \sigma_s, \delta_{S,k} - 3 \cdot \sigma_s]$ and $[\delta_{\theta,k}] = [\delta_{\theta,k} - 3 * \sigma_\theta, \delta_{\theta,k} - 3 * \sigma_\theta]$.
   - Calculate $[\mathbf{x}_{k+1}^i]$ using $[\delta_{S,k}]$, $[\delta_{\theta,k}]$ and (13).
5) GPS correction:
   a) From GPS measurement build a measurement box: $[\mathbf{z}_{k+1}] = ([x_{k+1,GPS}], [y_{k+1,GPS}])'$, $[x_{k+1,GPS}] = [x_{k+1,GPS} - 3 \cdot \sigma_x, x_{k+1,GPS} + 3 \cdot \sigma_x]$ and $[y_{k+1,GPS}] = [y_{k+1,GPS} - 3 \cdot \sigma_y, y_{k+1,GPS} + 3 \cdot \sigma_y]$.
   b) The innovation is given by: $[I^i] = [\mathbf{x}_{k+1}^i] \cap [\mathbf{z}_{k+1}]$.
   c) Contract $[\mathbf{x}_{k+1}^i]$ using $[I^i]$ and by applying Waltz algorithm according to system (13).
6) GIS correction:
   a) Update R_{k+1} and $m_1^{R_{k+1}}$:
      - **<u>IF</u>** the distance between the center of $[\mathbf{x}_k^i]$ and a node or a shape point of the road r^i is less then $\delta_{S,k}$, then:
        - Let $S(r^i)$ be the set of all roads directly linked to r^i including r^i. Let n_s be the number of roads including in $S(r^i)$.
        - Initialize $S^i = \phi$
        - For $j = 1 : n_s$
        - if $[r^j] \cap [\mathbf{x}_k^i]$ is not empty then $S^i = S^i \cup \{r^j\}$.
        - $[\mathbf{x}_{k+1}^j] = [\mathbf{x}_{k+1}^i] \cap [r^j]$.
        - ENDFOR_j
        - $R_{k+1} = R_{k+1} \cup S^i$.
        - $m_1^{R_{k+1}} = M.m_1^{R_{k+1}}$, where M is the transition matrix computed in such way that $m_1^{R_{k+1}}(\{r^i\})$ is transferred to $m_1^{R_{k+1}}(S^i)$.
      - **<u>ELSE</u>** R_{k+1} and $m_1^{R_{k+1}}$ remain unchanged and:
        - $[\mathbf{x}_{k+1}^i] = [\mathbf{x}_{k+1}^i] \cap [r^i]$.
      - **<u>ENDIF</u>**
   b) Construct a mass function $m_2^{R_{k+1}}$ using (14) and (15) on R_{k+1}.
7) ENDFOR_i
8) Compute $m^{R_{k+1}} = m_1^{R_{k+1}} \oplus m_2^{R_{k+1}}$
9) Overall estimation:
   a) The best road is that corresponding to the highest plausibility function computed from $m^{R_{k+1}}$.
   b) The state estimate is the center of $[\mathbf{x}_{k+1}^i]$ on the best road.
10) $k = k + 1$ go to 2 until $k = k_{end}$.

Figure 5. Belief map matching algorithm.

$[\mathbf{x}_k^i]$ is the minimal box englobing the intersection between the rectangular road $[r^i]$ and $[\mathbf{x}_k]$ as shown in Figure 4. L^i can be seen as a geometrical likelihood of the road given a state box $[\mathbf{x}_k]$. Using L^i, a mass function m_i can be computed using [23]:

$$\begin{cases} m_i(\{r^i\}) = & 0 \\ m_i(\overline{\{r^i\}}) = & \alpha_i(1 - a \cdot L^i) \\ m_i(\mathbf{R}_k) = & 1 - \alpha_i(1 - a \cdot L^i) \end{cases} \tag{14}$$

where $\overline{\{r^i\}}$ is the complement of $\{r^i\}$, α_i is a weakening coefficient associated with the road r^i and a is a normalization coefficient. The mass function $m_2^{R_k}$ is the combination of all m_i using Dempster rule of combination [25]:

$$m_2^{R_k} = \oplus_i m_i. \tag{15}$$

The mass functions $m_1^{R_k}$ and $m_2^{R_k}$ can be combined in order to compute m^{R_k} as:

$$m^{R_k} = m_1^{R_k} \cap m_2^{R_k}, \tag{16}$$

where $m^{R_k}(\{r^i\})$ represents the final part of belief given to the hypothesis that the vehicle is on road r^i and $m^{R_k}(\phi)$ represents the part of belief given to the hypothesis that the vehicle is moving on a road not included in the database of the map.

2) Initialization: At time step $k = 0$, a state box can be constructed using the GPS measurement and the standard deviations σ_x and σ_y estimated in real time by the GPS receiver: $[x_k] = [x_{k,GPS} - 3 \cdot \sigma_x, x_{k,GPS} + 3 \cdot \sigma_x]$ and $[y_k] = [y_{k,GPS} - 3 \cdot \sigma_y, y_{k,GPS} + 3 \cdot \sigma_y]$. Note that the heading angle θ is not directly observed, and is initialized as $[\theta_0] = [0, +2\pi]$. From $[\mathbf{x}_k]$ and the rectangular road map, a set R_k of CR is selected as explained in section V-D1. At time step $k = 0$, there is no prior information on the vehicle position and thus $m_1^{R_k}$ should be initialized as a vacuous mass function on R_k. The mass function $m_2^{R_k}$ can be calculated using (14) and (15). The final mass function m^{R_k} is thus the result of the combination of $m_1^{R_k}$ and $m_2^{R_k}$ according to (16). Note that from the fact that the vehicle should be on a road, $[\mathbf{x}_k]$ is substituted by $\{[\mathbf{x}_k^i]\}_{i=1}^{n_{R_k}}$.

3) Prediction: The state boxes $\{[\mathbf{x}_k^i]\}_{i=1}^{n_{R_k}}$ can be updated using $[\delta_{S,k}]$, $[\delta_{\theta,k}]$ and the evolution equation (13) thanks to the intervals tools [13]. Note here that, as inclusion functions are used, one may obtain *non-optimized* predicted state boxes, $\{[\mathbf{x}_{k+1}^i]\}_{i=1}^{n_{R_k}}$.

4) GPS correction: The GPS measurement box $[\mathbf{z}_{k+1}]$ can be used in order to adjust state boxes. The intersection between a state position box $([x_{k+1}^i], [y_{k+1}^i])^T$ and the GPS box $[\mathbf{z}_{k+1}]$ characterize the proximity between the prediction and the measurement. This intersection is used to contract $[\mathbf{x}_{k+1}^i]$ using the *Waltz algorithm* according to the constraints of system (13) [1] [13].

5) GIS correction: Regarding junctions situations, two cases should be considered as shown in Figure 6:

- If the distance between the center of $[\mathbf{x}_k^i]$ and a node or a shape point of the road r^i is less then the elementary movement $\delta_{S,k}$ given by the rear wheels ABS sensors, then it is possible that the vehicle leaves road r^i. For this reason, the set R_k of CR must be changed to a set R_{k+1}. This is done as follows. Let S^i be the set of all

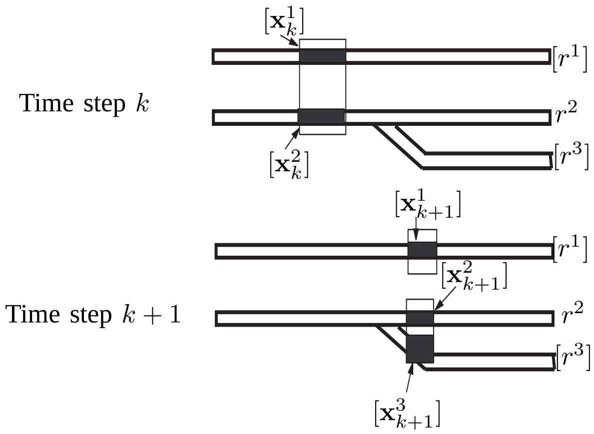

Figure 6. Possible scenarios when managing the multiple hypotheses case due to junctions. The black boxes represent the state after GIS correction.

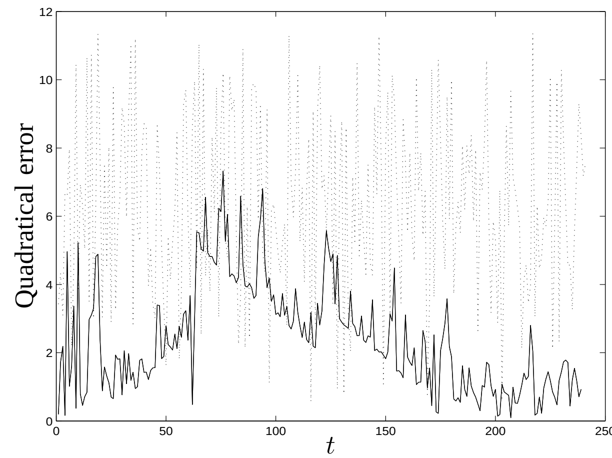

Figure 7. Quadratical error of the vehicle position; GPS error on dashed line and BMM error on black solid line.

roads directly linked to r^i ($r^i \in S^i$) and which has an intersection with $[\mathbf{x}_k^i]$. The set R_{k+1} is then updated by: R_{k+1}=$R_k \cup S^i$.

- If the distance between the center of the state box $[\mathbf{x}_k^i]$ and all nodes and shape points of the road r^i is higher than $\delta_{S,k}$, then $R_{k+1} = R_k$.

The mass function $m_1^{R_{k+1}}$ should be calculated using m^{R_k} and the relation between R_k and R_{k+1}. This is accomplished using conditional belief functions as introducing in section IV. This is explained by the follow example.

Consider the case of Figure 6, where $R_k = \{r^1, r^2\}$. The state boxes representing the vehicle position at time step k are $[\mathbf{x}_k^1]$ and $[\mathbf{x}_k^2]$. At time step $k+1$, three possible positions of the vehicle may occur, namely $\mathbf{x}_{k+1}^1, \mathbf{x}_{k+1}^2$ and $\mathbf{x}_{k+1}^3$ and thus $R_{k+1} = \{r^1, r^2, r^3\}$. From the fact that r^2 is linked to r^3 and r^1 is not linked to r^2 and r^3, the relation between m^{R_k} and $m_1^{R_{k+1}}$ is given by:

$$m_1^{R_{k+1}} = M \cdot m^{R_k}, \qquad (17)$$

Where M is a transition matrix defined as:

$$M = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

and m^{R_k} is the known mass function with focal elements $\{\phi, \{r^1\}, \{r^2\}, \{r^1, r^2\}\}$ and $m_1^{R_{k+1}}$ is the mass function to be calculated with focal elements $\{\phi, \{r^1\}, \{r^2\}, \{r^1, r^2\}, \{r^3\}, \{r^1, r^3\}, \{r^2, r^3\}, \{r^1, r^2, r^3\}\}$. This matrix is computed using the following relations under a conditional belief functions interpretation:

$$m^{R_{k+1}}[\{r^2\}](\{r^2, r^3\}) = 1,$$
$$m^{R_{k+1}}[\{r^1\}](\{r^1\}) = 1.$$

This transformation means that

- $m^{R_k}(\phi)$ is transferred to $m_1^{R_{k+1}}(\phi)$;
- $m^{R_k}(\{r^1\})$ is transferred to $m_1^{R_{k+1}}(\{r^1\})$;
- $m^{R_k}(\{r^2\})$ is transferred to $m_1^{R_{k+1}}(\{r^2, r^3\})$ as r^2 is linked to r^3: r^2 implies $\{r^2, r^3\}$;
- $m^{R_k}(\{r^1, r^2\})$ is transferred to $m_1^{R_{k+1}}(\{r^1, r^2, r^3\})$ as r^1 or r^2 implies $\{r^1, r^2, r^3\}$.

The mass function $m_2^{R_{k+1}}$ can be calculated using (14) and (15).

By combining $m_1^{R_{k+1}}$ and $m_2^{R_{k+1}}$ using (16), the final mass function $m^{R_{k+1}}$ can be computed.

Note that if there is no intersection between state boxes and all roads including in R_{k+1}, then the resulting state boxes $\{[\mathbf{x}_{k+1}^i]\}_{i=1}^{n_{R_k}}$ should be kept, e.g, the vehicle is moving on a road not included in map data. On the other hand, each available box $[\mathbf{x}_{k+1}^i]$ should be substituted by $[\mathbf{x}_{k+1}^j]$, which is the minimal box englobing the intersection between $[\mathbf{x}_{k+1}^i]$ and the rectangular road $[r^j]$.

6) Overall estimation: The overall estimate of the vehicle position should be computed using all state boxes and $m^{R_{k+1}}$. Thus, the best candidate road should be selected. Under the belief functions theory, several decision criteria such as the maximum of belief, the maximum of plausibility or the pignistic probability, can be chosen [7]. The estimate of the vehicle position can be chosen as the center of the box corresponding to the best candidate road. The confidence in this estimation is the width of the box.

The BMM algorithm is presented in Figure 5.

VI. Application

A. Simulated Results

In this Section, we present the results of applying the BMM algorithm on simulated Data. The vehicle position, the heading, the elementary movement and the elementary rotation were generated using the Matlab simulink toolbox. The GPS measurement noise was supposed to be white with $\sigma_x = 7$ m and $\sigma_y = 9$ m. The noise in the input data (elementary

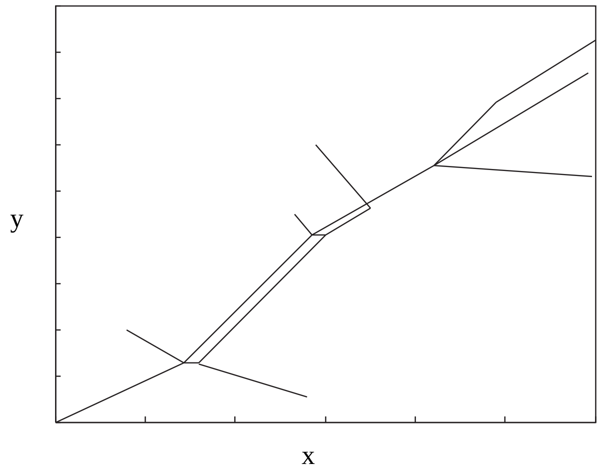

Figure 8. Simulated road map.

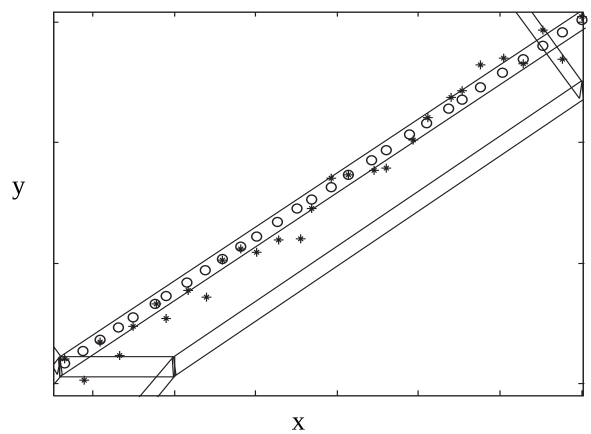

Figure 9. BMM results. GPS positions are represented by $(*)$, estimated positions are represented by (o).

movement and elementary rotation) was supposed to be white with $\sigma_s = 1/4$ m and $\sigma_\theta = 0.002$ degrees. In this application, we assumed that $\delta_{lo} = 1$ m and $\delta_{la} = l + 1$, where $l = 6$ m is the width of the road. Figure 7 displays the quadratic error of the BMM method compared to that of GPS. Figure 8 shows the simulated trajectory. Figure 9 shows the GPS in $(*)$ points and the estimated positions in (o) points. Although the BMM method can provide an estimate of the position on each CR, only the overall estimation positions are plotted using the maximum of plausibility criterion.

Figure 10 shows how several hypotheses are managed with the BMM method in order to handle junction situations. The estimated positions corresponding to all CR are represented by $(\triangle)$ points. The GPS positions are represented by $(*)$ points and the overall estimate positions are represented by (o) points. At time step $k-1$, there is only one possible position. At time step k, even if there is three possible positions caused by junction situation, the BMM method is able to choose

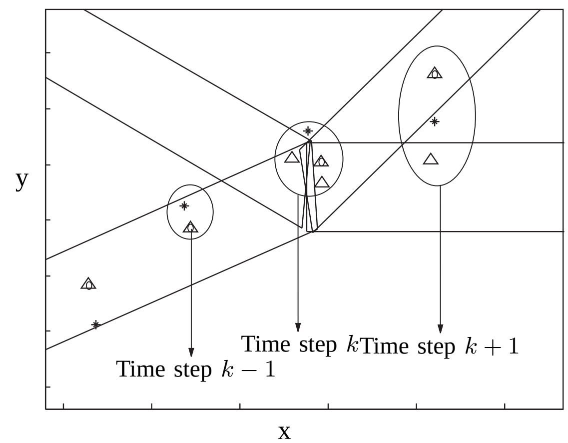

Figure 10. Multi-hypothesis managed with the BMM method in the case of junction situation. GPS positions are represented by $(*)$, overall estimate positions are represented by (o) and BMM possible positions are represented by $(\triangle)$.

a position and to save all the other positions for eventual correction. As can be seen, only two positions are kept at time step $k+1$.

B. *Results on real data*

In this section, the BMM method was applied to real data. The test trajectory was carried out in Compiègne, France with the experimental vehicle of the laboratory. The measurement of the position (x_{GPS}, y_{GPS}) was given by a GPS. The elementary rotation and displacement between two samples were obtained with *good precision* using a fiber optic gyrometer and two rears wheels ABS sensors. In this application, we assumed that $\delta_{lo} = 1m$ and $\delta_{la} = l + 1$, where $l = 6$ m is the width of the roads. Figure 11 shows the test trajectory. The zoomed part of the trajectory shows the GPS positions in $(*)$ points and estimated positions in (o) points. Note here that the BMM method may provide an estimate of the positions on each CR; however, only the overall estimation positions are plotted using the maximum of plausibility criterion. It is obvious that the resulting estimate is more adequate than the GPS position which is not on the road map.

VII. Conclusion

In this paper, a new method for map matching and state estimate has been presented. This method manages the output of existing bounded error estimation methods under the belief function framework by combining interval data with a rectangular road map. This method seems to be adequate to deal with some crucial situations of the map matching problem like multi hypothesis scenarios on junctions. Also, the implementation of this method is quite simple using geometrical properties of boxes and rectangular roads map. Results on simulated and real data have demonstrated the effectiveness of this method.

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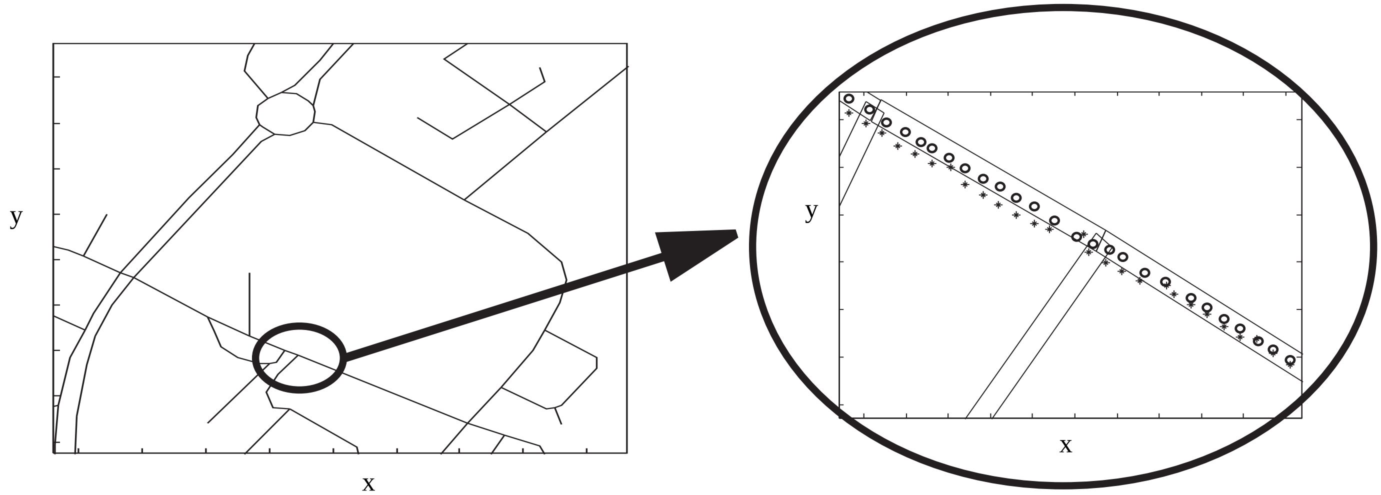

Figure 11. Experimental results. The estimation positions are represented by (o) points and GPS positions are represented by $(*)$ points.

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