Constructing Belief Functions from Qualitative Expert Opinions

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Abstract

A new method for constructing belief functions from elicited expert opinions is proposed. It consists in representing qualitatively expert opinions in terms of preference relations. These relations are transformed into constraints of an optimization problem whose resolution allows the generation of the least informative belief functions according to some uncertainty measures. Mono-objective and Multiobjective optimization techniques are used to optimize, respectively, one or simultaneously different uncertainty measures.

1. Introduction

When dealing with real-world problems, we can rarely avoid uncertainty. In general, uncertainty emerges whenever information pertaining to the situation is deficient in some respect. It may be incomplete, imprecise, contradictory, vague, unreliable, fragmentary, or deficient in some other way [5].

In such situations and especially when data needed for the considered problem are not all available, a way to complement missing information is to use opinions elicitated from experts in the problem domain, i.e., individuals who have special skills in a subject area and are recognized as qualified to address the problem at hand. Expert opinions are statements, based on knowledge and experience, that experts provide in response to a given question [2]. Hence, the elicitation of expert opinions may be defined as the process of collecting and representing expert's knowledge regarding the uncertainties of a problem.

For representing uncertainty, we should use appropriate frameworks such that probability theory, evidence theory or possibility theory. In this paper, we are interested in representing expert opinions in the evidence theory framework and precisely in the context of the Transferable Belief Model (TBM) [11]. In the last twenty years, this theory, also known as theory of belief functions (BFs) or Dempster-Shafer (DS) theory [9], has attracted considerable interest as a rich and flexible framework for representing and reasoning with imperfect information. The concept of BFs subsumes those of probability and possibility measures, making the theory very general. The TBM is a recent variant of DS theory developed by Smets which is considered to be a coherent and axiomatically justified interpretation of BF theory.

For collecting expert opinions, we can proceed quantitatively or qualitatively. In a quantitative manner, we may ask the expert to provide his opinions as numbers according to the uncertainty theory that will be used to represent them. This approach supposes that the expert should be familiar enough with the concepts of the theory framework to be able to correctly quantify his judgments. This is not always obvious. An alternative way is to elicit expert opinions qualitatively. This allows experts to express their opinions in a natural way, while deferring the use of numbers.

Recently, several authors have addressed the problem of eliciting qualitatively expert opinions and generating associated quantitative BFs [14], [3], [8].

In this paper, we propose a new method for constructing BFs from elicited expert opinions. Our method consists in representing qualitatively expert opinions in terms of preference relations that will be transformed into constraints of an optimization problem. The resolution of this problem allows the generation of the least informative BFs according to some uncertainty measures. Mono-objective and Multiobjective optimization techniques are used and different optimization models are proposed and discussed.

The rest of this paper is structured as follows. Section 2 summarizes the basic concepts of the TBM. Several uncertainty measures are then recalled in Section 3. In Section 4, we present some methods addressing the same problem considered in this paper. The new method we propose for constructing BFs from preference relations is presented in Section 5. Section 6 illustrates our method by an example and Section 7 concludes the paper.

2. The Transferable Belief Model

The Transferable Belief Model [11] is a subjective and non probabilistic interpretation of BF theory. The main concepts of the TBM are summarized here. More details can be found in Ref. [11]. The TBM is based on a two-level model: a credal level where beliefs are entertained, combined and updated, and a pignistic level where beliefs are converted into probabilities to make decisions.

2.1. Credal Level

Let Ω denote a finite set called the frame of discernment. A basic belief assignment (bba) or mass function is a function $m : 2^{\Omega} \rightarrow [0, 1]$, verifying:

$$\sum_{A \subseteq \Omega} m(A) = 1. \tag{1}$$

$m(A)$ measures the amount of belief that is exactly committed to A. A bba m such that $m(\emptyset) = 0$ is said to be normal. Notice that this condition is relaxed in the TBM: the allocation of a positive mass to the empty set ($m(\emptyset) > 0$) is interpreted as a consequence of the open-world assumption and can be viewed as the amount of belief allocated to none of the propositions of Ω. A bba verifying this condition is said to be subnormal, or unnormalized. The subsets A of Ω such that $m(A) > 0$ are called focal elements. Let $\mathcal{F}(m) \subseteq 2^{\Omega}$ denote the set of focal elements of a mass function m.

The belief function induced by m is a function bel: $2^{\Omega} \to [0, 1]$, defined as:

$$\text{bel}(A) = \sum_{\emptyset \neq B \subseteq A} m(B), \tag{2}$$

for all $A \subseteq \Omega$. bel(A) represents the amount of support given to A.

The plausibility function associated with a bba m is a function pl: $2^{\Omega} \to [0, 1]$, defined as:

$$\text{pl}(A) = \sum_{\emptyset \neq B \cap A} m(B). \tag{3}$$

pl(A) represents the total amount of potential specific support that could be given to A.

The commonality function associated with a bba m is a function q: $2^{\Omega} \to [0, 1]$, defined as:

$$\text{q}(A) = \sum_{B \supseteq A} m(B), \tag{4}$$

where $A, B \subseteq \Omega$.

Given two bba's m_1 and m_2 defined over the same frame of discernment Ω and induced by two distinct pieces of information, we can combine them using the conjunctive combination rule given by:

$$(m_1 \circledcirc m_2)(A) = \sum_{B \cap C = A} m_1(B) m_2(C), \tag{5}$$

for all $A \subseteq \Omega$.

If q_1, q_2 and $q_1 \circledcirc q_2$ are the commonality functions associated, respectively, to m_1, m_2 and $m_1 \circledcirc m_2$, we have

$$(q_1 \circledcirc q_2)(A) = q_1(A).q_2(A), \tag{6}$$

for all $A \subseteq \Omega$.

2.2. Pignistic Level

When a decision must be made, the beliefs held at the credal level induce a probability measure at the pignistic level. Hence, a transformation from belief functions to probability functions must be done. This transformation is called the pignistic transformation. Let m be a bba defined on Ω, the probability function induced by m at the pignistic level, denoted by BetP and also defined on Ω is given by:

$$\text{BetP}(\omega) = \sum_{A : \omega \in A} \frac{m(A)}{|A|}, \tag{7}$$

for all $\omega \in \Omega$ and where $|A|$ is the number of elements of Ω in A. This probability function can be used in order to make decisions using expected utility theory. Its justification is based on rationality requirements detailed in [11].

3. Uncertainty Measures

In the last twenty years, the question of measuring uncertainty within the theories of reasoning under uncertainty has been investigated [5]. Several measures were proposed to quantify the information content or the degree of uncertainty of a piece of information. In this section we will focus on some of these measures proposed within the theory of evidence.

Klir [5] noticed that in BFs theory two types of uncertainty are expressed, which are nonspecificity or imprecision, and discord or strife. Nonspecificity is connected with sizes (cardinalities) of relevant sets of alternatives while discord expresses conflicts among the various sets of alternatives. Several nonospecificity and conflict measures were proposed. Some of them are presented hereafter. For more details see Ref. [5], [6], [7].

3.1. Nonspecificity measures

Dubois and Prade [5] proposed to measure the nonspecificity of a normal bba by a function N defined as:

$$N(m) = \sum_{A \in \mathcal{F}(m)} m(A) \log_2 |A|. \tag{8}$$

The bba m is all the most imprecise (least informative) that $N(m)$ is large. The minimum ($N(m) = 0$) is obtained when m is a Bayesian BF (focal elements are singletons) and the maximum ($N(m) = \log_2 |\Omega|$) is reached when m is vacuous ($m(\Omega) = 1$). The function N is a generalization of the Hartley function ($H(A) = \log_2 |A|$ where A is a finite set).

Yager [5] also proposed a measure of nonspecificity defined as:

$$J(m) = 1 - \sum_{A \in \mathcal{F}(m)} \frac{m(A)}{|A|}. \tag{9}$$

3.2. Conflict measures

Conflict measures are considered as the generalized counterparts of the Shannon's entropy ($-\sum_{\omega \in \Omega} p(\omega) \log_2 p(\omega)$ where p is a probability measure). Yager, Hohle, and Klir and Ramer [5], [6], [7] defined different conflict measures that may be expressed as follows:

$$Conflict(m) = -\sum_{A \in \mathcal{F}(m)} m(A) \log_2 f(A), \tag{10}$$

where f is, respectively, pl, bel or BetP. These conflict measures are called, respectively, Dissonance (E), Confusion (C) and Discord (D).

Smets [6] proposed a different conflict measure, defined as:

$$I(m) = -\sum_{A \subseteq \Omega} \log_2 q(A). \tag{11}$$

Notice that this measure is not a generalization of the Shannon's entropy and it exists only if $m(\Omega) > 0$. An interesting property of $I(m)$ is that it is additive:

$$I(m_1 \circledcirc m_2) = I(m_1) + I(m_2), \tag{12}$$

which is a consequence of equation (6).

3.3. Composite measures

Different global measures have been, respectively, proposed by Lamata and Moral, Klir and Ramer, Pal, Bezdek and Hemasinha, and Smets [5], [6], [7]. These measures are defined, respectively, as:

$$G_1(m) = E(m) + N(m), \tag{13}$$

$$T(m) = D(m) + N(m), \tag{14}$$

$$H(m) = \sum_{A \in \mathcal{F}(m)} m(A) \log_2 \left(\frac{|A|}{m(A)} \right), \tag{15}$$

$$EP(m) = -\sum_{\omega \in \Omega} \mathrm{BetP}(\omega) \log_2 \mathrm{BetP}(\omega). \tag{16}$$

The interesting feature of $H(m)$ is that it has a unique maximum.

4. Previous Works

Several authors [14], [8], [3] have been interested in eliciting qualitatively expert opinions and generating associated quantitative BFs. In the sequel, some of these works are summarized.

4.1. Wong and Lingras' method

Wong and Lingras [14] proposed a method for generating BFs from qualitative preference relations. The idea behind this method is that given a pair of propositions, an expert can usually express which of the propositions is more likely to be true, or may judge the two propositions equally likely to be true. Hence, two binary relations $\cdot >$ and $\sim$ called, respectively, preference relation and indifference relation were defined on 2^Ω. Given these binary relations, the objective of Wong and Lingras' method is to represent them by a BF, such that:

$$A\cdot > B \Leftrightarrow bel(A) > bel(B), \tag{17}$$

$$A \sim B \Leftrightarrow bel(A) = bel(B), \tag{18}$$

where $A, B \in 2^\Omega$.

Notice that this method does not require that the expert supply the preference relations between all pairs of propositions in $2^\Omega \times 2^\Omega$. In fact, it allows the generation of BFs using *incomplete* qualitative preference relations. The issue is whether this BF exists. It has been shown [13] that this depends on the structure of the preference relation $\cdot >$. In fact, such BF exist when $\cdot >$ satisfies the following axioms:
1) *Asymmetry*: $A \cdot > B \Rightarrow \neg(B \cdot > A)$.
2) *Negative Transitivity*: $\neg(A\cdot > B)$ and $\neg(B\cdot > C) \Rightarrow \neg(A\cdot > C)$.
3) *Dominance*: For all $A, B \in 2^\Omega, A \supseteq B \Rightarrow A\cdot > B$ or $A \sim B$.
4) *Partial monotonicity*: For all $A, B, C \in 2^\Omega$, if $A \supset B$ and $A \cap C \neq 0$, then $A \cdot > B \Rightarrow (A \cup C) \cdot > (B \cup C)$.
5) *Nontriviality*: $\Omega \cdot > \emptyset$.

Since the preference relation $\cdot >$ is asymmetric and negatively transitive, $\cdot >$ is a *weak order* [12], [13]. It should be noted that Axioms 1 and 2 imply that $\cdot >$ is transitive (if $A\cdot > B$ and $B\cdot > C \Rightarrow A\cdot > C$). Given a preference relation $\cdot >$ satisfying axioms 1 and 2, It has also been shown [13] that the binary relation $\sim$ defined by $A \sim B \Leftrightarrow (\neg(A\cdot > B), \neg(B\cdot > A))$ is an *equivalence relation* on 2^Ω, i.e., it is reflexive ($A \sim A$), symmetric (if $A \sim B \Rightarrow B \sim A$) and transitive (if $A \sim B$ and $B \sim C \Rightarrow A \sim C$).

Let $S = \cdot > \cup \sim$, defined on 2^Ω. Notice that since $\cdot >$ is a weak order and $\sim$ is an equivalence relation, S is a *complete preorder* [12].

To generate a BF from such preference relations, Wong and Lingras proceeded in two steps: Determine the focal elements, and Compute the bba. The first step consists in considering that all the propositions that appear in the preference relations are potential focal sets. Then, some of them are eliminated according to the condition: if $A \sim B$ for some $B \subset A$, then A is not a focal element. The second step enables the generation of a bba from the preference relations through the resolution of the system of equalities and inequalities defined by equations (17) and (18) using a perceptron algorithm. It should be noted that several BFs may satisfy this equality and inequality system. However, the perceptron algorithm selects arbitrary only one of them.

Moreover, Wong and Lingras suggest that the expert may provide different levels of preference relations, depending on how much more likely a proposition is compared to another proposition. Their method allows also the use of additional numeric constraints.

It has been noted [3] that this method does not address the issue of inconsistency in the pairwise comparisons. In fact, the expert may provide inconsistent preference relations ($A\cdot > B$, $B\cdot > C$, and $C\cdot > A$).

4.2. Bryson et al.' method

Bryson, *et al.* [3] proposed a method called "Qualitative discrimination process" (QDP) for generating belief functions from qualitative preferences. The QDP was originally developed for multicriteria qualitative scoring and assigning relevant numeric estimates in decision making. Then, it was extended to the problem of generating belief functions for evidential reasoning in expert and intelligent decision support systems.

The QDP is a multi-step process. First, it involves a qualitative scoring step in which the expert assign propositions first into a *Broad* category bucket, then to a corresponding *Intermediate* bucket, and finally to a corresponding *Narrow* category bucket. The qualitative scoring is done using a table where each *Broad* category is a linguistic quantifier in the sense of Parsons [3], [8]. Hence, it allows to the expert to progressively refine the qualitative distinctions in the strength of his beliefs in the propositions. In the second step, the qualitative scoring table from step 1 is used to identify and remove non-focal propositions by determining if the expert is indifferent in his strength of belief of any propositions and their subsets in the same or lower *Narrow* category bucket. It should be noted that this step is consistent with Wong and Lingras' approach presented in the previous section. Step 3 is called "imprecise pairwise comparisons" because the expert is required to provide numeric intervals to express his beliefs on the relative truthfulness of the propositions. In step 4, the consistency of the belief information provided by the expert is checked. Then, the belief function is generated in step 5 by providing a bba interval for each focal element. Finally, in step 6, the expert examines the generated BF and stops the QDP if it is acceptable, otherwise the process is repeated.

It should be noted that the QDP, in spite of being proposed as a qualitative approach for generating BFs from qualitative information, involves numeric intervals to provide them.

5. Constructing Belief Functions from Qualitative Preferences

In this section we propose a new method for constructing BFs from elicited expert opinions expressed in terms of qualitative preference relations. Our method allows the generation of *optimized* BFs in the sense of one or several uncertainty measures.

Expressing expert opinions in terms of qualitative relations as proposed by Wong and Lingras [14] seems to be very attractive. In fact, it is natural and quite easy to make pairwise comparisons between propositions of a frame of discernment modeling a certain problem. Convinced of this motivation, we also propose, in our method, to use the preference and the indifference relations ($\cdot >, \sim$) defined by Wong and Lingras to express expert judgments. We assume also that $\cdot >$ satisfies axioms (1)-(5) introduced in Section 4.1. Given such binary relations, we propose to convert them into constraints of an optimization problem whose resolution allows the generation of optimized BFs.

A crucial step for generating BFs before solving such optimization problem is to determine BF focal elements. We propose to consider that all the propositions existing in the preference and the indifference relations expressed by the expert are potential focal elements. Furthermore, we assume that Ω should always be considered as a potential focal element, which seems to us to be more coherent with BF theory.

5.1. Mono-objective optimization model

Conventionally, a constrained mono-objective optimization problem has the following form:

minimize / maximize ***criterion***
subject to
constraints

The criterion is also called the objective function. In our method, we propose to maximize an uncertainty measure (UM) (or entropy measure) of the BF to be generated. Hence, we generate the *least informative* or the *most uncertain* BFs as it is commanded by the Least Commitment Principle [10], also referred to as the principle of Maximum Uncertainty [5], which play a role similar to the Maximum Entropy principle in Bayesian theory.

The constraints are derived from the expert preferences, as defined in equations (17) and (18). They have the following form:

$$A\cdot > B \Leftrightarrow bel(A) - bel(B) \geq \varepsilon \qquad (19)$$

$$A \sim B \Leftrightarrow -\varepsilon \leq bel(A) - bel(B) \leq \varepsilon \qquad (20)$$

where $\varepsilon > 0$ is considered to be the smallest gap that the expert may discern between the degrees of belief in two propositions A and B. Note that ε is a constant specified by the expert before beginning the optimization process. Consequently, our constrained optimization problem will be formulated as follows:

Model 1

$$\begin{aligned}
&\mathrm{Max}_m\ UM(m)\\
&\text{s.t.}\\
&bel(A) - bel(B) \geq \varepsilon \qquad \forall\ A\cdot > B\\
&bel(A) - bel(B) \leq \varepsilon \qquad \forall\ A \sim B\\
&bel(A) - bel(B) \geq -\varepsilon \qquad \forall\ A \sim B\\
&\textstyle\sum_{A\in\mathcal{F}(m)} m(A) = 1;\ m(A) \geq 0\ \ \forall A \subseteq \Omega;\ m(\emptyset) = 0
\end{aligned}$$

where the first, second and third constraints of Model 1 are derived from equations (19) and (20), which represent the quantitative constraints corresponding to the qualitative preference relations. The fourth constraint ensures that the total amount of masses allocated to the focal elements of the bba is equal to one, the fifth constraint specifies that masses are nonnegative and the last constraint imposes that the bba to be generated must be normalized.

Therefore, considering the problem of generating quantitative BFs from qualitative preference relations as an optimization problem, allows us to integrate the issue of *quality* of the constructed BFs in our method. It should be noted that none of the methods presented in Section 4 address this issue. Furthermore, our method addresses the inconsistency of the preference relations provided by the expert. In fact, if these relations are consistent, then the above constrained optimization problem is feasible. Otherwise no solutions will be found. Thus, the expert may be guided to reformulate his preferences.

In some situations, we can fail to attain a global maximum or also they may be several maxima. In other words, there are several BFs that maximize the optimized UM. Although this model allows the construction of BFs from qualitative preference relations, we consider that having only these preferences constitute too weak information to generate BFs.

5.2. Multiobjective optimization models

As an alternative formulation of the BF generation problem, we propose to use multiobjective optimization techniques [4]. One of the well-known multiobjective methods is goal programming. This model allows to take into account simultaneously several objectives in a problem for choosing the most satisfactory solution within a set of feasible solutions [1].

The idea behind the use of goal programming to formulate our problem is to be able to integrate additional information about the BFs to be generated. We may do this by asking the expert to give besides the preference relations, his certainty degree for the considered problem. Hence, we consider the certainty degree of the expert as a goal to be reached and formulate the problem by the following goal programming model:

Model 2

$$\mathrm{Min}_{m,\delta^+,\delta^-}\ \ (\delta^+ + \delta^-)$$

s.t.
$UM(m) - \delta^+ + \delta^- = G$
$bel(A) - bel(B) \geq \varepsilon \;\; \forall A \cdot > B$
$bel(A) - bel(B) \leq \varepsilon \;\; \forall A \sim B$
$bel(A) - bel(B) \geq -\varepsilon \;\; \forall A \sim B$
$\sum_{A \in \mathcal{F}(m)} m(A) = 1;\ m(A) \geq 0 \;\; \forall A \subseteq \Omega;\ m(\emptyset) = 0$

where δ^+ and δ^- indicate, respectively, positive and negative deviations of the achievement level from aspirated level [1]. This model allows us to restrict the search space to the proximity of the goal G (level of aspiration) associated with the objective (UM). Notice that the expert may provide his certainty degree quantitatively in terms of numbers or qualitatively by selecting in a linguistic scale. It should be noted that the goal G may be attained in several points which means that there are several BFs are solutions of Model 2.

To overcome the problem encountered with the two previous models, we propose to integrate in the objective function of Model 2, the nonspecificity measure. Hence, we optimize simultaneously the two UMs according to the following model:

Model 3
$\text{Min}_{m,\delta^+,\delta^-} \;\; (\delta^+ + \delta^-) - N(m)$
s.t.
$UM(m) - \delta^+ + \delta^- = G$
$bel(A) - bel(B) \geq \varepsilon \;\; \forall A \cdot > B$
$bel(A) - bel(B) \leq \varepsilon \;\; \forall A \sim B$
$bel(A) - bel(B) \geq -\varepsilon \;\; \forall A \sim B$
$\sum_{A \in \mathcal{F}(m)} m(A) = 1;\ m(A) \geq 0 \;\; \forall A \subseteq \Omega;\ m(\emptyset) = 0$
$\delta^+, \delta^- \geq 0$

Solving this model allows us to generate a tradeoff solution. The BF constructed is the *least specific* and the *least informative* BF in the neighborhood of G. Notice that this model may be transformed, depending on the problem at hand, to optimize simultaneously more than two UMs by including additional constraints for these UM and their associated goals.

We may also propose a different goal programming model allowing us to construct BFs while tolerating inconsistency of some preference relations if it is needed. This is done by relaxing, in the formulated problem, the constraints derived from these relations. So, we introduce slack variables in the constraints to be relaxed so that we accept, in some situations, to violate them. Hence, the problem is formulated as follows:

Model 4
$\text{Min} \; \sum_{i=1}^{j} (\delta_i^+ + \delta_i^-) + \eta_{AB} + \varphi_{AB} + \varphi'_{AB}$
s.t.
$UM(m) - \delta_1^+ + \delta_1^- = G_1$
$\vdots$
$UM(m) - \delta_j^+ + \delta_j^- = G_j$
$bel(A) - bel(B) + \eta_{AB} \geq \varepsilon \;\;\; \forall A \cdot > B$
$bel(A) - bel(B) \leq \varepsilon + \varphi_{AB} \;\;\; \forall A \sim B$
$bel(A) - bel(B) + \varphi'_{AB} \geq \varepsilon \;\; \forall A \sim B$
$\sum_{A \in \mathcal{F}(m)} m(A) = 1;\ m(A) \geq 0 \;\;\; \forall A \subseteq \Omega;$
$m(\emptyset) = 0;\ \delta_i^+, \delta_j^-, \eta_{AB}, \varphi_{AB}, \varphi'_{AB} \geq 0;$
$i \in \{1, \cdots, j\}$

Notice that this model allows the optimization of several UMs. N(m) may also be considered in the objective function as it was in Model 3.

6. Example

Let us consider the example proposed by Bryson et al. [3]. The example involves a medical problem in which the observed symptoms from a patient suggest the possibility of five different subsets of the following five diseases: gastric cancer (gc), peptic ulcer (pu), functional disorder (fd), and gallstones (gs). Let $\Omega = \{gc, pu, fd, gs\}$. The propositions of interest are: $P_1 = \{gc\}$, $P_2 = \{gc, pu\}$, $P_3 = \{gc, pu, fd\}$, $P_4 = \{gs, fd\}$ and $P_5 = \{gc, pu, gs\}$.

According to the qualitative scoring done by the medical analyst presented in [3], we can derive the following preference relations:

$$P_5 \cdot > P_1 \quad P_3 \cdot > P_1 \quad P_2 \cdot > P_1$$
$$P_1 \cdot > P_4 \quad P_5 \sim P_3 \quad P_3 \sim P_2$$

Given these preference relations, we use the new method we proposed in the previous section to generate associated BFs. First, we should identify the potential focal elements of the BF. As suggested above, we have $\mathcal{F}(m) = \{P_1, P_2, P_3, P_4, P_5, \Omega\}$. Then, we formulate the optimization problem according to one of the optimization models proposed. Notice that the choice of the optimization model to be used depends on the problem at hand and the expert objectives.

Let us formulate the problem according to Model 1. Assume that $\varepsilon = 0.01$. Suppose that we are maximizing, respectively, the measure of total uncertainty H (eq. 15), the nonspecificity measure N (eq. 8)and the pignistic entropy EP (eq. 16). The resolution of the formulated problems produces, respectively, the bbas m_1 ($UM = H$), m_2 ($UM = N$) and m_3 ($UM = EP$) (see Table 1).

Table 1.Generated BFs

	P_1	P_2	P_3	P_4	P_5	Ω
m_1	**0.160**	**0.220**	**0.010**	**0.151**	**0.020**	**0.439**
bel_1	**0.160**	**0.380**	**0.390**	**0.151**	**0.400**	**1.000**
m_2	**0.010**	**0.010**	0.000	0.000	0.000	**0.980**
bel_2	**0.010**	**0.020**	0.020	0.000	0.020	**1.000**
m_3	**0.010**	**0.010**	0.000	0.000	0.000	**0.980**
bel_3	**0.010**	**0.020**	0.020	0.000	0.020	**1.000**
m_4	**0.160**	**0.219**	**0.010**	**0.150**	**0.020**	**0.441**
bel_4	**0.160**	**0.379**	**0.389**	**0.150**	**0.399**	**1.000**
m_5	**0.247**	**0.242**	**0.010**	**0.237**	**0.020**	**0.244**
bel_5	**0.247**	**0.489**	**0.499**	**0.237**	**0.509**	**1.000**
m_6	0.000	**0.010**	0.000	0.000	0.000	**0.990**
bel_6	0.000	**0.010**	0.001	0.000	0.010	**1.000**
m_W	**0.500**	**0.250**	0.000	**0.250**	0.000	0.000
bel_W	**0.500**	**0.750**	0.750	**0.250**	0.750	1.000
m_B	**0.400**	**0.171**	0.000	**0.286**	**0.143**	0.000
bel_B	**0.400**	**0.571**	0.571	**0.286**	**0.714**	1.000

Notice that, in the three cases, P_3 and P_5 have very small (close to ε) or null masses. So may conclude that they are non-focal elements or elements that do not have a considerable effect for the considered problem. This proves that superfluous propositions do not pose a problem to our method while constructing BFs from preference relations (Wong and Lingras have also done the same remark for their method [13]). This could be considered as a good feature for our method as we do not need to determine the focal elements of the BFs before proceeding to their computation as it was proposed in the two methods presented in Section 4. Consequently, we may say that the quantitative BFs constructed represent "exactly" the qualitative preference relations provided by the expert.

If we use Wong and Lingras' method to generate BFs for this example, P_3 is eliminated from the list of potential focal elements in the first step since we have $P_3 \sim P_2$ and $P_2 \subset P_3$. In the second step, P_5 is also eliminated when simplifying the inequality matrix using the Gaussian elimination technique. This can be explained as a consequence of the transitivity of the indifference relation: $P_5 \sim P_3$ and $P_3 \sim P_2$ $\Rightarrow P_5 \sim P_2$, and as $P_2 \subset P_5$. Let m_W (see Table 1) be the BF generated using Wong and Lingras' method.

Let m_B (see Table 1) be the BF generated by Bryson et al.' [3]. It should be noted that the relation $P_5 \sim P_3 \sim P_2$ exists in the qualitative scoring table they proposed for this example. This relation has been used to conclude that P_3 is not a focal element. However, they do not eliminate P_5 and consider it as a focal element. This seems to us ambiguous because they do not take into account the transitivity of the relation $\sim$. Consequently, positive interval masses was affected to P_5. This shows that this method do not discover superfluous propositions while generating BFs which may be considered as a major drawback.

Let us reformulate this problem, according to the multiobjective models, Model 2 and Model 3, proposed above. Suppose that the certainty degree of the expert is equal to 3.5 ($G = 3.5$). Suppose that UM optimized is H. Assume that $\varepsilon = 0.01$. The resolution of the formulated models produces, respectively, the bbas m_4 with $\delta^- = 0.187$ and m_5 with $\delta^- = 0.325$ (see Table 1). Notice that, in the two problems, the goal is underachieved as $\delta^- > 0$.

Finally, let us formulate the problem according to Model 4. Assume that $\varepsilon = 0.01$.Suppose that the UM optimized is H with $G1 = 2$ and that we accept to relax the first constraint, i.e., bel(P_1)-bel(P_4)$-\delta_2^+ + \delta_2^- = G2$. Let m_6 be the generated BF (see Table 1) such that $\delta_1^+ = 0.707$ and $\delta_2^- = 0.510$.

7. Conclusion

A new method for constructing BFs from elicited expert opinions expressed in terms of qualitative preference relations has been defined. Our method consists in transforming the preference relations provided by the expert into constraints of an optimization problem involving one or several uncertainty measures. Mono-objective and Multiobjective optimization techniques were used to optimize, respectively, one or simultanously different uncertainty measures. Our method allows the construction of the least informative BFs according to some uncertainty measures. Different constrained optimization models were proposed and discussed. Our method had also been illustrated by an example. The BFs constructed was compared with those generated by previous methods adressing the same problem considerd in this paper. Further work is under way to extend our method for combining multi-expert qualitative opinions.

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8. References

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