Output coding of spatially dependent subclassifiers in evidential framework. Application to the diagnosis of railway track/vehicle transmission system.

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Abstract - ***This paper addresses the problem of fault detection in a complex system made up of several spatially dependent subsystems. The diagnosis method consists of both detecting and localizing a defect on the system by combining the outputs scores of subclassifiers within the framework of belief function theory. This paper is focused on the coding and the combination of classifier outputs that can reflect the spatial relationship between the subsystems. In the particular case of upstream/downstream dependency, two strategies of output coding are detailed. The proposed methodology is illustrated on a railway device diagnosis application. It will be shown that the choice of an appropriate coding scheme improves the classification results.***

Keywords: Classification, data fusion, belief functions, diagnosis, neural network, Dempster-Shafer theory.

1 Introduction

The diagnosis of a complex system consists in identifying its working state from one or more measurements. When a pattern recognition approach is adopted, the goal is to assign any measurement signal represented by a feature vector to one of the labelled classes [1]. In the particular case when the system is composed of several subsystems, the diagnosis also involves the isolation of the defective subsystem. We can either choose to build one global classifier or as many classifiers as subsystems. In this case a fusion stage is required to combine the individual classifier outputs. This partitioning approach is interesting because it allows to design each classifier independently (choice of the structure, input space, etc.). In addition, it seems to be suitable in the diagnosis applications involving spatially dependent subsystems where the global approach fails.

Figure 1 shows an example of upstream/downstream spatial dependency: a presence of a defect on one subsystem modifies the information related to downstream subsystems.

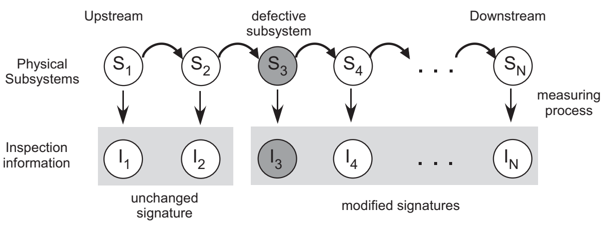

Figure 1: Downstream spatial dependency

When the global diagnosis problem is split into several subproblems, a coding scheme has to be chosen, and classifier outputs have to be combined, taking into account the spatial relationship between the subsystems. The aim is first to assign the working state of each subsystem to a reliable or a defective case and then, to combine the individual classifier outputs to determine the defective subsystem. A myriad of methods for fusing output classifiers have been proposed [7], [5], [6]. The method investigated here is based on the belief function theory in relation with a particular output coding. The interest of the belief function theory is that it makes it possible to handle problems of uncertainty and conflicts between several classifiers.

This paper is organized as follows. First, the basics of belief function theory will be recalled. Then, two coding schemes for the classifier outputs will be presented, and fusion formulas associated to each one of them will be given in the particular case of upstream/downstream dependency. In Section 4, this approach will be applied to the diagnosis of the track/vehicle transmission system called the track circuit, in the railway domain. The performance of the approach will be presented in Section 5, and Section 6 will conclude the paper.

2 Belief Function Theory

In this section, we briefly recall the bases of Belief Function Theory, first introduced by Dempster [4], and formalized by Shafer [8]. The interest of this theory is that it generalizes the probability theory, by introducing an explicit representation of uncertainty. A subjectivist interpretation of Dempster-Shafer theory was proposed by Smets, under the name of Transferable Belief Model (TBM). In the TBM theory, there exists a two-level structure composed of a credal level where beliefs are entertained, and a pignistic level where decisions are made. In this section, we only define the concepts that are used in our diagnosis method. Further details can be found in [2], [10], [11], and [12].

2.1 Representation of Beliefs: the Credal Level

2.1.1 Frame of Discernment

In Dempster-Shafer theory, a problem is represented by a set Θ of mutually exclusive and exhaustive hypotheses θ_i. Θ is called the frame of discernment.

$$\Theta = \{\theta_1, \theta_2, \ldots, \theta_n\} \tag{1}$$

2.1.2 Basic belief assignment (bba)

A Basic Belief Assignment (denoted bba) is a function m from 2^Θ to $[0,1]$ that assigns a value to each conjunction in the frame of discernment:

$$m:\ 2^\Theta \rightarrow [0,1], \tag{2}$$

such that

$$\sum_{A \subseteq \Theta} m(A) = 1 \tag{3}$$

The basic belief mass $m(A)$ represents the measure of the belief that is committed exactly to A, given the available evidence, and that cannot be committed to any strict subset of A because of lack of information. Every $A \subseteq \Theta$ such that $m(A) > 0$ is called a focal proposition. A bba verifying $m(\emptyset) = 0$ is said to be normal. If $m(\emptyset) \neq 0$, $m(\emptyset)$ can be interpreted as the part of belief committed to the assumption that none of the hypotheses in Θ might be true (open-world assumption).

2.1.3 Combination of Several Bbas

To combine several bbas over the same frame of discernment, Smets introduced the conjunctive rule of combination [10]. In order to use it, the different bbas must be based upon distinct pieces of evidence. Let m_1 and m_2 be 2 bbas, the bba that results from their conjunctive combination, denoted $m_1 \textcircled{\cap} m_2$, is defined for all $A \subseteq \Theta$ as:

$$m_1 \textcircled{\cap} m_2(A) = \sum_{B,C \subseteq \Theta : B \cap C = A} m_1(B) m_2(C) \tag{4}$$

Followed by a normalization step, this combination rule is equal to Dempster's rule of combination [4].

2.2 Decision making: pignistic level

In the TBM, when a decision has to be made, the bbas are transformed into probabilities. To do that, we build a pignistic probability function $BetP$ from the bba m, using the pignistic transformation defined as [12],[9]:

$$BetP(A) = \sum_{B \subseteq \Theta} \frac{|A \cap B|}{|B|} \frac{m(B)}{1 - m(\emptyset)}, \quad \forall A \subseteq \Theta, \tag{5}$$

where $|X|$ denotes the cardinality of X

This definition relies on the idea that, in the absence of additional information, $m(A)$ should be equally distributed among the elements of A. This solution is a classical probability measure from which expected criteria can be computed in order to take optimal decisions (for example, the maximum of pignistic probability criterion).

3 Combination of Classifier Outputs

Considering a system Σ composed of N subsystems that are spatially dependent, the global diagnosis problem can be split into N subproblems (N classifiers), each one dedicated to the diagnosis of one subsystem. The elementary classifier outputs are then combined to assign the system to a nominal or defective working state.

In this paper, we consider probabilistic classifiers, whose outputs p_i, are probabilities. The frame of discernment is $\mathcal{Y} = \{1, \ldots, N, N+1\}$ where N is the number of subsystems. Each singleton $\{i\}$, $i = 1, \ldots, N+1$ corresponds to a possible position of the defect. The virtual position $N+1$ corresponds to the absence of defect. In the following sub-sections, we describe two output coding schemes and we detail the fusion formulas obtained for each one of them.

3.1 Output coding 1

In this standard coding, the classifier output related to the defective subsystem is equal to 1 and the other classifier outputs are 0. Let l be the location of the defective subsystem and Z_i the output of the i^{th} classifier ($i \in [1, N]$):

$$Z_i = \begin{cases} 0 & \text{if } i \neq l \\ 1 & \text{if } i = l. \end{cases} \tag{6}$$

3.2 Fusion formulas for coding 1

Considering the coding described above, if the output of the i^{th} classifier is 1, then the defect is on the subsystem number i. The i^{th} classifier gives a bba on each singleton $\{i\}$ and on its complementary subset $\mathcal{Y}\backslash\{i\}$ as follows:

$$\begin{array}{ll} m_i^{\mathcal{Y}}(\{i\}) & = p_i \\ m_i^{\mathcal{Y}}(\mathcal{Y}\backslash\{i\}) & = 1 - p_i \end{array} \tag{7}$$

These bbas are combined through the Dempster conjunctive rule to obtain the bba associated to each singleton and to the empty set $\emptyset$ as follows:

$$\begin{array}{lll} m^{\mathcal{Y}}(\{i\}) & = & p_i \prod_{j \neq i} (1 - p_j) \;\; \forall\, i = 1, \ldots, N \\ m^{\mathcal{Y}}(\{N+1\}) & = & \prod_{i=1}^{N} (1 - p_i) \\ m^{\mathcal{Y}}(\emptyset) & = & 1 - \sum_{i=1}^{N+1} m^{\mathcal{Y}}(\{i\}) \end{array} \tag{8}$$

Example: Let us consider a system with $N = 3$ subsystems. Then, the frame of discernment is $\mathcal{Y} = \{1, 2, 3, 4\}$ and the different bbas are given by:

$$\begin{array}{ll} m_1^{\mathcal{Y}}(\{1\}) & = p_1 \\ m_1^{\mathcal{Y}}(\{2,3,4\}) & = 1 - p_1 \\ & \\ m_2^{\mathcal{Y}}(\{2\}) & = p_2 \\ m_2^{\mathcal{Y}}(\{1,3,4\}) & = 1 - p_2 \\ & \\ m_3^{\mathcal{Y}}(\{3\}) & = p_3 \\ m_3^{\mathcal{Y}}(\{1,2,4\}) & = 1 - p_3. \end{array}$$

Using Dempster's rule of combination, we obtain:

$$\begin{array}{ll} m^{\mathcal{Y}}(\{1\}) & = p_1(1-p_2)(1-p_3) \\ m^{\mathcal{Y}}(\{2\}) & = p_2(1-p_1)(1-p_3) \\ m^{\mathcal{Y}}(\{3\}) & = p_3(1-p_1)(1-p_2) \\ & \\ m^{\mathcal{Y}}(\{4\}) & = (1-p_1)(1-p_2)(1-p_3) \\ & \\ m^{\mathcal{Y}}(\emptyset) & = 1 - \sum_{i=1}^{N+1} m^{\mathcal{Y}}(\{i\}). \end{array}$$

We can notice that, if $p_1 = p_3 = 1$, and $p_2 = 0$, we have a contradiction between classifier 1 and classifier 3. Then, applying the formula, we obtain $m^{\mathcal{Y}}(\{1\}) = m^{\mathcal{Y}}(\{2\}) = m^{\mathcal{Y}}(\{3\}) = 0$ and $m^{\mathcal{Y}}(\emptyset) = 1$, which means that the highest mass after fusion is attributed to uncertainty.

3.3 Output Coding 2

The second coding uses the basic idea that when a defect occurs on one subsystem, it modifies the information obtained for all subsystems located downstream from it. The information related to subsystems located upstream from it are not affected. Let l be the index of the defective subsystem, and Z_i the output of the i^{th} classifier ($i \in [1, N]$):

$$Z_i = \begin{cases} 0 & \text{if } i < l \\ 1 & \text{if } i \geq l. \end{cases} \tag{9}$$

Figure 2 illustrates the two coding schemes.

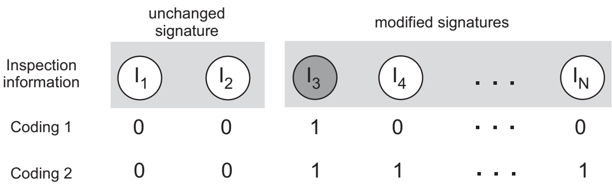

Figure 2: Classifier output codings

Coding 2 is called thermometric code and has been used into neural networks scheme [3]. It belongs to the binary Gray codes family and it is often used in analog-to-digital converters.

3.3.1 Fusion Formulas

Considering N subsystems $S_1, ..., S_N$, this coding means that a desired output equal to 0 corresponds to the fact that there is no defect between S_1 and S_i, whereas a desired output equal to 1 means that a defect is located between S_1 and S_i. So, classifier number i gives a bba on each subset $\{1, \ldots, i\}$ on each subset $\{i+1, \ldots, N+1\}$ as follows:

$$\begin{array}{ll} m_i^{\mathcal{Y}}(\{1, \ldots, i\}) & = p_i \\ m_i^{\mathcal{Y}}(\{i+1, \ldots, N+1\}) & = 1 - p_i \end{array} \tag{10}$$

We can then combine these bbas using the Dempster conjunctive combination:

$$\begin{array}{lll} m^{\mathcal{Y}}(\{i\}) & = & \prod_{j=1}^{i-1} (1 - p_j) \prod_{k=i}^{N} p_k, \;\; i = 1, \ldots, N \\ m^{\mathcal{Y}}(\{N+1\}) & = & \prod_{i=1}^{N} (1 - p_i) \end{array} \tag{11}$$

$$m^{\mathcal{Y}}(\emptyset) = 1 - \left[\left(\sum_{i=1}^{N} \prod_{j=1}^{i-1} (1 - p_j) \prod_{k=i}^{N} p_k \right) + \prod_{i=1}^{N} (1 - p_i) \right]. \tag{12}$$

Example: Let us consider a system with $N = 3$ subsystems. Then, the frame of discernment is $\mathcal{Y} = \{1, 2, 3, 4\}$ and the different bbas are given by:

$$\begin{array}{ll} m_1^{\mathcal{Y}}(\{1\}) & = p_1 \\ m_1^{\mathcal{Y}}(\{2,3,4\}) & = 1 - p_1 \\ & \\ m_2^{\mathcal{Y}}(\{1,2\}) & = p_2 \\ m_2^{\mathcal{Y}}(\{3,4\}) & = 1 - p_2 \\ & \\ m_3^{\mathcal{Y}}(\{1,2,3\}) & = p_3 \\ m_3^{\mathcal{Y}}(\{4\}) & = 1 - p_3 \end{array}$$

$$
\begin{aligned}
m^{\mathcal{Y}}(\{1\}) &= p_1 p_2 p_3 \\
m^{\mathcal{Y}}(\{2\}) &= (1-p_1)p_2 p_3 \\
m^{\mathcal{Y}}(\{3\}) &= (1-p_1)(1-p_2)p_3 \\
m^{\mathcal{Y}}(\{4\}) &= (1-p_1)(1-p_2)(1-p_3) \\
m^{\mathcal{Y}}(\emptyset) &= 1 - \sum_{i=1}^{N+1} m^{\mathcal{Y}}(\{i\}).
\end{aligned}
$$

We can notice that, if $p_1 = p_3 = 1$, and $p_2 = 0$, we have a contradiction between classifier 1 and classifier 3. Then, applying the formula, we obtain $m^{\mathcal{Y}}(\{1\}) = m^{\mathcal{Y}}(\{2\}) = m^{\mathcal{Y}}(\{3\}) = 0$ and $m^{\mathcal{Y}}(\emptyset) = 1$, which means that the highest mass after fusion is attributed to uncertainty.

The decision rule is very simple and corresponds to the maximum pignistic probability, that takes into account a normalization step. The position of the defect is assigned to one of the $N+1$ classes ($N+1$ is used when there is no defect).

4 Railway Application

4.1 Track Circuit Principle

Track circuit is an essential element of automatic train control. Its main function is to detect the presence or absence of a train on a given railway section of track. For the French high speed lines, track circuit is also a fundamental element of track/vehicle transmission system (TVM). It is used to transmit, over a specific carrier frequency, coded data to the train such as the maximum authorized speed on given section with safety constraints.

The railway track is divided into different sections by means of electrical separation joints. A track circuit associated to a specific section, consists of the following components (see Figure 3):

- a transmitter which supplies a FM alternating current;
- the two rails that can be considered as a transmission line;
- a receiver that is connected to the opposite end from the transmitter. It mainly consists of a trap circuit used to avoid the transmission of information to the neighboring section;
- trimming capacitors connected between the two rails at constant spacing to compensate the inductive behaviour of the track. An electric tuning is then achieved that limits the attenuation of the emitted current and improve the transmission level. The number of capacitors depends on the carrier frequency and the length of the track section.

Characteristics of track circuit equipment may change because of aging, atmospheric conditions or track maintenance operations, that induce an unfortunate attenuation of the transmitted signal. If the carrier level becomes too low, the train automatically stops, therefore it is important to detect system dysfunctions as soon as possible in order to maintain it at required safety and availability levels.

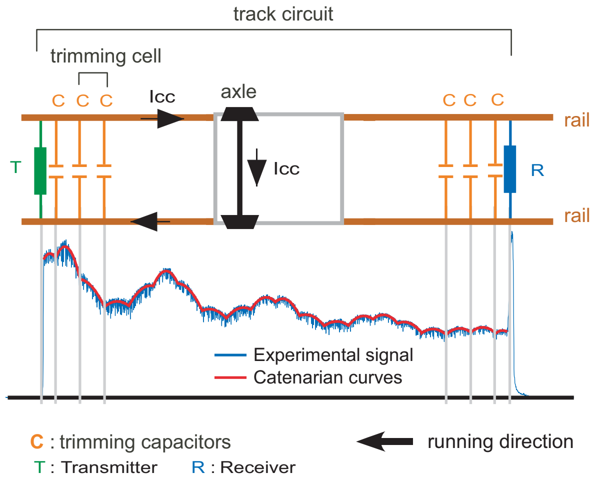

Figure 3: Track circuit representation and inspection signals.

For this purpose, an inspection car is able to deliver a measurement signal linked to electrical track circuit characteristics. Current track circuit inspection techniques use relatively basic decision rules to activate the maintenance procedures. Most of them are based on thresholding of the specific recorded signal. This simple rule enables detection of major defects but it is not suitable to give an accurate diagnosis for predictive maintenance. The improvement of the diagnosis system requires more complex techniques. Here, we present an automatic diagnosis system dedicated to detect and localize defects, especially trimming capacitor defects (removed or resistive capacitors).

4.2 Description of the diagnosis method

Figure 4 shows simulated signals (called I_{cc} signals) in the case of one resistive trimming capacitor.

Different observations can be made:

- each position of a trimming capacitor coincides with a discontinuity of the derivative curve. So, between trimming capacitors the signal can be considered as succession of catenarian curves (local arches) that can be fitted by a second degree polynomials;
- the presence of a defect on the system only affects the signal between the defect and the receiver while it is unchanged upstream.

Hence the idea to consider the track circuit as the system Σ, and each of the N trimming cells as a subsystem S_i. At first, we build one classifier per trimming cell, taking into account the signal from the transmitter up to the capacitor corresponding to the considered catenarian curve. According to the coding we use

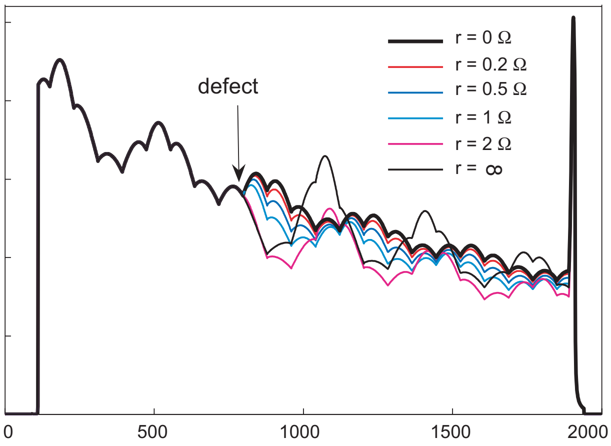

Figure 4: Example of simulated signal obtained when the 9^{th} capacitor is defective

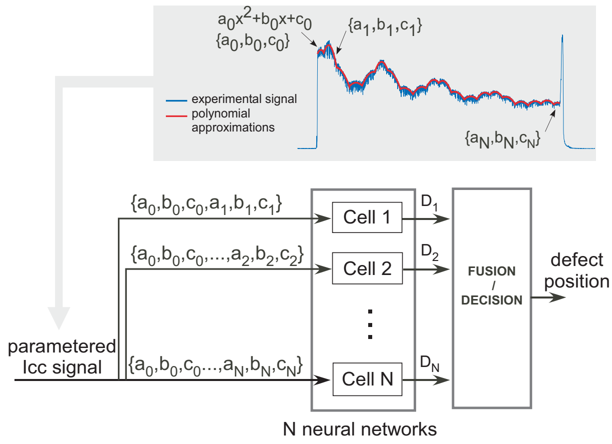

Figure 6: Architecture of the diagnosis system.

as shown in Figure 5, each classifier gives information about the presence of a defect either on this capacitor or between this capacitor and the transmitter. The re-

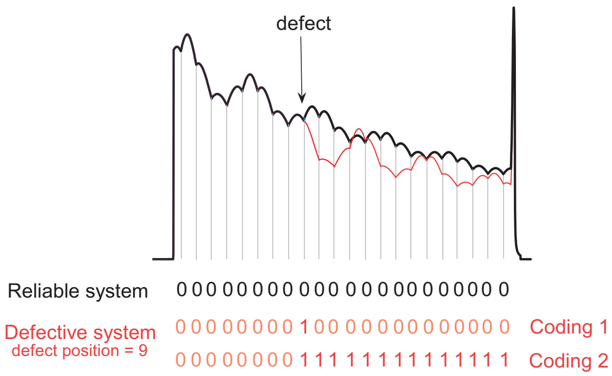

Figure 5: Principle of the two coding schemes for an experimental signal with and without defect.

sponses of all the classifiers are combined within the TBM theory and a decision is made by computing the pignistic probability. In the case of defective working state, this method gives also the position of the defect. Figure 6 presents the principle of the complete diagnosis system.

5 Results and Discussion

To assess the performances of the above approach, we considered a track circuit of $N = 19$ trimming cells, and we built a data base of 4256 simulated noised signals obtained for different values of each capacitor resistance, and for different values of global track parameters. Among these signals, 608 were reliable, and 3648 had one defective capacitor with a resistance between $r = 1\ \Omega$ and $r = \infty$ (removed capacitor).

As shown in Figure 6, a pattern recognition approach was used to design each subclassifier.

Starting with an experimental signal, a parametric representation was obtained by means of polynomial approximations. Each local arch was fitted by a second degree polynomial. The signal was thus described by a total of 60 variables. The inputs of the i^{th} classifier are the $3 \times (i+1)$ parameters corresponding to the $i+1$ catenarian curves between the transmitter and the i^{th} trimming capacitor. Its output gives information on the presence (or not) of a defect on the i^{th} subsystem if the coding 1 is chosen, or between S_1 and S_i in the case of coding 2.

The classification task was then performed by a 2-layer neural network with tan-sigmoïd hidden layer of no more than 7 nodes and one output linear layer of one neuron. The classifier performances were estimated by splitting the whole data base into 3 subsets (training, validation, test).

The analysis of the results was achieved by computing the rates of good detections (GD), false alarms (FA) and non detections (ND) on the whole data set including both defective and reliable signals. When a defect was detected, we distinguished between good localizations ($GL = GD_0 + GD_1$) and false localizations (GD_2). A defect can indeed be well detected but wrongly localized within the system. These definitions are summarized in Table 1.

Table 1: Definition of good detection, false alame and non detection rates.

		Truth	
		Defect position i	No Defect
Decision	Position i	GD_1	FA_1
	Position $\neq i$	GD_2	FA_2
	No Defect	ND	GD_0

If N_0 denotes the number of reliable cases and N_1 the number of defective cases within the database, the different rates are estimated as follows:

$$t_{GD} = \frac{GD_0 + GD_1 + GD_2}{N_0 + N_1}$$
$$t_{GL} = \frac{GD_0 + GD_1}{GD_0 + GD_1 + GD_2}$$
$$t_{ND} = \frac{ND}{N_1}$$
$$t_{FA} = \frac{FA_1 + FA_2}{N_0}.$$

The results are reported in Table 2, as compared to a reference method. This reference method is a regression (R) using a multilayer perceptron with one hidden layer of 7 neurons. The inputs are the 60 parameters of the I_{cc} signals, and the output is the position of the defect. Whatever the coding scheme chosen, the results obtained when fusing local classifiers are much better than those obtained by the regression method. In terms of detection, the good detection rate is significantly improved ,while false alarms almost disappear. *Coding* 1 leads to a higher non detection rate than *coding* 2. This can be explained by the unbalanced number of training instances from each class in the database used to train subclassifiers. Indeed, when using *Coding* 1, each subclassifier learns much more 0s (no defect) than 1s. Moreover, when a defect is detected, we have quite few false localizations for the two codings. In most cases, the localization error is equal to 1, which is satisfactory. Discounting subclassifier outputs before the fusion procedure was not found to bring any significative improvement to these results.

Table 2: Performances of the different coding schemes.

Rates (%)	R	Coding 1	Coding 2
t_{GD}	92.69	94.27	99.15
t_{GL}	63.04	99.8	93.14
t_{FA}	42.18	1.97	0.66
t_{ND}	2.30	6.36	0.88

6 Conclusions

In this paper, the application of the TBM to the diagnosis of a complex system made up by several subsystems with upstream/downstream spatial relationship has been investigated. When the global classification task is performed by combining the individual classifier responses, it has been shown that an appropriate coding scheme and classifier output fusion mechanism allows one to take into account this kind of dependency. This approach was applied to the diagnosis of railway infrastructure components. Good results were obtained on noisy simulated signals, which demonstrates the efficiency of the method. Satisfying tests were also achieved on real signals, but the problem is that very few signals are labelled. Further studies are carried out to compare this TBM approach with other fusion methods, and also to take into account the presence of multiple defects and assess their seriousness, which can be useful in a predictive maintenance context.

References

[1] C.M. Bishop. *Neural Networks for Pattern Recognition.* Oxford university press edition, 1995.

[2] I. Bloch. *Fusion d'informations en traitement du signal et des images.* Hermes science publications edition, Janvier 2003.

[3] D. Bonnet, A. Grumbach, and V. Labouisse. On the parsimony of the multi-layer perceptrons when processing encoded symbolic variables. *Neural Processing Letters*, 8(2):p.145–153, 1998.

[4] A.P. Dempster. A generalization of bayesian inference. *Journal of the Royal Statistical Society*, 30:p.205–247, 1968.

[5] T. Denœux. Analysis of evidence-theoretic decision rules for pattern classification. *Pattern Recognition*, 30(7):1095–1107, 1997.

[6] T. Denoeux, M. Masson, and B. Dubuisson. Advanced pattern recognition techniques for system monitoring and diagnosis: a survey. *Journal Européen des Systèmes Automatisés (RAIRO-APII-JESA)*, 31(9-10):p.1509–1539, 1998.

[7] J. Kittler. Combining classifiers: a theoretical framework. *Pattern Analysis and Applications*, 1:p.18–27, 1998.

[8] G. Shafer. *A Mathematical Theroy of Evidence.* Princeton university press edition, 1976.

[9] Ph. Smets. *Decision making in a context where uncertainty is represented by belief functions.* R.P.Srivastava and T.J. Mock, Ed. Physica-Verlag, Forthcoming.

[10] Ph. Smets. The combination of evidence in the transferable belief model. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol.12(5):p.447–458, 1990.

[11] Ph. Smets and R. Kennes. The transferable belief model. *Artificial Intelligence*, vol.66:p.191–234, 1994.

[12] Ph. Smets and R. Kruse. The transferable belief model for quantified belief representation. In A. Motro and P. Smets, editors, *Uncertainty in Information Systems: From Needs to Solutions*, pages 343–368. Kluwer, Boston, MA., 1997.