A state estimation method for multiple model systems using belief function theory

Ghalia Nassreddine
HEUDIASYC, UMR CNRS 6599
Université de Technologie de Compiègne
Compiègne - France
gnassred@hds.utc.fr

Fahed Abdallah
HEUDIASYC, UMR CNRS 6599
Université de Technologie de Compiègne
Compiègne - France
fabdalla@hds.utc.fr

Thierry Denœux
HEUDIASYC, UMR CNRS 6599
Université de Technologie de Compiègne
Compiègne - France
tdenoeux@hds.utc.fr

Abstract − *Multiple model methods have been generally considered as the mainstream approach for estimating the state of dynamic systems under motion model uncertainty. In this paper, a multiple model method based on belief function theory is proposed. This method handles the case of systems with an unknown and variant motion model. First, a set of candidate models is selected and an associated Dempster-Shafer mass function is computed based on the measurement likelihood of possible motion models. The estimated state of the system is then derived by computing the expectation with respect to the pignistic probability. In order to validate our work, we applied the proposed method to a vehicle localization problem. The comparison with other methods demonstrates the effectiveness of the proposed method.*

Keywords: State estimation, multiple model approaches, multi-sensor fusion, belief function theory, mobile localization, Dempster-Shafer theory, evidence theory.

1 Introduction

Dynamic system state estimation is an important task in many applications, e.g, vehicle localization, navigation and target tracking. In addition to the measurement uncertainty, the state estimation problem faces two interrelated main challenges: non linearity and motion model uncertainty. Non linearity is best handled by non linear filtering techniques. The most common technique for non linear filtering is the Extended Kalman Filter (EKF). The basic idea of EKF is to linearize the state and measurement equations and then apply the Kalman filter (the optimal linear state estimator under Gaussian noises) in order to obtain the state estimates, assuming the process and observation noises to be normal. The state posterior probability distribution is approximated by a Gaussian distribution that is propagated analytically through the linearized system equations [2]. However, in the presence of motion uncertainty, the state estimation problem is better handled by a more complex approach, referred to as the Multiple model approach (MM). This approach gets around the difficulty due to the motion model uncertainty by using more then one motion model [3] [4] [8] [7]. Generally, MM methods are developed using probability theory. Thus, motion model uncertainty and state estimation are considered under the same framework.

Recently, the theory of belief functions, also known as Dempster-Shafer or Evidence theory, has emerged as an important tool to manage and handle uncertainty and imprecision or even lack of information [13]. The use of belief theory steadily spreads out, mostly because it is flexible enough to model the wide range of uncertainties that are inherent to natural environments [13].

In this paper, we propose a state estimation method that manages motion model uncertainty using the formalism of belief function theory. This method handles the case of systems with unknown and variant motion model and uses both belief function theory and existing filtering techniques in order to manage uncertainty and to compute an accurate state estimate. This is done by selecting a set of Candidate Models (CMs) based on prior knowledge of the system and by using after a bank of basic filters, where each filter is associated to a CM. As the true motion model of the system can change during time, the CMs have to work together in order to compute an accurate state of system. Thus, a cooperative step involving the different CMs is performed using belief function theory. Finally, an overall estimation of the state is computed using the outputs from the basic filters and the belief function on the set of models.

This paper is organized as follows. First, an overview of existing multiple model methods is presented in Section 2. In Section 3, the background on belief functions theory is recalled. The MM method based on belief function theory is then introduced in Section 4. In Section 5, we show the results of applying the proposed method to a dynamic vehicle localization problem. Finally, in Section 6 we conclude and discuss the main contributions of this paper.

2 Multiple model methods

Multiple model estimation techniques have recently received a great deal of attention due to their power and great success in handling problems with both measurement and motion uncertainties. They are used as a way to decompose a complex problem into simpler subproblems [8]. MM approaches select a set of candidate models and run a bank of filters, each filter being associated to a CM. By using the measurement likelihood according to CMs, an overall state estimate can be computed [3][4].

In [8], three generations of MM formulations have been identified, with different structures, limitations and capabilities. The first generation of methods, referred to as autonomous MM (AMM), considers the case of dynamic systems with unknown and invariant motion model. In this formulation, the filters associated to CMs operate individually and independently. Consequently, this simple approach cannot resolve the estimation problem for dynamic systems with time-varying motion model. The second formulation, known by cooperative MMs (CMMs), assumes a system with motion model varying among the CMs. Therefore, the true motion model of the system can switch between possible CMs. For this reason, filters associated to CMs work together as a team via a supplementary cooperative step. The interacting multiple model (IMM), developed by Blom [5] is one of the most popular of these methods. The IMM filter has been used in many real-life applications that involve dynamic systems with motion uncertainty, such as traffic control systems and target tracking. The performance of this method has been well demonstrated by Bar-Shalom and others [3][4]. Both AMM and CMM have a fixed structure as they use an invariant set of CMs. However, a system with many different motion models may not be represented accurately by a small set of CMs. To get more accurate results a large bank of filters should be used, thereby increasing the complexity of the method. As a consequence, a third formulation seems to be useful. The variable structure MMs (VSMMs) use a variant set of CMs in order to resolve the estimation problem [8]. The VSMMs generation introduces new motion models if the existing CMs are not good enough and is able to eliminate the worst CMs at each time step.

In this paper, a new state estimation method for dynamic system with Markov switching, called *Belief interacting multiple model* (BIMM), will be presented. This method can be used as an alternative to the IMM method. The BIMM method uses belief function theory in order to represent the motion model uncertainty and the switching between different candidate models. Belief function theory is considered as a formal tool suitable for representing the inaccuracy, uncertainty and unavailability of knowledge. It is an alternative to Bayesian theory, which does not rely on probabilistic quantification, but on a more general formalism based on belief functions. This formalism is summarized in the next section.

3 Belief function theory

In this section, we briefly review the main concepts of Belief function theory that will be used in the rest of this work.

3.1 Basic definitions

Let Ω be a finite set of mutually exclusive and exhaustive hypotheses, called the frame of discernment. The set of all subsets of Ω is denoted by 2^Ω. The impact of a piece of evidence on the different subsets of the frame of discernment Ω is represented by a basic belief assignment (bba). The bba is a function, noted m, from 2^Ω to $[0,1]$, verifying: $\sum_{A\subseteq\Omega} m(A) = 1$. The value $m(A)$, called a basic belief mass, represents the portion of belief committed exactly to the subset A. Every subset A of Ω such that $m(A) > 0$ is called a *focal element* of m. A bba m is said to be

- normal if $\emptyset$ is not a focal element,
- categorical if it has one focal element and
- vacuous if m is categorical and Ω is a focal element.

The belief function, noted *bel*, associated to a bba m, is a function *bel*: $2^\Omega \longmapsto [0,1]$ verifying:

$$bel(A) = \sum_{\emptyset\neq B\subseteq A} m(B) \text{ for all } A \subseteq \Omega. \qquad (1)$$

The belief function *bel* assigns to every subset A of Ω the sum of masses of belief committed to every subset of A by the bba m. The *plausibility function*, denoted *pl*, quantifies the maximum amount of potential specific support that could be given to $A \subseteq \Omega$. It is obtained by adding all the masses given to focal elements B that verify $B \cap A \neq \emptyset$:

$$pl(A) = \sum_{B\cap A\neq\emptyset} m(B) = bel(\Omega) - bel(\bar{A}). \qquad (2)$$

3.2 Discounting

Assume that a source of information provides a mass function m, and we have a degree of confidence $\alpha \in [0,1]$ in the reliability of that source. Then, m can be discounted with a discount rate $1-\alpha$, resulting in the following discounted mass function [11]:

$${}^{\alpha}m(A) = \begin{cases} \alpha\, m(A) & \text{if } A \subset \Omega, \\ 1-\alpha(1-m(\Omega)) & \text{if } A = \Omega. \end{cases}$$

3.3 Combination rules

Let us now consider two bbas m_1 and m_2 defined on the same frame of discernment and induced by two distinct and independent pieces of evidence. These bbas can be combined using the *conjunctive rule* [13] defined by:

$$\forall m_{12}(A) = m_1 \circledcirc m_2(A) = \sum_{B \cap C = A} m_1(B) m_2(C), \quad (3)$$

for all $A \subseteq \Omega$. We observe that the conjunctive rule of combination may provide a non normal bba, even if the combined bbas are normal. The mass $m_{12}(\emptyset)$ is called the degree of conflict between m_1 and m_2. If the degree of conflict is not equal to 1, a normal bba may be obtained by a normalization step. This is the definition of *Dempster's rule* of combination [13], denoted by $\oplus$:

$$m_{12}(A) = m_1 \oplus m_2(A) = \frac{\sum_{B \cap C = A} m_1(B) m_2(C)}{\sum_{B \cap C \neq \emptyset} m_1(B) m_2(C)}.$$

3.4 Marginalisation and vacuous extension operators

Let Ω and Θ be two frames of discernment. Let $m^{\Omega \times \Theta}$ be a bba defined on the Cartesian product $\Omega \times \Theta$. *The marginal* bba, denoted $m^{\Omega \times \Theta \downarrow \Omega}$, is defined for all $A \subseteq \Omega$, as:

$$m^{\Omega \times \Theta \downarrow \Omega}(A) = \sum_{\{B \subseteq \Omega \times \Theta / proj(B \downarrow \Omega) = A\}} m^{\Omega \times \Theta}(B), \quad (4)$$

where $proj(B \downarrow \Omega)$ is the projection of B on Ω, defined as: $proj(B \downarrow \Omega) = \{w_1 \in \Omega / \exists w_2 \in \Theta; (w_1, w_2) \in B\}$. Conversely, let m^{Ω} be a bba defined on Ω. Its *vacuous extension* on $\Omega \times \Theta$ is defined by:

$$m^{\Omega \uparrow \Omega \times \Theta}(B) = \begin{cases} m^{\Omega}(A) & \text{if } B = A \times \Theta \\ 0 & \text{otherwise.} \end{cases} \quad (5)$$

3.5 Conditioning and ballooning extension operators

Let $m^{\Omega \times \Theta}$ be a bba on the product space $\Omega \times \Theta$, and $h \subseteq \Omega$ an hypothesis of Ω. Suppose that we learn that h is true, then $m^{\Omega \times \Theta}$ should change for representing the new belief on $\Omega \times \Theta$. This is done using the conditioning operator defined as:

$$m^{\Omega \times \Theta}[h] = m^{\Omega \times \Theta} \circledcirc m_h^{\Omega \uparrow \Omega \times \Theta}. \quad (6)$$

where m_h^{Ω} is a categorical bba supporting the hypothesis h, $m_h^{\Omega}(h) = 1$.

The opposite of the conditioning operation is the ballooning extension on $\Omega \times \Theta$ which is given by: $m^{\Theta}[h]^{\Uparrow(\Omega \times \Theta)}(A) = \begin{cases} m^{\Theta}[h](B) & \text{if } A = (B \times h) \cup (\Theta \times (\Omega \backslash h)) \\ 0 & \text{otherwise.} \end{cases}$

3.6 Pignistic probability

Let m be a mass function on Ω obtained after combining all available items of evidence. Assume that we have to select an element of Ω. In the DS theory, different rules of decision have been proposed. One could select the element with maximum belief, highest plausibility or highest *pignistic probability* [13]. For a mass function m, the pignistic probability function, noted *betp*, is given by

$$betp(\omega) = \sum_{\{A \subseteq \Omega | \omega \in A\}} \frac{m(A)}{(1 - m(\emptyset))|A|}, \quad \forall \omega \in \Omega, \quad (7)$$

where $|A|$ is the cardinality of A. The pignistic probability function is thus obtained from m by distributing equally each normalized mass $m(A)/(1 - m(\emptyset))$ among the elements of A.

3.7 The Generalized Bayes Theorem

If we know the bbas on Θ given each $\omega_k \in \Omega$, we can compute the bba on Ω conditionally on any $\theta \in \Theta$ by using the Generalized Bayes Theorem (GBT). The GBT works in three steps by [12]:

- computing the ballooning extensions of the conditional bbas $m^{\Theta}[\omega_k], \forall \omega_k \in \Omega$, to get $m^{\Theta}[\omega_k]^{\Uparrow \Omega \times \Theta}$,
- combining these bbas conjunctively to get $m^{\Theta \times \Omega} = \circledcirc_k m^{\Theta}[\omega_k]^{\Uparrow \Omega \times \Theta}$,
- conditioning $m^{\Omega \times \Theta}$ on θ and marginalizing the result on Ω. Then, the final result is defined as follows [12]:

$$m^{\Omega}[\theta] = (\circledcirc_k m^{\Theta}[\omega_k]^{\Uparrow \Omega \times \Theta})[\theta]^{\downarrow \Omega}. \quad (8)$$

3.8 Implication rules within the belief function framework

Implication rules can be expressed in the belief function framework using conditional bbas. Let Ω and Θ be two frames of discernment. An implication rule between these frames is an expression of the form

$$R: \text{ if } A \text{ then } B \text{ with belief mass equal to } \beta,$$

where A and B are two hypotheses of Ω and Θ, respectively, and $\beta \in [0, 1]$. The expression 'if A then B' is logically equivalent to 'not A or B' (by definition of implication). The bba related to this expression is given by [10]: $m^{\Theta}[A](B) = \beta$ and $m^{\Theta}[A](\Theta) = 1 - \beta$, where $m^{\Theta}[A](B)$ is the conditional belief in B given A.

4 Multiple model estimation using belief function theory

4.1 Introduction

As mentioned in Section 2, the existing multiple model methods use probability theory in order to represent and handle motion model uncertainty. In this

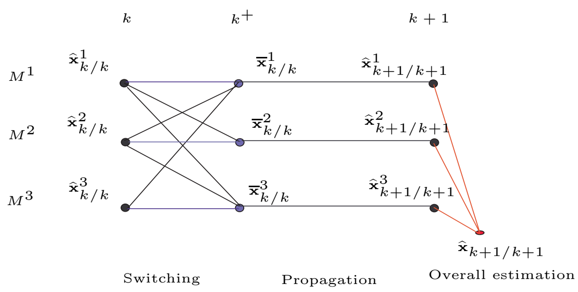

Figure 1: The BIMM main steps during time interval $[k, k+1]$.

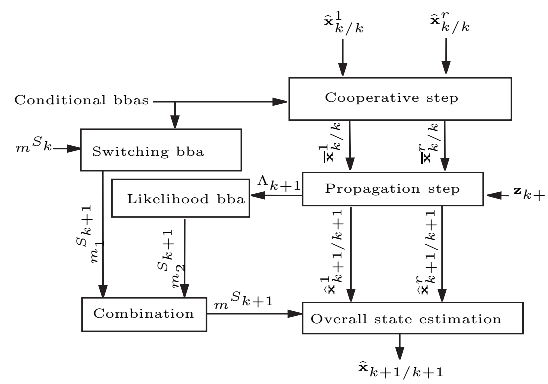

Figure 2: Block diagram of the BIMM with r CMs.

section, we will present the Belief Interacting Multiple Model (BIMM), based on belief function theory. As mentioned in Section 2, this method belongs to CMM approaches. It considers the case of dynamic systems with unknown and variant motion model among a set of CMs.

Let $S_k = \{M_k^1, M_k^2, \cdots, M_k^r\}$ be the set of r CMs at time step k. Note that $S_k = S_{k+1}$ and $M_k^i = M_{k+1}^i$, $\forall k$. Figure 1 shows an example of a system with three possible CMs $\{M_k^1, M_k^2, M_k^3\}$, where the state estimate at time step k for model i is $\widehat{\mathbf{x}}^i_{k/k}$ and the overall state estimate is $\widehat{\mathbf{x}}_{k/k}$. As illustrated in this figure, all $\{\widehat{\mathbf{x}}^i_{k/k}\}_{i=1}^3$ should be updated to $\{\overline{\mathbf{x}}^i_{k/k}\}_{i=1}^2$ after the system changes its motion model. During $[k, k+1]$ the system can change its motion model at $k_1 \in [k, k+1]$. As done in the IMM method, we assume that this is done at time step k^+; thereby, the period $[k, k+1]$ can be decomposed in three main steps [3]:

- Switching step (during $[k, k^+]$) or *Cooperative step*. In this step, the system changes its motion model and the states of CMs are updated according to our belief in the switching between CMs.
- Propagation step (during $[k^+, k+1]$): In this step, the states of CMs are updated according to the corresponding motion model using the measurement vectors.
- Overall estimation (at time step $k+1$): using $m^{S_{k+1}}$, the states of all CMs and the pignistic probability distribution on S_{k+1} derived from mass function computed in previous steps, an overall state estimation of the system can be computed.

According to the IMM principle, Figure 2 shows the block diagram of the BIMM method. In this figure, $\widehat{\mathbf{x}}^i_{k/k}$ is the state associated to M_k^i, m^{S_k} is the bba on CMs, $\mathbf{z}_k$ is the measurement vector and $\Lambda_{k+1} = (\Lambda^1_{k+1}, \cdots, \Lambda^i_{k+1})^T$ is a vector representing the measurement likelihood according to CMs. In the next section, we will present how bbas on the available models are handled and how these bbas are combined under the belief theory framework in order to introduce the BIMM.

4.2 Mass Function Construction

Let m^{S_k} be the bba on the CMs at time step k. The bba at time step $k+1$, $m^{S_{k+1}}$, is computed using a switching mass function and a likelihood mass function described hereafter.

- **Switching mass function**: Suppose that the only available information on the possible change of the motion model is that, with a given belief coefficient α, the model remains invariant between two time steps. The knowledge on switching between CMs can be then represented by r implication rules relating S_k and S_{k+1}: $R_i : M_k = M_k^i \Rightarrow M_k = M_{k+1}^i$ with a belief coefficient $\beta_i \in [0, 1]$, where R_i is the implication rule associated to M_k^i and where M_k is the true motion model of the system at time step k. These rules can be represented in the belief framework by conditional mass functions as explained in Section 3.8. The corresponding conditional mass function is given by:

$$\begin{cases} m^{S_{k+1}}[\{M_k^i\}](\{M_{k+1}^i\}) & = \beta_i \\ m^{S_{k+1}}[\{M_k^i\}](S_{k+1}) & = 1 - \beta_i \end{cases}$$

where $m^{S_{k+1}}[M_k^i](\{M_{k+1}^j\})$ represents the part of belief given to the hypothesis that $M_{k+1} = M_{k+1}^j$ knowing that $M_k = M_k^i$. Using these conditional bbas and the mass function m^{S_k}, a mass function $m_1^{S_{k+1}}$ representing our belief given to the hypothesis that the system is governed by each CM in S_{k+1} can be computed as:

$$m_1^{S_{k+1}} = \sum_{i=1}^{r} m^{S_{k+1}}[M_k^i] m^{S_k}(\{M_k^i\}). \qquad (9)$$

Using [14], the previous relation between m^{S_k} and $m_1^{S_{k+1}}$ can be written as

$$m_1^{S_{k+1}} = M_t \cdot m^{S_k}, \qquad (10)$$

where M_t is a transition matrix computed using the conditional bbas $\{m^{S_{k+1}}[M_k^i]\}_{i=1}^r$.

Example 1 *Consider a system with two possible CMs,* $S_k = \{M_k^1, M_k^2\}$. *Let* $m^{S_k} = \{0, 0.45, 0.20, 0.35\}$ *be a prior information about the CMs. Now suppose that the following implication rules have been provided to represent our prior knowledge on the possible change of the motion model of the system:*

- R_1*: If* $M_k = M_k^1$ *then* $M_{k+1} = M_{k+1}^1$ *with* $\beta_1 = 0.9$,
- R_2*: If* $M_k = M_k^2$ *then* $M_{k+1} = M_{k+1}^2$ *with* $\beta_2 = 0.89$.

The related conditional bbas are given by:

- $m^{S_{k+1}}[M_k^1](\{M_{k+1}^1\}) = 0.9$ *and* $m^{S_{k+1}}[M_k^1](S_{k+1}) = 0.1$,
- $m^{S_{k+1}}[M_k^2](\{M_{k+1}^2\}) = 0.89$ *and* $m^{S_{k+1}}[M_k^2](S_{k+1}) = 0.11$.

Using, these conditional mass functions, the transition matrix is given by

M=

	ϕ	$\{M_{k+1}^1\}$	$\{M_{k+1}^2\}$	S_k
ϕ	1	0	0	0
$\{M_k^1\}$	0	0.9	0	0.1
$\{M_k^2\}$	0	0	0.89	0.11
S_k	0	0	0	1

Using m^{S_k}, *the switching mass function* $m_1^{S_{k+1}}$ *can be calculated according to (10), which gives* $m_1^{S_{k+1}} = \{0, 0.44, 0.21, 0.35\}$.

- **Likelihood mass function**: using the the measurement likelihoods according to CMs, a mass function $m_2^{S_{k+1}}$ on S_{k+1} can be computed [1]. Let Λ_{k+1}^i be the likelihood of the model M_{k+1}^i calculated using the measurements. From Λ_{k+1}^i we can build a mass function on S_{k+1} as follows [1]:

$$\left\{\begin{array}{rl} m_l^i(\{M_{k+1}^i\}) = & 0 \\ m_l^i(\overline{\{M_{k+1}^i\}}) = & \alpha_i(1 - R\Lambda_{k+1}^i) \\ m_l^i(S_{k+1}) = & 1 - \alpha_i(1 - R\Lambda_{k+1}^i), \end{array}\right. \tag{11}$$

where $\overline{\{M_{k+1}^i\}}$ is the complement of $\{M_{k+1}^i\}$, α_i is a discounting coefficient associated with the model M_{k+1}^i and R is a normalization coefficient. By combining the mass functions $\{m_l^i\}_{i=1}^r$ using the conjunctive rule of combination, the mass function $m_2^{S_{k+1}}$ is calculated as:

$$m_2^{S_{k+1}} = \textcircled{\cap}_i m_l^i \tag{12}$$

Finally, the mass functions $m_1^{S_{k+1}}$ and $m_2^{S_{k+1}}$ can be combined in order to compute $m^{S_{k+1}}$ as

$$m^{S_{k+1}} = m_1^{S_{k+1}} \textcircled{\cap} m_2^{S_{k+1}}. \tag{13}$$

4.3 Sketch of belief interacting multiple model method

Consider a non linear dynamic system with r possible CMs. For all CMs in $S_k = \{M_k^1, \cdots, M_k^r\}$, the motion and observation equations for a model j can be represented by:

$$\left\{\begin{array}{rl} \mathbf{x}_{k+1} = & \mathbf{f}^j(\mathbf{x}_k, \mathbf{u}_k, \mathbf{v}_k^j) \\ \mathbf{z}_{k+1} = & \mathbf{g}^j(\mathbf{x}_{k+1}, \mathbf{w}_{k+1}^j), \end{array}\right. \tag{14}$$

where $\mathbf{f}^j$ is a possibly non-linear function which relates the state at time $k+1$ to the previous state at time k, the input $\mathbf{u}_k$ and an independent identically distributed (i.i.d) process noise sequence $\mathbf{v}_k^j$. The function $\mathbf{g}^j$ defines the relation between the measurement or the observation $\mathbf{z}_{k+1}$, the state $\mathbf{x}_{k+1}$ and an i.i.d observation noise sequence $\mathbf{w}_{k+1}^j$.

4.3.1 Initialization

At time step $k = 0$, and for all CMs, the initial states are considered to be normal with mean $\widehat{\mathbf{x}}_0^i$ and covariance $\widehat{\mathbf{P}}_0^i$. As there is no prior knowledge about the true motion model of the system, the motion model uncertainty is represented by a vacuous mass function on S_k. Assume that, at each time step, the system can change its motion model according to r implication rules which can be described by r known conditional mass functions on S_{k+1}, $\{m^{S_{k+1}}[\{M_k^i\}]\}_{i=1}^r$. These conditional mass functions can be represented by a transition matrix M_t as explained in Section 4.2. M_t is considered to be known and time invariant.

Algorithm 1 BIMM implementation

% Initialization %
$k \leftarrow 0$
Select a set of r CMs: $S_k \leftarrow \{M_k^1, \cdots, M_k^r\}$
$m^{S_k} \leftarrow$ vacuous mass function
Construct $\{m^{S_{k+1}}[\{M_k^i\}]\}_{i=1}^r$, representing the prior knowledge on switching between CMs
for $i = 1$ to n_{data} **do**
  % Cooperative step%
  $m_1^{S_{k+1}} \leftarrow M_t \cdot m^{S_k}$
  $m^{S_k}[\{M_{k+1}^i\}] \leftarrow (\textcircled{\cap} m^{S_{k+1}}[w_k^i]^{\Uparrow S_k \times S_{k+1}})[\{M_{k+1}^i\}]^{\downarrow S_k}$
  Update the states and covariance matrix of CMs using equations (16)and (17)
  % Propagation step%
  Compute the states, $\{\widehat{\mathbf{x}}_{k+1/k+1}^i\}_{i=1}^r$ and the corresponding covariance matrixes $\{\widehat{\mathbf{P}}_{k+1/k+1}^i\}_{i=1}^r$ using the measurement vector and the motion models of CMs
  $\Lambda_{k+1}^i \leftarrow \frac{1}{\sqrt{2\pi I_{k+1}^i}} \exp\{-\frac{1}{2}[\nu_{k+1}^i]^T [I_{k+1}^i]^{-1} [\nu_{k+1}^i]\}$
  Compute $m_2^{S_{k+1}}$ from the likelihoods using equation (11).
  $m^{S_{k+1}} \leftarrow m_1^{S_{k+1}} \textcircled{\cap} m_2^{S_{k+1}}$
  Compute the overall state estimation and the corresponding covariance matrix by using equations (19) and (20).
  $k = k + 1$
end for

4.3.2 Cooperative step

At time step k, the states $\{\widehat{\mathbf{x}}^i_{k/k}\}_{i=1}^r$ corresponding to CMs should be updated according to the conditional mass functions, $\{m^{S_{k+1}}[\{M^i_k\}]\}_{i=1}^r$. This is done as follows.

Consider the case of model i. One should use the states associated to all CMs at time step k, assuming that the true model at time step $k+1$ is M^i_{k+1}. Therefore, the conditional mass function $m^{S_k}[\{M^i_{k+1}\}]$ on S_k should be computed using the GBT and $\{m^{S_{k+1}}[\{M^i_k\}]\}_{i=1}^r$, as:

$$m^{S_k}[\{M^i_{k+1}\}] = (\circledcirc m^{S_{k+1}}[A]^{\Uparrow S_k \times S_{k+1}})[\{M^i_{k+1}\}]^{\downarrow S_k} \quad (15)$$

where A is a focal element of S_k. Now, the state associated to M^i_{k+1} can be updated using the pignistic probability, $Betp_i$, of $m^{S_k}[M^i_{k+1}]$. A mixed state and a covariance matrix of M^i_{k+1} can be thus computed as:

$$\overline{\mathbf{x}}^i_{k/k} = \sum_j^r Betp_i(\{M^j_k\}) \cdot \widehat{\mathbf{x}}^j_{k/k}, \quad (16)$$

$$\overline{\mathbf{P}}^i_{k/k} = \sum_j^r Betp_i(\{M^j_k\}) \cdot (\widehat{\mathbf{P}}^j_{k/k} + \mathbf{C}), \quad (17)$$

where $\mathbf{C} = [\widehat{x}^j_{k/k} - \overline{x}^i_{k/k}][\widehat{x}^j_{k/k} - \overline{x}^i_{k/k}]^T$. The switching mass function, $m_1^{S_{k+1}}$, is computed from M_t and m^{S_k} using equation (10).

4.3.3 Propagation step

When the measurement vector $\mathbf{z}_{k+1}$ is available, the states $\{\widehat{\mathbf{x}}^i_{k+1/k+1}\}_{i=1}^r$ and the covariance matrices $\{\widehat{\mathbf{P}}^i_{k+1/k+1}\}_{i=1}^r$ associated to all CMs can be computed according to the motion equations of CMs. The measurement likelihood according to CM i is computed as:

$$\Lambda^i_{k+1} = \frac{1}{\sqrt{2\pi I^i_{k+1}}} \exp\{-\frac{1}{2}[\nu^i_{k+1}]^T [I^i_{k+1}]^{-1} [\nu^i_{k+1}]\} \quad (18)$$

where Λ^i_{k+1} is the likelihood of model M^i_{k+1}, $\nu^i_{k+1} = \mathbf{z}_{k+1} - g^i(\widehat{\mathbf{x}}^i_{k+1/k+1}, \mathbf{w}^i_{k+1})$ is the innovation and I^i_{k+1} is the associated covariance matrix. Using the likelihoods of CMs as in Section 4.2, a likelihood mass function $m_2^{S_{k+1}}$ can be computed using (11) and (12). By combining $m_1^{S_{k+1}}$ and $m_2^{S_{k+1}}$, a final mass function on S_k can be calculated using equation (13).

4.3.4 Overall estimation

In this final step, the pignistic probability $BetP$ associated to $m^{S_{k+1}}$ is computed, and overall state and covariance estimates are obtained by taking the expectation of the state vector and covariance matrix with respect to $BetP$:

$$\widehat{\mathbf{x}}_{k+1/k+1} = \sum_j^r Betp(\{M^j_k\}) \cdot \widehat{\mathbf{x}}^j_{k+1/k+1}, \quad (19)$$

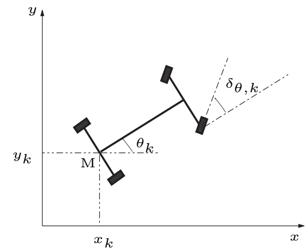

Figure 3: Definition of the frames.

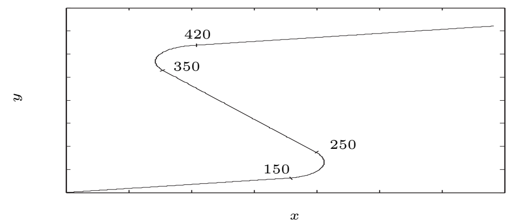

Figure 4: Simulink trajectory. In this figure appear the time in second starting at $t_0 = 0$.

$$\widehat{\mathbf{P}}_{k+1/k+1} = \sum_j^r Betp(\{M^j_k\}) \cdot (\widehat{\mathbf{P}}^j_{k+1/k+1} + \mathbf{C}'), \quad (20)$$

where $\mathbf{C}' = [\widehat{x}^j_{k+1/k+1} - \widehat{\mathbf{x}}_{k+1/k+1}][\widehat{x}^j_{k+1/k+1} - \widehat{\mathbf{x}}_{k+1/k+1}]^T$.

The BIMM method is summarized in Algorithm 1.

5 Application

In this Section, we present the application of the BIMM to a vehicle localization problem. The vehicle position is represented by the Cartesian coordinates (x_k, y_k) of the point M attached to the center of the rear axle as shown in Figure 3. The heading angle is denoted θ_k. The state $\mathbf{x}_k = (x_k, y_k, \theta_k)^T$ is calculated at each time step k thanks to the following discrete representation:

$$\begin{cases} x_{k+1} = & x_k + \delta_{S,k}\cos(\theta_k + \frac{\delta_{\theta,k}}{2}) \\ y_{k+1} = & y_k + \delta_{S,k}\sin(\theta_k + \frac{\delta_{\theta,k}}{2}) \\ \theta_{k+1} = & \theta_k + \delta_{\theta,k} \end{cases} \quad (21)$$

where $\delta_{S,k}$ is the elementary linear displacement and $\delta_{\theta,k}$ is the measure of the elementary rotation. The vehicle position, the heading, the elementary movement and the elementary rotation were generated using the Matlab simulink toolbox with fixed time step equal to 1 second. The GPS measurement noise was supposed to be white with $\sigma_x = 7$ m and $\sigma_y = 9$ m. The noise in the input data (elementary movement and elementary rotation) was supposed to be white with $\sigma_s = 1/4$ m and $\sigma_\theta = 0.002$ degrees. Figure 4 shows the simulated trajectory. As shown in this figure, the motion of the vehicle can be described by two models: a linear model M^1

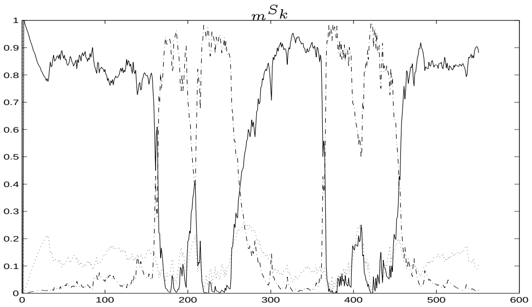

Figure 5: Mass function associated to the vehicle trajectory of Figure 4. $m^{S_k}(\{M^1\})$, $m^{S_k}(\{M^2\})$ and $m^{S_k}(S_k)$ are represented by solid, dashed and dotted lines, respectively.

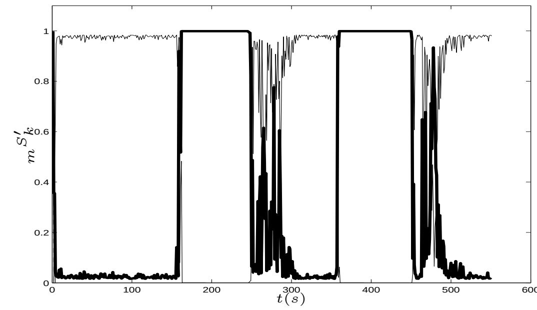

Figure 6: Mass function on $S'_k = \{M^1, M^3\}$. $m^{S'_k}(\{M^1\})$ and $m^{S'_k}(\emptyset)$ are plotted by solid line and bold line, respectively.

and a turning model M^2, therefore $S_k = \{M^1, M^2\}$. Model M^1 is characterized by a constant heading of the vehicle ($\theta_{k+1} = \theta_k \; \forall k$) while for M^2 θ_k changes according to: $\theta_{k+1} = \theta_k + \delta_{\theta,k}$. In this application we assume that the change of the motion model of the vehicle can be represented by two implication rules:

- R_1: If M^1 then M^1 with $\beta_1 = 0.99$,
- R_2: If M^2 then M^2 with $\beta_2 = 0.97$.

Table 1: Mean square errors for GPS, BIMM, EKF with model M^1 and EKF with model M^2.

	GPS	BIMM	EKF M^1	EKF M^2	IMM
$E_x(m)$	1.632	0.561	0.871	0.692	0.670
$E_y(m)$	2.785	0.912	1.418	1.106	1.013

The initial mass function on S_k was assumed to be vacuous. Figure 5 shows the computed mass function on S_k, which represents our belief given to the hypothesis that the vehicle is moving according to CMs in S_k. As shown in this figure, the BIMM succeeds to detect the change of the motion model of the vehicle. Figure 7 shows the quadratic errors (QE) of BIMM and EKF of linear and turning models. In this figure, the QE of the BIMM method, EKF with model M^1 and EKF with model M^2 are plotted by bold black line, dotted line and dashed line respectively. As we can see in this figure, the QE of BIMM follows the QE of the model with the largest mass function. Consequently, this figure confirms that the BIMM method detects the true model of the vehicle displacement. Table 1 shows a comparison between mean square errors obtained by BIMM, EKF with model M^1, EKF with model M^2 and the interacting multiple model method (IMM) applied to the same CMs (M^1, M^2) and with the following parameters: $\mu_0^1 = 0.5$, $\mu_0^2 = 0.5$, $\pi_{11} = 0.99$, $\pi_{12} = 0.01$, $\pi_{21} = 0.1$ and $\pi_{22} = 0.9$, where μ_k^i is the probability that M^i is the true model at time step k and π_{ij} is the probability of switching between M^i and M^j. As illustrated in this table, the BIMM method has better performance than EKF of M^1, M^2 and IMM. As a conclusion, the BIMM method succeeds in combining the outputs of EKF M^1 and M^2 under belief theory framework in order to compute an accurate estimation of the vehicle position. Figure 6 shows the computed bba on $S'_k = \{M^1, M^3\}$, where M^3 is the stationary model of the vehicle displacement. It is characterized by: $x_{k+1} = x_k$ and $y_{k+1} = y_k \; \forall k$. As we can see in this figure, during $[150, 250]$ and $[350, 420]$, the mass given to $\emptyset$ is equal to 1 ($m^{S'_{k+1}}(\emptyset) = 1$) since the vehicle motion is governed by the turning model which is not included in S'_k; the BIMM thus succeeds to detect the fact that the true model is not included in the set of CMs.

6 Conclusion

In this paper, a new method for state estimation of dynamic systems with unknown and variant motion model has been presented. This method uses belief function theory and existing state estimation methods in order to represent the motion model uncertainty and to compute an accurate estimate of the system state. This is done by selecting a set of candidate models and by using a bank of basic filters associated to CMs. As the motion model of the system can change, a cooperative step between different models is performed under the belief theory framework. Finally, the pignistic probability over CMs is used for compute an overall state of the system. The proposed method seems to be adequate to deal with systems with frequent model switch. Also, the BIMM method can detect the fact that the true model is not included in the set of CMs. Furthermore, the implementation of this method is quite simple. Results on simulated data of vehicle localization problem and the comparison with other methods like the Extended Kalman filter and interacting multiple model methods show the effectiveness of this method

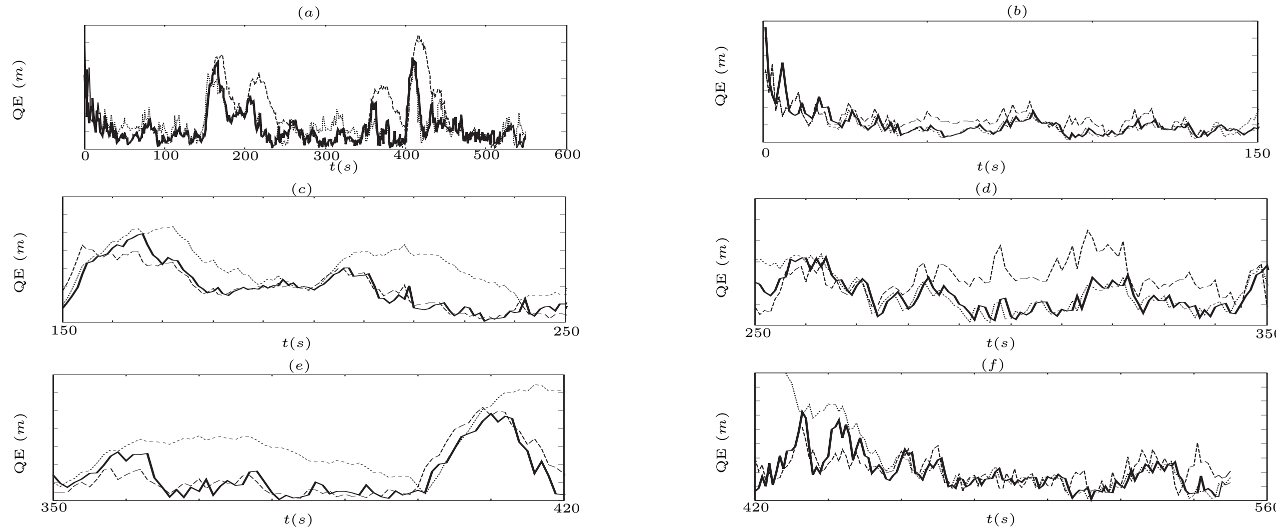

Figure 7: Quadratic errors of BIMM (solid line), EKF with model M^1 (dotted line) and EKF with model M^2 (dashed line).

to manage multiple model problems and suggest that it can be considered as a good alternative to the existing multiple model methods. A further advantage of this method is the possibility to combine the final mass function on possible models with additional information on the models or on the switching step given by a supplementary sensors (or information source) such as digital road network data, which can also be represented efficiently in the belief function framework [9].

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