# Application of fuzzy preference modelling to the fusion of sensory profile data

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*Abstract*— We propose a new analysis method to deal with sensory profile data. Such data are composed of scores attributed by human experts (or judges) in order to describe a set of products according to a given sensory descriptor. All assessments are repeated, usually three times. The first step consists in extracting and encoding the relevant information of each replicate into a fuzzy weak dominance relation. Then an aggregation procedure over the replicates allows synthesis of the perception of each judge into a new fuzzy relation. In a similar way, a consensual relation is finally obtained by fusing the relations of the judges. The proposed analysis tools are based on a particular objective of the fuzzy preference modelling: the decomposition of a fuzzy weak preference relation into a fuzzy preference structure. An example of application illustrates the interest of the method.

*Index Terms*— sensory profile data analysis, fuzzy logic, fuzzy preference modelling, aggregation.

## I. INTRODUCTION

### A. Sensory profile data analysis

We focus in this paper on sensory profile data, i.e., data gathered from a group of persons, in order to describe the way they perceive a set of products according to a given sensory descriptor. Acquisition is managed according to the following protocol: in the course of $n_r$ spaced screenings, a *panel* of $n_j$ persons called *judges* evaluate each one of the $n_p$ *products* according to the descriptor, by giving a score $u_{pjr} \in [0, 10]$. These values are asserted using a graphical user interface, by moving a cursor on to a continuous finite scale.

The main objective of sensory profile data analysis is to describe how the products are perceived by the judges. But it has also to describe the own performances of the judges, notably their ability to replicate their scores and to be discriminant. A more global performance indicator can then be provided for the panel, in order to measure the agreement of the judges.

A particular difficulty of such data is due to the imprecision of the assessments. In spite of training, a perfect similarity among the $n_r$ replicates is not plausible. For this reason, the analysis clearly needs to take into account this imprecision. A common solution consists in averaging the scores over the replicates and applying an analysis of variance.

We claim that this approach is not completely suitable. Fundamentally, we may consider that each judge does not exactly assert the same information during the $n_r$ replicates. On the one hand, if two products are only slightly different, their difference may or may not be perceived. Indeed, we may reasonably suppose that the ability of a judge to discriminate the products is not constant, especially when the products are almost similar. On the other hand, the evaluation of the intensity of a sensation is difficult, and the judge may deliver erroneous scores. In these two situations, the differences between the scores of two products should not be necessarily understood as a difference of intensity. Averaging the scores could be suitable for a large number of replicates, but only three replicates are generally available.

Based on a compromise between a quantitative and an ordinal approach, the proposed method uses a radically different way of fusing sensory profile data, first over the replicates to summarize the perception of each judge, and then over the judges so as to express a consensus.

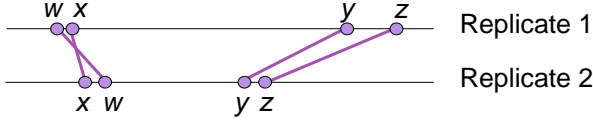## B. Compromise between a quantitative and an ordinal approach



Fig. 1. Example of scores asserted by a judge to a set of 4 products $\{w, x, y, z\}$ in the course of two replicates

The approach we propose consists in extracting the information expressed by a judge in each replicate, and then in aggregating it over the replicates; a similar procedure will then be used to obtain a consensual response. A fundamental question is about the nature of the relevant information delivered by the judges. A typical example of two replicates of a same judge is shown in Figure 1. We observe that the quantitative values of scores are not clearly preserved, in particular for the products $y$ and $z$. The ordinal information appears to be more preserved, except for the pair $(w, x)$, whose ordering relation is reversed over the replicates. The difference of their scores is close to zero in both replicates, and this pair of products is probably not discriminated by the judge. The difference in scores should therefore be taken into account to assess the relevance of ordering relations: the more important the difference, the more relevant the ordering relation. Consequently, our objective is to use quantitative information so as to weight the certainty of the ordinal information. This compromise, clearly more informative than a crisp ordering relation, will be achieved thanks to fuzzy ordering relations.

## C. Fuzzy preference theory

For each replicate, information given by a judge is summarized into a fuzzy ordering relation that synthesizes the way the judge discriminate the products. The data analysis we propose is exclusively based on these relations: after this modelling step, scores will no longer be used. Fuzzy preference theory [7], originally developed as a multicriteria decision tool, is then used to interpret the obtained relation. It proposes a well-suited decomposition of these relations, into three subrelations: dominance, and non-dominance that is itself divided into indifference and incomparability.

## D. Overview

This paper explains the whole process of this data analysis method. Section II describes a method to extract the relevant information of each replicate into a fuzzy weak dominance relation. Then more synthetic relations are built over the replicates or over the judges. This process is obtained through two consecutive stages. Section III describes a raw aggregation procedure, and section IV proposes a correction of the obtained raw relation, that consists in making its dominance subrelation transitive.

## II. EXTRACTION OF FUZZY RELATIONS

### A. Comparison of fuzzy intervals

Let $j \in \{1, \ldots, n_j\}$ be a judge, and let $r \in \{1, \ldots, n_r\}$ be a given replicate. Each product $p \in X$ where $X = \{1, \ldots, n_p\}$ is the set of products, is then described by a score $u_{pjr} \in [0, 10]$. In this section, we simply denote this value by $u_p$.

Because of the continuous scale, we may assume non tied values in the set $\{u_1, \ldots, u_p\}$. Consequently, the direct comparison of the $n_p$ values induces a total order on $X$: this corresponds to a full ordinal approach, that is not suitable. In particular, for all pairs of products $(x, y)$, a difference $(u_x - u_y) > 0$ very close to zero induces the *(strict) dominance* of $x$ over $y$.

To take into account the imprecision of the values, it could be better to consider intervals instead of single-values. For each value $u_x$, we could define an interval $\bar{u}_x$ centered on $u_x$, as the set of values the judge could have assigned to $x$. Non-empty intersection between $\bar{u}_x$ and $\bar{u}_y$ would induce *indifference* instead of *dominance* for the pair of products $(x, y)$. All products are no more necessarily discriminated: the intervals induce an *interval order*. But the former problem occurs again: a very small modification of the bounds of the intervals may change indifference into dominance and reciprocally.

Fuzzy logic allows a more suitable modelling of imprecision. We consider fuzzy intervals: to each value $u_x$, we associate a fuzzy interval $\tilde{u}_x$, characterized by a possibility distribution $\mu_x : [0, 10] \to [0, 1]$. For all $a \in [0, 10]$, the value $\mu_x(a)$ may be interpreted as the possibility for the value $a$ to be the intensity perceived by the judge. We choose to use identical trapezoidal fuzzy intervals, whose kernel and support are centered on the value $u_x$ given by the judge. The ratio between the sizes of the kernel and the support is arbitrarily set to $1/3$. The last parameter is then the size of the support, that could be set arbitrarily too. But in a next section, we will propose a method to estimate the most suitable size for each judge.

A fuzzy ordering relation $S$ finally results from the comparison by pair of the fuzzy intervals, based on the Zadeh's extension principle [12]:

$$\forall (x, y) \in X^2, \ S(x, y) \ = \ \pi(\tilde{u}_x \geq \tilde{u}_y)$$
$$= \ \sup_{\{a, b / a \geq b\}} \min\{\mu_x(a), \mu_y(b)\}.$$

If $u_x \leq u_y$ then $S(x, y)$ is equal to the height of the intersection of both trapezoids. Otherwise, $S(x, y)$ is equal to 1: it is totally possible for the product $x$ to dominate the product $y$.

In classical preference theory, $S$ is referred to as a *weak preference relation* (or as a *fuzzy outranking relation*) [9]: $S(x, y)$ is the degree to which the proposition "$x$ is not worse than $y$" is true. The only expected formal constraint is *reflexivity*, that is clearly satisfied by $S$. But it can be shown that $S$ is $W'_\varphi$-*strongly complete* (where $W'_\varphi$ is a Lukasiewicz's t-conorm), and $min - max$-*Ferrers*: according to Bufardi [2], it is *fuzzy interval order*.

### B. Decomposition and interpretation

Preference modelling allows to decompose a *crisp weak preference relation* $S$ defined on $X$ into three subrelations of *(strict) preference* $(P)$, *indifference* $(I)$ and *incomparability* $(J)$:

$$\begin{aligned} P &= p(S) = S \cap S^c \\ I &= i(S) = S \cap S^{-1} \\ J &= r(S) = S \cap (S^c)^{-1}, \end{aligned}$$

where $S^{-1}$ is the inverse relation of $S$ (i.e. $(x,y) \in S^{-1} \Leftrightarrow (y,x) \in S$), and $S^c$ is the negation of $S$ (i.e. $(x,y) \in S^c \Leftrightarrow (x,y) \notin S$).

It is a decomposition, because each non-ordered pair $(x,y)$ or $(y,x)$ necessarily belongs to one and only one of these subrelations:

$$\left( P \cup P^{-1} \right) \cup J \cup I = X \times X.$$

For our application, we prefer to interpret $P$ as a subrelation of *dominance*, and $S$ as a *weak dominance* relation.

Fuzzy preference theory extends this decomposition to the *fuzzy weak dominance relation*. The principle consists in extending the logical operators union and disjunction. Different solutions are mentioned by Fodor & Roubens [7, p. 78:81], but we select the one that makes the relation $P$ min-asymmetric:

$$\begin{aligned} P(x,y) &= W_\varphi \left( S(x,y), c_\varphi(S(y,x)) \right) \\ I(x,y) &= \min\{S(x,y), S(y,x)\} \\ J(x,y) &= \min \left\{ c_\varphi(S(x,y)), c_\varphi(S(y,x)) \right\}, \end{aligned}$$

where $W_\varphi$ is a Lukasiewicz's t-norm depending of an automorphism $\varphi : [0,1] \to [0,1]$, and $c_\varphi$ is the following function of *strong negation*: $c_\varphi(x) = \varphi^{-1}(1 - \varphi(x))$, $\forall x \in [0,1]$. Apart from the min-asymmetry of $P$, we have the symmetry of $I$ and $J$, the reflexivity of $I$ and the irreflexivity of $J$: it is postulated that each object is indifferent to itself.

Next we simply opt for the identity function as the function $\varphi$, such that:

$$\begin{aligned} P(x,y) &= \max\{S(x,y) - S(y,x), 0\} \\ I(x,y) &= \min\{S(x,y), S(y,x)\} \\ J(x,y) &= 1 - \max\{S(x,y), S(y,x)\}. \end{aligned}$$

As in the crisp case, we denote respectively $p$, $i$, and $r$ the functions that transforms $S$ into $P$, $I$, and $J$.

In the case where relation $S$ results from the comparison of fuzzy intervals, the *incomparability* subrelation is empty $(J(x,y) = 0, \forall (x,y) \in X^2)$. We observe that $I(x,y)$ is equal to the height of the intersection of the fuzzy intervals associated to the products $x$ and $y$: the closer the intervals of the products, the higher their indifference. At last, $P$ is min-asymmetric and min-transitive [4]. It may be interpreted as a necessity measure [3]:

$$P(x,y) = N(\tilde{u}_x > \tilde{u}_y).$$

The structure $(P,I,J)$ is called a *fuzzy preference structure* [10]. Bufardi [1] gives a concise definition of such a structure,

depending on an automorphism $\varphi$ (that parameterizes the Lukasiewicz's t-norm previously used):

*Definition 2.1:* : a structure $(P,I,J)$ of fuzzy relations defined on $X$ is a fuzzy preference structure relation if and only if following conditions are satisfied:

1) $I$ is reflexive,
2) $I$ is symmetric,
3) $\varphi(P) + \varphi(P^{-1}) + \varphi(I) + \varphi(J) = \mathbf{I}$,

where $\mathbf{I}$ denotes the identity relation defined on $X$ ($\mathbf{I}(x,y) = 1 \Leftrightarrow x = y$).

The third equation underlines the fact $(P,I,J)$ is a decomposition of $X \times X$. As proposed in Figure 3, this property is useful to build a graphical representation of $S$, based on a grid of size $n_p \times n_p$, whose each square is horizontally divided into three parts. Heights of these rectangles are selected to be proportional to the values $\max \{P(x,y), P(y,x)\}$, $I(x,y)$ and $J(x,y)$, respectively the degrees of *dominance*, *indifference* and *incomparability*. To make the reading easier, we sort the products by order of decreasing dominance, and we associate distinct colors or grey levels to the three subrelations. Moreover, *white* may be used to reveal antisymmetry of $P$: if $P(x,y) > 0$ then a particular color would be used for the pair $(x,y)$ ("$x$ is more likely to dominate $y$") and *white* for the inverse pair $(y,x)$. Next, *black* may be used to reveal symmetry of $I$: $I(x,x) = 1$ does not bring any information because of its necessity, and this black diagonal makes the reading easier.

The decomposition can also be used to define global indicators of the relation. Indicators of *dominance*, *indifference* and *incomparability* show the way the descriptor was used by the judge(s), during the replicate(s). We propose the following definitions, proportional to the fuzzy cardinals of $P$, $I$ and $J$, summing to 1:

$$\begin{aligned} Q^P &= \frac{2}{(n_p(n_p - 1))} \sum_{(x,y) \in X \times X} P(x,y) \\ Q^I &= \frac{1}{(n_p(n_p - 1))} \sum_{(x,y) \in X \times X} (-n_p + I(x,y)) \\ Q^J &= \frac{1}{(n_p(n_p - 1))} \sum_{(x,y) \in X \times X} J(x,y). \end{aligned}$$

## III. RAW AGGREGATION OF FUZZY RELATIONS

### A. Introduction to the raw aggregation

*1) Objective:* we now consider $n_s$ fuzzy weak dominance relations $S_i$. These relations may either correspond to several replicates of a unique judge, or to aggregated relations of several judges. In both cases, we need to aggregate these $n_s$ relations into a new one $M$, in order to synthesize the difference of perception of each pair $(x,y)$ of products through both values $M(x,y)$ and $M(y,x)$.

This process is called a "raw" aggregation because each pair of products is separately processed: both values $M(x,y)$ and $M(y,x)$ only depend on the values $S_i(x,y)$ and $S_i(y,x)$. Section IV will therefore describe a correction of this raw

aggregated relation so as to make it consistent over all pairs of products $(x, y)$.

*2) Simplification:* Our aggregation does not manage the two kinds of non-dominance that may appear in the relations $S_i$: indifference and incomparability. These two notions are then merged: incomparability is converted to indifference. The whole information is then contained in the subrelations of dominance $P_i = d(S_i)$ and indifference ($I_i = i(S_i)$).

We remark that this conversion is useless in the aggregation of replicates, because of the empty incomparability subrelation. The incomparability notion only results from the confrontation of antagonistic dominances: for any pair $(x, y) \in X^2$, this means: $\exists i, j \in \{1, \ldots, n_s\}/\ P_i(x, y) > 0$ and $P_j(y, x) > 0$).

*3) Two kinds of fusion:* we distinguish the fusion of judges and the fusion of replicates. With the latter, we make $M(x, y)$ increasing with the frequency and the degree of certainty (equal to $P_i(x, y)$) of the replicates asserting the dominance of $x$ over $y$. Ordered weighted averaging (*owa*) operators [11] will allow an adjustment of the measure of frequency, from "in at least one replicate" to "in all replicates". In this case, for any $(x, y) \in X^2$, the degrees $P_i(x, y)$ and $P_i(y, x)$ will not be weighted in the same way.

With judges we deal with another possibility, by considering that relations are a priori weighted. In our example we set the weight of each judge proportional to $1 - Q^J$, the complement to 1 of its indicator of conflict.

Note that both aggregating techniques agree in a special case: *owa* as a mean for the first case, and equal weights in the second one.

*4) Basis of the fusion process:* our fusion procedures are based on the following proposition.

*Proposition 3.1:* if $(P, I, J)$ is a fuzzy preference structure such that preference $P$ is min-asymmetric, then the preference decomposition, which fulfills min-asymmetry of $P$, of the corresponding weak preference $S = P \cup_\varphi I$ is $(P, I, J)$.
This result shows the equivalence between such $(P, I, J)$ structures and their corresponding weak preference $S$. This explains the possibility of inferring $S$ from $(P, I, J)$. Therefore we propose to build a structure $(P, I, J)$ defined by three separately aggregated subrelations, from which we will finally infer our fused weak dominance relation: $P \cup_\varphi I$.

*B. Fusion of replicates*

In case of crisp relations $S_i$, we could define how many replicates asserting the dominance of $x$ over $y$ are necessary to finally decide $(x, y) \in M$. In the fuzzy case, a particular class of *owa* [11] was proposed by O'Hagan [8] as fuzzy quantifiers of frequency. An *owa* operator consists of a set of positive weights $\{w_1, \ldots, w_{n_s}\}$ summing to 1; it is used to fuse a set of $n_s$ values by averaging this values weighted in descending order (i.e. $w_i$ is associated to the $i^{\text{st}}$ greatest value).

Each *owa* of this class only depends on the size $n_s$ of values to aggregate, and on a degree of disjunction (*orness degree* [6]) $\rho \in [0, 1]$. Extreme operators are the min ($\rho = 0 : w_{n_s} = 1$), and the max ($\rho = 1 : w_1 = 1$). An operator of intermediate

orness degree like $\rho = 0.25$, for $n_s = 3$ defines weights: $[w_1 = 0.1, w_2 = 0.3, w_3 = 0.6]$.

We are mainly interested in an *owa* of orness degree close to 1, i.e. close to the max operator. Indeed, as explained, we do not expect the judges to be able to maintain their discriminations over all the replicates. For each pair $(x, y)$, we may consequently focus on the $\max_{i \in \{1, \ldots, n_s\}} P_i(x, y)$ value. Moreover, it emphasizes *conflict*, defined as the existence of antagonistic dominances asserted by the judge for the given pair of products. This conflict is encoded through the incomparability subrelation. The operator max is very suitable for small numbers of replicates; when this number increases, decreasing the orness degree appears more convenient ($\rho \in [0.5, 1]$).

We denote by $M_1$ the fuzzy irreflexive relation defined on $X$ by:

$$
\begin{aligned}
M_1(x, y) &= \text{owa}_{i \in \{1, \ldots, n_s\}}(P_i, \rho) \\
&= \sum_{i \in \{1, \ldots, n_s\}} w_i P_{(i)}(x, y), \ \forall (x, y) \in X^2,
\end{aligned}
$$

where $P_{(i)}(x, y)$ defines the $i^{\text{st}}$ greatest value of $\{P_i(x, y), \ i \in \{1, \ldots, n_s\}\}$, and where weights $\{w_i, \ i \in \{1, \ldots, n_s\}$ are defined by the number of replicates $n_s$ and the orness degree $\rho$.

The subrelation $p(M_1)$ reveals the way one pair $(x, y)$ or $(y, x)$ is more dominant than the other. The subrelation $i(M_1)$ is the symmetric part of the aggregated dominances: it reveals the conflict. But $M_1$ is not satisfactory, because the third subrelation $r(M_1)$ does not represent a relevant indifference. Indeed, if we consider an even $n_s$, with half $P_i(x, y) = 1$ ($\Rightarrow P_i(y, x) = 0$) and half $P_i(y, x) = 1$ ($\Rightarrow P_i(x, y) = 0$), dominance and indifference are expected to be empty: it is a situation of *maximal conflict*. However, $M_1(x, y) = M_1(y, x)$ does not generally reach the value 1: $M_1$ must be modified so to as to build $M$. But this is not necessary with the max operator ($\rho = 1$): in this special case, the following modification has to be neutral.

We propose to keep the dominance subrelation obtained:

$$
M_P = p(M_1),
$$

and to compute separately the aggregated indifference:

$$
M_I = \text{owa}_{i \in \{1, \ldots, n_s\}}(I_i, \rho').
$$

In order to preserve the neutrality with max operator, let $\rho' = 1 - \rho$: this simply reverses the order of the weights ($w_i$ becomes $w_{n_s - i + 1}$).

Then, it is easy to prove that $M_I \subseteq M_1$, where $\subseteq$ denotes Zadeh's inclusion [12]:

$$
M_I(x, y) \le r(M_1)(x, y), \ \forall (x, y) \in X^2.
$$

This means that the new indifference degree is lower than the former one: incomparability (or conflict) is increased. It was expected: in case of maximal conflict for the pair $(x, y)$, $M_1(x, y)$ does not reach 1, because non-zero weights are associated to zero-values $P_i(x, y)$ such that $P_i(y, x) = 1$. It

is so processed as indifference, although this is only due to some conflict.

At last, to preserve the equation of decomposition, we let:

$$M_J(x, y) = 1 - (M_I(x, y) + \max\{M_P(x, y), M_P(y, x)\}),$$

that includes initial conflict $i(M_1)$.

Therefore $(M_P, M_I, M_J)$ is a fuzzy preference structure, whose subrelation $P$ is min-asymmetric. However, instead of building the inferred weak dominance relation, we consider its dual relation $M$:

$$M = M_J + M_P.$$

The relation $M$ is a raw aggregated relation, whose membership degrees directly refer to the dominance expressed by the judge: the symmetric part $i(M)$ encodes the notion of conflict, and its asymmetric part encodes the non-conflicting dominance. As initially specified, $M$ is called a "raw" relation because it still needs to be refined so as to ensure a consistency of the dominance among all pairs of products $(x, y)$, through the property of transitivity (see section IV).

## C. Fusion of judges

Like in the previous section, we deal with $n_s$ weak dominance relations $S_i$ whose subrelation of incomparability are empty. We now assume each relation $S_i$ to be weighted with a $w_i \geq 0$, such that the sum of weights is equal to 1: $\sum_{i \in \{1, \ldots, n_s\}} w_i = 1$.

The only formal difference lies in replacing the *owa* operator with a simple *weighted averaging*. The interpretation is slightly modified: antagonistic dominances are now interpreted as *disagreement* instead of *conflict*.

We consequently let:

$$M_1(x, y) = \sum_{i \in \{1, \ldots, n_s\}} w_i P_i(x, y), \ \forall (x, y) \in X^2,$$

from which we extract the raw aggregated dominance subrelation:

$$M_P = p(M_1).$$

Then the aggregated indifference is computed:

$$M_I = \sum_{i \in \{1, \ldots, n_s\}} w_i I_i(x, y),$$

that is again included in $r(M_1)$ (i.e. with Zadeh's inclusion). That is why the new disagreement $M_J$ includes $i(M_1)$:

$$M_J(x, y) = 1 - (M_I(x, y) + \max\{M_P(x, y), M_P(y, x)\}).$$

As previously, $(M_P, M_I, M_J)$ is a fuzzy preference structure that satisfies min-asymmetry of $P$. It is easy to prove that the disagreement subrelation is proportional to the symmetric part of the initial fused relation $M_1$:

$$M_J = 2 * i(M_1).$$

The raw aggregated relation is finally obtained by:

$$M = M_P + M_J.$$

## IV. CORRECTION FOR TRANSITIVITY

### A. Problem

Formally, nothing ensures the preservation of the property of transitivity of the fused dominance subrelation: transitivity of $P_i$ does not imply transitivity of $M_P = p(M)$. This result was expected: when fusing replicates, we do not expect judges to systematically repeat their perception, and we use an *owa* close to the max so as to maximize the discriminant information over the replicates. The transitivity represents a global consistency of the dominance relation on all pairs of products, that can not be obtained by such a process.

The objective of the correction for transitivity is first to build an aggregated subrelation of dominance $P$. But this restored consistency has then to be extended to the two other subrelations of the fuzzy preference structure $(P, I, J)$.

### B. Definition of the corrected dominance $P$

We now consider the raw aggregated dominance $M$ as a set of *assertions*: $M(x, y)$ is interpreted as the assertion "$x$ dominates $y$" with a degree of certainty $M(x, y)$. Its *inverse* assertion is "$y$ dominates $x$" with a degree of certainty $M(y, x)$. The assertions specified by $M$ are said to be *raw assertions*.

Let $P$ denote the final aggregated dominance subrelation. The first problem is to define the subset of assertions of $M$ on which $P$ will depend on: the whole or only a part. We propose to ignore the subset of *conflicting* assertions, and we denote by $U$ the relation defined by the remaining assertions:

$$U = p(M) = M_P.$$

More precisely, $U$ is the subset of *explicit useful assertions*:
- *explicit*, because it is a subset of $M$, that includes all the assertions provided by the judge(s);
- *useful*, because these assertions will take part in the building of $P$, contrary to the assertions included in $i(M)$.

We then deduce the *implicit useful assertions* by transitive closure. Let $V$ denote the set of *useful assertions* which are either *implicit* or *explicit useful assertions*:

$$V = \text{FT}(U) \cap_{\min} \mathbf{I}^c,$$

where the intersection with $\mathbf{I}^c$ maintains the impossibility of the judge to assert that a product dominates itself. $\text{FT}(U)$ denotes the min-transitive closure of $U$ [7, p. 56].

We now define $P$ as the subset of the *non-conflicting useful assertions*, which are also *explicit*. The generation of $P$ is clearly based on both $U$ and $V$ sets of assertions. But the *conflicting* assertions of $V$ have to be excluded:

$$V_P = p(V) = \min\{V, V^{-1}\}.$$

From this, we obtain the following definition of $P$:

$$\begin{aligned} P &= \text{FT}(V_P \cap_{\min} U) \\ &= \text{FT}(\min\{V_P, U\}). \end{aligned}$$

To justify the choice of the t-norm, we first need to define the *ideal case* as the case in which the set of *useful* assertions is already consistent, i.e.: $U$ is min-transitive. It implies too:

$$U \min -\text{transitive} \quad \Rightarrow \quad V = \text{FT}(U) = U$$
$$\Rightarrow \quad U = V_P = M_P.$$

In this *ideal case*, we impose our aggregation to be neutral. For $P$, this means:

$$M_P = P \quad \Leftrightarrow \quad M_P = \text{FT}(V_P \cap U)$$
$$\Leftrightarrow \quad M_P = \text{FT}(M_P \cap M_P).$$

This equation is clearly solved by the idempotency [7, p. 16] of the t-norm: now min is the only idempotent t-norm.

### C. Definition of the corrected incomparability $J$

We proceed in the same way to build the fused subrelation of conflict. It is defined as the subset of either *useful* or simply *raw assertions* which are not kept in the fused dominance $P$:

$$
\begin{aligned}
J &= (P \cup_{\max} P^{-1})^c \ldots \\
&\quad \cap_W \left[ (V \cup_{\max} V^{-1}) \cup_{\max} (M \cup_{\max} M^{-1}) \right] \\
&= \max \left\{ \max \left\{ V, V^{-1}, M, M^{-1} \right\} \ldots \\
&\quad - \max\{P, P^{-1}\}, 0 \right\}.
\end{aligned}
$$

To justify necessary t-norms and t-conorms, we first write the definition of $M_J$ in the *ideal case*:

$$
\begin{aligned}
M_J &= \mathbf{I} - (M_P + M_P^{-1} + M_I) \\
&= (M_P \cup_{\max} M_P^{-1})^c \cap_W (M \cup_{\max} M^{-1}) \\
&= (P \cup_{\max} P^{-1})^c \ldots \\
&\quad \cap_W \left[ (V \cup_{\max} V^{-1}) \cup_{\max} (M \cup_{\max} M^{-1}) \right],
\end{aligned}
$$

because in this case: $U = V$ and $U \subseteq M$ implies $V \subseteq M$ (Zadeh's inclusion). All these t-norms and t-conorms are necessary to ensure the neutrality $J = M_J$ in this *ideal case*.

### D. Deduction of the corrected structure $(P, I, J)$

Finally, the new aggregated subrelation of indifference is simply determined thanks to the decomposition equation:

$$I = \mathbf{I} - (\max\{P, P^{-1}\} + J).$$

From now on, we get three complementary subrelations $(P, I, J)$ such that $I$ is symmetric and reflexive, and such that $P$ is min-asymmetric. This last property must still be proved.

$V_P$ is at the same time min-transitive and min-asymmetric [7, p. 83]. For this reason, $U \cap_{\min} V_P$ is included into a min-transitive and min-asymmetric relation; necessarily, its min-transitive closure $P$ satisfies this same property. So $P$ is min-asymmetric (as well as min-transitive).

Therefore, the final aggregated structure $(P, I, J)$ is a fuzzy preference structure whose subrelation $P$ is min-asymmetric. According to the previous proposition, it corresponds to the decomposition of the following aggregated weak preference relation:

$$
\begin{aligned}
S' &= P \cup_W I \\
&= P + I.
\end{aligned}
$$

## V. APPLICATION TO A SENSORY PROFILE DATA

We consider a sensory profile data set provided by the sensory laboratory of *PSA Peugeot-Citroën*. It consists of scores asserted to 8 transparent plastics by 12 judges in the course of 3 screenings. Evaluations were made according to the following visual descriptor: "granular".
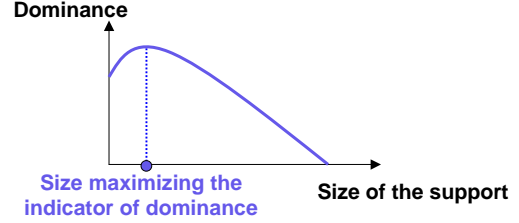


Fig. 2. Typical variation of $Q^P$ as a function of the size of the support of the fuzzy interval

### A. Parameters of the fusion procedures

To parameterize the extraction procedure, we choose to make the size (of the support) of the fuzzy intervals depend only on the judge (i.e. not on the products or on the replicates). We consider that each judge may have its own imprecision in using the scale. Then we apply a principle of maximization of the discriminant information. For each judge, the size of the support is set to the size maximizing its indicator of dominance $Q^P$. Indeed, too small a size results in an overestimation of the conflict: because of the continuous scale, the judge can not exactly assign the same scores to a pair of similar products; for this reason, these scores may be inversed over the replicates, and a too small size would induce a conflict instead of an indifference or even a dominance. Next, too great a size results in an overestimation of the indifference, because of the large overlaps between the fuzzy intervals. In both cases the dominance may be underestimated. Consequently, the higher the indicator of dominance, the more adjusted the size of the support. Figure 2 describes how the indicator of dominance $Q^P$ typically varies with the size of the support: this has been observed in a large number of sensory profile data.

In the aggregation procedures, we choose the second definition $(U_2)$ of the *useful* assertions, in order to minimize the influence of the antagonistic dominances.

The fusion of replicates is processed with an *owa* close to the max ($\rho = 0.2$). The weights of the judges are set proportional to their own indicator $1 - Q^J$.

### B. Fusion of the 3 replicates

We consider here the responses of Judge 1. The variation of its scores is represented in the upper side of Figure 3: for each replicate, the eight products are located on the scale according to their scores; the left and right extremities of the scale respectively correspond to the 0 and 10 values. The size of the support of the fuzzy interval used in the fusion is reported in the middle of the figure by a tiny bar.
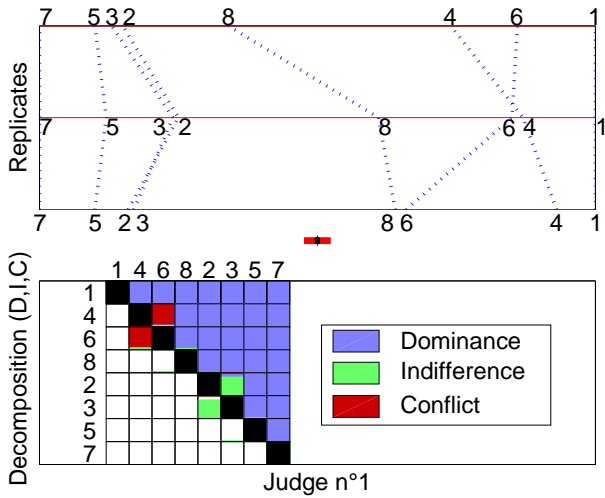
Fig. 3.   Scores and fuzzy aggregated weak dominance relation of Judge 1

We observe a slight dispersion of the values over the replicates, in particular for the products 8, 6 and 4. But according to the fused weak dominance relation drawn in the lower side of the figure (whose representation is described at the end of Section II-B), the judge appears to be coherent and discriminant: the dominance subrelation is the most important one. However, we note two exceptions:

- the pair $(2, 3)$ appears to be indifferent: their scores are very close in all replicates, in spite of their variations;
- the pair $(4, 6)$ is almost totally conflicting: this is due to the inversion of their scores in the first replicate in comparison to the last replicates; the conclusion is the following: knowledge over this pair of products is too poor to assert either a dominance or an indifference for this pair.

In most cases the impact of the correction for transitivity is not discernible. That is why we do not focus on the differences between the raw aggregated weak dominance and its corrected version.
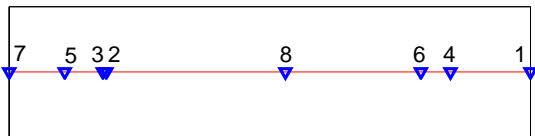


Fig. 4.   Averaged scores of judge 1 over the 3 replicates

The averaged scores of judge 12 are shown in Figure 4. Usual methods based on a probabilistic modelling use these averages. Our main contribution is to manage the notion of conflict. This introduces a second level of non-discrimination, that is opposed to the *indifference*. The indifference underlines the similarity of the scores of the products, while the conflict underlines erroneous assessments (i.e. non ambiguous inversions of dominance over the replicates): neither indifference nor dominance can be decided, so the conflict is interpreted as

a lack of information. This concept makes the interpretation more precise. For example if we only consider the averaged scores, it is impossible to differentiate the pairs $(3, 5)$ and $(4, 6)$. The differences of their scores are indeed very similar, although they reveal two distinct cases: a clear dominance for the pair $(3, 5)$ and a conflict for the pair $(4, 6)$. Moreover, the dominance relation between the products is classically obtained thanks to a multiple comparisons test, like Duncan's method [5]. We remark that such tests are based on a measure of dispersion of the scores over the replicates: the higher the dispersion, the lower the number of ordered pairs of products. Judges with great dispersions may consequently be considered as less discriminant. It is not necessarily the case with our descriptive method, that only focus on the fuzzy ordinal relations between the scores.
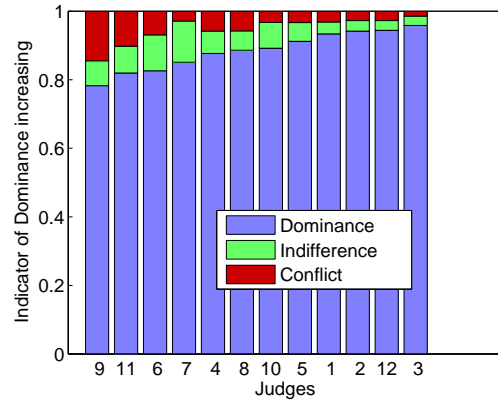


Fig. 5.   Indicators of the 12 judges

The three global indicators of each judge are represented in Figure 5, by increasing order of their indicator of dominance. This result gives a precise differentiation of the judges. We remark that the group composed of judges $\{3, 12, 2, 1\}$ has a great ability to discriminate the products with almost no conflict. However, other judges have a quite good performance.
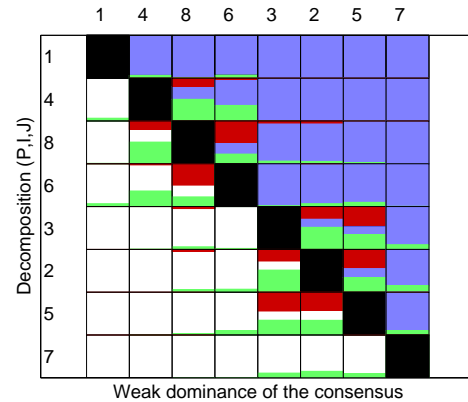
### C. Fusion of the 12 judges



Fig. 6.   Weak dominance of the consensus

The obtained weak dominance is shown in Figure 6. As a relation of consensus, it synthesizes the way the 12 judges globally perceive the set of products. In this particular case, we observe that the notions of indifference and disagreement are very correlated. In summary, we obtain 4 groups of unanimously distinct products:

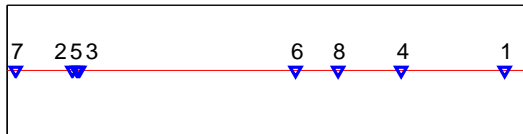$$\{1\} \succ \{4, 8, 6\} \succ \{3, 2, 5\} \succ \{7\}.$$



Fig. 7. Averaged scores over replicates and judges

In Figure 7, we observe that the averages of the scores over the replicates and judges do not exactly give the same information: in particular, the difference between the averaged scores of the pair $(4, 8)$ is greater than the one of the pair $(7, 2)$. However, product 2 unanimously dominates the product 7; and the pair $(4, 8)$ seems inversely not to be distinguished.

## VI. CONCLUSION

A new method has been developed to manage the relevant information of the sensory profile data. Our choice of a compromise between a quantitative approach and a full ordinal approach has led us to use fuzzy weak dominance relations, that proved very convenient. Two similar weighted aggregation procedures were presented to synthesize these relations over the replicates as well as over the judges. The consistency of the obtained relations is stated as a property of transitivity, and it is obtained thanks to procedure consisting in revealing implicit assertions of the judges. The first main contribution of this method is the suitability of the proposed modelling, principally because of the differentiation between *indifference* and *conflict*. The second one certainly consists in its easy interpretation allowed by some simple graphical tools.

Our main perspectives are to process several descriptors, and to develop more detailed tools of interpretation.

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