Elicitation of Expert Opinions for Constructing Belief Functions

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Abstract

This paper presents a method for constructing belief functions from elicited expert opinions expressed in terms of qualitative preference relations. These preferences are transformed into constraints of an optimization problem whose resolution allows the generation of the least informative belief functions according to some uncertainty measures. Mono-objective and Multiobjective optimization techniques are used to optimize one or different uncertainty measures simultaneously.

Keywords: Elicitation, belief functions, preference relations, optimization, uncertainty measures, Dempster-Shafer theory, Evidence theory.

1 Introduction

When dealing with real-world problems, we can rarely avoid uncertainty. In general, uncertainty emerges whenever information pertaining to the situation is deficient in some respect. It may be incomplete, imprecise, contradictory, vague, unreliable, fragmentary, or deficient in some other way [8].

In such situations and especially when data needed for the considered problem are not all available, a way to complement missing information is to use opinions elicitated from experts in the problem domain, i.e., individuals who have special skills in a subject area and are recognized as qualified to address the problem at hand. Expert opinions are statements, based on knowledge and experience, that experts provide in response to a given question [2]. Hence, the elicitation of expert opinions may be defined as the process of collecting and representing expert's knowledge regarding the uncertainties of a problem.

For representing uncertainty, we can use appropriate frameworks such that probability theory, evidence theory or possibility theory. In this paper, we are interested in representing expert opinions in the evidence theory framework and precisely in the context of the Transferable Belief Model (TBM) [14]. In the last twenty years, this theory, also known as theory of belief functions (BFs) or Dempster-Shafer (DS) theory [12], has attracted considerable interest as a rich and flexible framework for representing and reasoning with imperfect information. The concept of BFs subsumes those of probability and possibility measures, making the theory very general. The TBM is a recent variant of DS theory developed by Smets which is considered to be a coherent and axiomatically justified interpretation of BF theory.

For collecting expert opinions, we can proceed quantitatively or qualitatively. In a quantitative manner, we may ask the expert to provide his opinions as numbers according to the uncertainty theory that will be used to represent them. This approach supposes that the expert should be familiar enough with the concepts of the theory framework to be able to correctly quantify his judgments. This is not always obvious. An alternative way is to elicit expert opinions qualitatively. This allows experts to express their opinions in a natural way, while deferring the use of numbers.

Recently, several authors have addressed the problem of eliciting qualitatively expert opinions and generating associated quantitative BFs [16, 4, 11].

In this paper, we propose a new method for constructing BFs from elicited expert opinions. Our method consists in representing qualitatively expert opinions in terms of preference relations that will be transformed into constraints of an optimization problem. The resolution of this problem allows the generation of the least informative BFs according to some uncertainty measures (UMs). Mono-objective and Multiobjective optimization techniques are used and different optimization models are proposed and discussed.

The following Section summarizes the background concepts related to the TBM, uncertainty measures, and the Least Commitment Principle (LCP). In Section 3, we summarize previous work addressing the problem considered in this paper. Section 4 presents the new method proposed for constructing BFs from qualitative expert opinions. The optimization models introduced in this section are illustrated by examples. Section 5 concludes the paper.

2 Background

2.1 The Transferable Belief Model

The Transferable Belief Model [14] is a subjective and non probabilistic interpretation of BF theory. The main concepts of the TBM are summarized here. More details can be found in Ref. [14]. The TBM is based on a two-level model: a credal level where beliefs are entertained, combined and updated, and a pignistic level where beliefs are converted to probabilities to make decisions.

2.1.1 Credal Level

Let Ω denote a finite set called the frame of discernment. A basic belief assignment (bba) or mass function is a function $m : 2^\Omega \to [0,1]$ verifying:

$$\sum_{A\subseteq\Omega} m(A) = 1. \qquad (1)$$

$m(A)$ measures the amount of belief that is exactly committed to A. A bba m such that $m(\emptyset) = 0$ is said to be normal. Notice that this condition is relaxed in the TBM: the allocation of a positive mass to the empty set ($m(\emptyset) > 0$) is interpreted as a consequence of the open-world assumption and can be viewed as the amount of belief allocated to none of the propositions of Ω. A bba verifying this condition is said to be subnormal, or unnormalized. The subsets A of Ω such that $m(A) > 0$ are called focal elements. Let $\mathcal{F}(m) \subseteq 2^\Omega$ denote the set of focal elements of a mass function m.

The belief function induced by m is a function bel: $2^\Omega \to [0,1]$, defined as:

$$\text{bel}(A) = \sum_{\emptyset\neq B\subseteq A} m(B), \qquad (2)$$

for all $A \subseteq \Omega$. bel(A) represents the amount of support given to A.

The plausibility function associated with a bba m is a function pl: $2^\Omega \to [0,1]$ defined as:

$$\text{pl}(A) = \sum_{\emptyset\neq B\cap A} m(B). \qquad (3)$$

pl(A) represents the total amount of potential specific support that could be given to A.

The commonality function associated with a bba m is a function q: $2^\Omega \to [0,1]$ defined as:

$$\text{q}(A) = \sum_{B\supseteq A} m(B), \qquad (4)$$

where $A, B \subseteq \Omega$.

2.1.2 Pignistic Level

At this level, beliefs are used to make decisions. When a decision must be made, the beliefs held at the credal level induce a probability measure at the pignistic level. Hence, a transformation from belief functions to probability functions must be done. This transformation is called the pignistic transformation. Let m be a bba defined on Ω, the probability function induced by m at the pignistic level, denoted by BetP and also defined on Ω is given by:

$$\mathrm{BetP}(\omega) = \sum_{A:\omega\in A} \frac{m(A)}{|A|}, \tag{5}$$

for all $\omega \in \Omega$ and where $|A|$ is the number of elements of Ω in A. This probability function can be used in order to make decisions using expected utility theory. Its justification is based on rationality requirements detailed in [14].

2.2 Uncertainty Measures

Several measures have been proposed to quantify the information content or the degree of uncertainty of a piece of information [8]. In this section we will focus on some of these measures proposed within the theory of evidence. For more details see Ref. [8, 9, 10].

Klir [8] noticed that in BFs theory two types of uncertainty can be expressed: nonspecificity or imprecision, and discord or strife. Nonspecificity is connected with sizes (cardinalities) of relevant sets of alternatives while discord expresses conflicts among the various sets of alternatives. Composite measures, referred to as global or total measures of uncertainty, have also been proposed. They attempt to capture both nonspecificity and conflict.

2.2.1 Nonspecificity measures

Dubois and Prade [8] proposed to measure the nonspecificity of a normal bba by a function N defined as:

$$N(m) = \sum_{A\in\mathcal{F}(m)} m(A)\log_2|A|. \tag{6}$$

The bba m is all the most imprecise (least informative) that $N(m)$ is large. The minimum ($N(m) = 0$) is obtained when m is a Bayesian BF (focal elements are singletons) and the maximum ($N(m) = \log_2|\Omega|$) is reached when m is vacuous ($m(\Omega) = 1$). The function N is a generalization of the Hartley function ($H(A) = \log_2|A|$ where A is a finite set).

Yager [8] proposed another measure of nonspecificity defined as:

$$J(m) = 1 - \sum_{A\in\mathcal{F}(m)} \frac{m(A)}{|A|}. \tag{7}$$

2.2.2 Conflict measures

Conflict measures are considered as the generalized counterparts of the Shannon's entropy ($-\sum_{\omega\in\Omega} p(\omega)\log_2 p(\omega)$ where p is a probability measure). Yager, Hohle, and Klir and Ramer [8, 9, 10] defined different conflict measures that may be expressed as follows:

$$Conflict(m) = -\sum_{A\in\mathcal{F}(m)} m(A)\log_2 f(A), \tag{8}$$

where f is, respectively, pl, bel or BetP. These conflict measures are called, respectively, Dissonance (E), Confusion (C) and Discord (D).

Smets [9] proposed a different conflict measure, defined as:

$$I(m) = -\sum_{A\subseteq\Omega} \log_2 q(A). \tag{9}$$

Notice that this measure is not a generalization of the Shannon's entropy and it exists only if $m(\Omega) > 0$.

George and Pal [7], proposed a measure of total conflict which is a generalization of Vajda's quadratic entropy, given by:

$$TC(m) = \sum_{A,B\in\mathcal{F}(m)} m(A)m(B)CON(A,B), \tag{10}$$

where $CON(A,B) = 1 - \frac{|A\cap B|}{|A\cup B|}$ represents the conflict between propositions A and B.

2.2.3 Composite measures

The following uncertainty measures, have been proposed, respectively, by Lamata and Moral, Klir and Ramer, Pal, Bezdek and Hemasinha, and Smets [8, 9, 10]. They are called, respectively, global uncertainty (G1), total uncertainty (T, H), and pignistic entropy (EP). These measures are defined, respectively, as:

$$G_1(m) = E(m) + N(m), \tag{11}$$

$$T(m) = D(m) + N(m), \tag{12}$$

$$EP(m) = -\sum_{\omega\in\Omega} \text{BetP}(\omega)\log_2 \text{BetP}(\omega), \tag{13}$$

$$H(m) = \sum_{A\in\mathcal{F}(m)} m(A)\log_2\left(\frac{|A|}{m(A)}\right). \tag{14}$$

The interesting feature of $H(m)$ is that it has a unique maximum.

2.3 Least Commitment principle

The Principle of Least Commitment [6] plays a central role in the TBM. It formalizes the idea that one should never presuppose more beliefs than justified. In fact, given a family of BFs compatible with a set of constraints, depending on how their "information content" is compared, the LCP indicates that the most appropriate is the *least committed.* Dubois and Prade [6] have made three proposals to order BFs according to their informational content: pl-ordering, q-ordering, and s-ordering [6]. In this paper, however, we propose to apply the LCP using UMs for comparing BFs informational contents. In fact, Klir [8] have pointed out that the degree of uncertainty of a piece of information is intimately connected to its information content and have proposed an equivalent uncertainty principle to the LCP referred to as the principle of maximum uncertainty. These principles play a role similar to the Maximum Entropy principle in Bayesian theory.

3 Previous Works

Several authors [16, 11, 4] have been interested in eliciting qualitatively expert opinions and generating associated quantitative BFs. In the sequel, some of these works are summarized.

3.1 Wong and Lingras' method

Wong and Lingras [16] proposed a method for generating BFs from qualitative preference relations. To express expert opinions, they defined two binary relations $\cdot >$ and $\sim$ defined on 2^Ω and called, respectively, preference relation and indifference relation . The objective of Wong and Lingras' method is to represent these preference relations by a BF, such that:

$$A\cdot > B \Leftrightarrow bel(A) > bel(B), \tag{15}$$

$$A \sim B \Leftrightarrow bel(A) = bel(B). \tag{16}$$

where $A, B \in 2^\Omega$.

Note that this method does not require that the expert supply the preference relations between all pairs of propositions in $2^\Omega \times 2^\Omega$. In fact, it allows the generation of BFs using *incomplete* qualitative preference relations. It has been shown that the existence of such BF depends on the structure of the preference relation $\cdot >$ that should satisfies the following axioms:

1) *Asymmetry*: $A\cdot > B \Rightarrow \neg(B\cdot > A)$.

2) *Negative Transitivity*: $\neg(A\cdot > B)$ and $\neg(B\cdot > C) \Rightarrow \neg(A\cdot > C)$.

3) *Dominance*: For all $A, B \in 2^\Omega, A \supseteq B \Rightarrow A\cdot > B$ or $A \sim B$.

4) *Partial monotonicity*: For all $A, B, C \in 2^\Omega$, if $A \supset B$ and $A \cap C \neq 0$, then $A\cdot > B \Rightarrow (A \cup C)\cdot > (B \cup C)$.

5) *Nontriviality*: $\Omega \cdot > \emptyset$.

Since the preference relation $\cdot >$ is asymmetric and negatively transitive, $\cdot >$ is a *weak order* [15]. It should be noted that Axioms 1 and 2 imply that $\cdot >$ is transitive (if $A\cdot > B$ and $B\cdot > C \Rightarrow A\cdot > C$). Given a preference relation $\cdot >$ satisfying axioms 1 and 2, it has also been shown that the binary relation $\sim$ defined by $A \sim B \Leftrightarrow (\neg(A\cdot > B), \neg(B\cdot > A))$ is an *equivalence relation* on 2^Ω, i.e., it is reflexive ($A \sim A$), symmetric (if $A \sim B \Rightarrow B \sim A$) and transitive (if $A \sim B$ and $B \sim C \Rightarrow A \sim C$). Let $S = \cdot > \cup \sim$, defined on 2^Ω. Notice that since $\cdot >$ is a weak order and $\sim$ is an equivalence relation, S is a *complete preorder* [15].

To generate a BF from such preference relations, Wong and Lingras proceeded in two steps: Determine the focal elements, and Compute the bba. The first step consists in considering that all the propositions that appear in the preference relations are potential focal sets. Then, some of them are eliminated according to the following condition: if $A \sim B$ for some $B \subset A$, then A is not a focal element. The second step enables the generation of a bba from the preference relations through the resolution of the system of equalities and inequalities defined by equations (15) and (16) using a perceptron algorithm. It should be noted that several BFs may be solutions of this system. However, the perceptron algorithm selects arbitrary only one of them.

It has been noted [4] that this method does not address the issue of inconsistency in the pairwise comparisons. In fact, the expert may provide inconsistent preference relations ($A\cdot > B$, $B\cdot > C$, and $C\cdot > A$).

3.2 Bryson et al.' method

Bryson, *et al.* [4] proposed a method called "Qualitative discrimination process" (QDP) for generating belief functions from qualitative preferences. The QDP is a multi-step process. First, it involves a qualitative scoring step in which the expert assign propositions first into a *Broad* category bucket, then to a corresponding *Intermediate* bucket, and finally to a corresponding *Narrow* category bucket. The qualitative scoring is done using a table where each *Broad* category is a linguistic quantifier in the sense of Parsons [4, 11]. Hence, it allows to the expert to progressively refine the qualitative distinctions in the strength of his beliefs in the propositions. In the second step, the qualitative scoring table from step 1 is used to identify and remove non-focal propositions by determining if the expert is indifferent in his strength of belief of any propositions and their subsets in the same or lower *Narrow* category bucket. It should be noted that this step is consistent with Wong and Lingras' approach presented in the previous section. Step 3 is called "imprecise pairwise comparisons" because the expert is required to provide numeric intervals to express his beliefs on the relative truthfulness of the propositions. In step 4, the consistency of the belief information provided by the expert is checked. Then, the belief function is generated in step 5 by providing a bba interval for each focal element. Finally, in step 6, the expert examines the generated BF and stops the QDP if it is acceptable, otherwise the process is repeated.

It should be noted that the QDP, in spite of being proposed as a qualitative approach for generating BFs from qualitative information, involves numeric intervals in the elicitation process.

4 Constructing Belief Functions from Qualitative Preferences

In this section we propose a new method for constructing BFs from qualitative expert opinions expressed in terms of preference relations. Our method allows the generation of *optimized* BFs in the sense of one or several UMs. We first present the main ideas behind our method, then we propose the optimization techniques and models used for deriving BFs along with illustrative examples. We also point out the main differences between our method and those presented in the previous section.

4.1 Main Ideas

Expressing expert opinions in terms of qualitative relations as proposed by Wong and Lingras [16] seems to be very attractive. In fact, it is natural and quite easy to make pairwise comparisons between propositions of a frame of discernment modeling a certain problem. Convinced of this motivation, we also propose to use the preference and indifference relations $(\cdot >, \sim)$ defined by Wong and Lingras to express expert judgments in our method. We assume that the preference relation satisfies axioms (1)-(5) introduced in Section 3.1. Given such binary relations, we propose to convert them into constraints of an optimization problem whose resolution allows the generation of optimized BFs.

A crucial step for generating BFs before solving such optimization problem is to determine BF focal elements. We propose to consider that all the propositions existing in the preference and the indifference relations expressed by the expert are potential focal elements. Furthermore, we assume that Ω should always be considered as a potential focal element, which seems to us to be more coherent with BF theory.

Conventionally, a constrained optimization problem has the following form:

minimize / maximize ***criterion***
subject to
constraints

The criterion is also called the objective function. In our method, we propose to maximize an UM (entropy measure) of the BF to be generated. Hence, we generate the *least informative* or the *most uncertain* BFs as it is commanded by the LCP recalled in Section 2.3. The constraints are derived from the expert preferences, as defined in equations (15) and (16). They have the following form:

$$A\cdot > B \Leftrightarrow bel(A) - bel(B) \geq \varepsilon \quad (17)$$

$$A \sim B \Leftrightarrow |bel(A) - bel(B)| \leq \varepsilon \quad (18)$$

where $\varepsilon > 0$ is considered to be the smallest gap that the expert may discern between the degrees of belief in two propositions A and B. Note that ε is a constant specified by the expert before beginning the optimization process. Consequently, our constrained optimization problem will be formulated as follows:

Model 1
$\text{Max}_m \, UM(m)$
s.t.
$bel(A) - bel(B) \geq \varepsilon \quad \forall\, A\cdot > B$
$bel(A) - bel(B) \leq \varepsilon \quad \forall\, A \sim B$
$bel(A) - bel(B) \geq -\varepsilon \quad \forall\, A \sim B$
$\sum_{A \in \mathcal{F}(m)} m(A) = 1;\ m(A) \geq 0\ \forall A \subseteq \Omega;$
$m(\emptyset) = 0$

where the first, second and third constraints of Model 1 are derived from equations (17) and (18), which represent the quantitative constraints corresponding to the qualitative preference relations.

Therefore, considering the problem of generating quantitative BFs from qualitative preference relations as an optimization problem, allows us to integrate the issue of *quality* of the constructed BFs in our method. It should be noted that none of the methods presented in Section 3 address this issue. Furthermore, our method addresses the inconsistency of the preference relations provided by the expert. In fact, if these relations are consistent, then Model 1 is feasible. Otherwise no solutions will be found. Thus, the expert may be guided to reformulate his preferences.

Besides Model 1, we propose several others constrained optimization models to formulate the problem of generating BFs from preference relations. Several optimization techniques are also used to solve them. In the sequel, these models are presented and discussed.

4.2 Mono-objective optimization model

The mono-objective optimization model we use is that formulated in Model 1. This model allows the construction of BFs that maximize one UM.

Example 1 *Let $\Omega = \{a, b\}$ be a frame of discernment and let $\{a\} \cdot > \{b\}$ be the preference relation given by an expert. To construct a BF from this preference relation, we first define the potential focal elements of the BF, as proposed in Section 4.1. Hence, $\mathcal{F}(m_1) = \{\{a\}, \{b\}, \Omega\}$. Then, we propose to optimize the composite UM proposed by Pal and Bezdek given by equation (14). Let us consider Model 1 to formulate this problem. Assume that $\varepsilon = 0.01$. Notice that $H(m)$ is a non-linear function so we use non-linear programming to solve this problem. The obtained solution is a bba m_1 such that: $m_1(\{a\}) = 0.260$, $m_1(\{b\}) = 0.240$ and $m_1(\Omega) = 0.5$. Given the previous statements, Wong and Lingras' method generates the bba m_2 such that: $m_2(\{a\}) = 0.667$, $m_2(\{b\}) = 0.333$.*

Suppose that we want to construct a BF optimizing the nonspecificity measure given by equation (6). The obtained solution is a bba

m_3 such that: $m_3(\{a\}) = 0.01$, $m_3(\Omega) = 0.99$. If we optimize the pignistic entropy given by equation (13), then we generate m_4 such that: $m_4(\{a\}) = 0.25$, $m_4(\{b\}) = 0.24$, $m_4(\Omega) = 0.51$.

All the bbas generated for Example 1 are represented in Figure 1. Each BF is represented as a point in an equilateral triangle using barycentric coordinates. The lower left corner, the lower right corner, and the upper corner correspond, respectively, to $\{a\}$, $\{b\}$, and Ω. The orthogonal distance to the lower side of the triangle is thus proportional to $m(\Omega)$, while the distances to the right-hand and the left-hand sides are, respectively, proportional to $m(\{a\})$ and $m(\{b\})$.

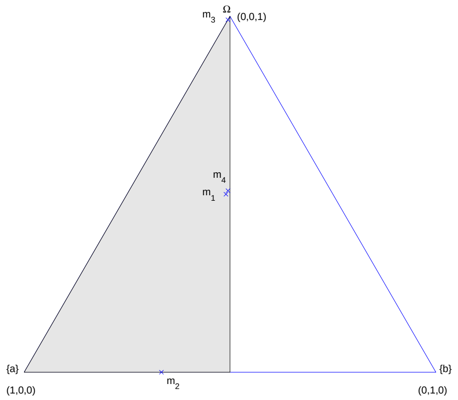

Figure 1: Example 1

Notice that these bbas are all close to the frontier of the feasible solution domain (in gray) which seems to us to be not very satisfactory solutions. Moreover, in some situations, we can fail to attain a global maximum or also there may be several maxima. In other words, there are several BFs that maximize the optimized UM. Although this model allows the construction of BFs from qualitative preference relations, we consider that having only these preferences constitute too weak information to generate BFs.

4.3 Multiobjective optimization models

As an alternative formulation of the BF generation problem, we propose to use multiobjective optimization techniques [5]. One of the well-known multiobjective methods is goal programming. This model allows to take into account simultaneously several objectives in a problem for choosing the most satisfactory solution within a set of feasible solutions [1].

The idea behind the use of goal programming to formulate our problem is to be able to integrate additional information about the BFs to be generated. We can do this by asking the expert to give besides the preference relations, his certainty degree for the considered problem. Hence, we consider this additional information as a goal to be reached and formulate the problem by the following goal programming model:

Model 2
$\text{Min}_{m,\delta^+,\delta^-} \quad (\delta^+ + \delta^-)$
s.t.
$UM(m) - \delta^+ + \delta^- = G$
$bel(A) - bel(B) \geq \varepsilon \;\; \forall A \cdot > B$
$bel(A) - bel(B) \leq \varepsilon \;\; \forall A \sim B$
$bel(A) - bel(B) \geq -\varepsilon \;\; \forall A \sim B$
$\sum_{A \in \mathcal{F}(m)} m(A) = 1; \; m(A) \geq 0 \;\; \forall A \subseteq \Omega;$
$m(\emptyset) = 0$

where δ^+ and δ^- indicate, respectively, positive and negative deviations of the achievement level from aspirated level [1]. This model allows us to restrict the search space to the proximity of the goal G (level of aspiration) associated with the objective (UM). Notice that the expert may provide his certainty degree quantitatively in terms of numbers or qualitatively by selecting in a linguistic scale.

Example 2 *Let us consider the setting of example 1. Suppose that we are optimizing $H(m)$ and that the degree of certainty of the expert is equal to 75%. the resolution of this problem, Formulated according to Model 2, allows the generation of a bba m such that: $m(\{a\}) = 0.06$, $m(\{b\}) = 0.05$ $m(\Omega) = 0.89$. Figure 2 represents this solution and the isolines of the H(m) function.*

It should be noted that the goal G may be attained in several points. In fact, Figure 2 shows that any bba such that $H(m)$ are points of the isoline of H equal to G are solutions of Model 2. Notice that Model 2 may

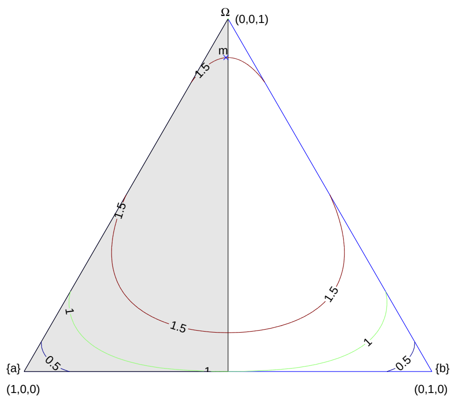

Figure 2: Example 2

be transformed, depending on the problem at hand, to optimize simultaneously several UMs by including additional constraints for these UMs and their associated goals. The objective function may also be weighted so that to express different importance for the different objectives.

To overcome the problem encountered with the two previous models, we propose to integrate in the objective function of Model 2, the nonspecificity measure. So, the model is transformed as follows:

Model 3
$\text{Min}_{m,\delta^+,\delta^-} \ (\delta^+ + \delta^-) - N(m)$
s.t.
$UM(m) - \delta^+ + \delta^- = G$
$bel(A) - bel(B) \geq \varepsilon \ \ \forall \ A \cdot > B$
$bel(A) - bel(B) \leq \varepsilon \ \ \forall \ A \sim B$
$bel(A) - bel(B) \geq -\varepsilon \ \ \forall \ A \sim B$
$\sum_{A \in \mathcal{F}(m)} m(A) = 1; \ m(A) \geq 0 \ \forall A \subseteq \Omega;$
$m(\emptyset) = 0; \ \delta^+, \delta^- \geq 0$

Solving this model allows us to generate a tradeoff solution. In fact, the BF constructed is the *least specific* and the *least informative* BF in the neighborhood of G. As the nonspecificity measure is maximized in the objective function of Model 3, a natural question to be asked here is which UM should be considered in the constraint? This UM should represent the certainty degree of the expert behind the considered problem, as explained above. We propose to represent this information by a composite UM because it captures the whole uncertainty of the problem. As there are several composite UM, we have analyzed and compared solutions optimizing Model 3 using different composite UM from that introduced in Section 2.2.3. We noticed that the Pignistic entropy, given by equation (13), allows to generate satisfactory solutions. Consequently, we propose to use it to represent the certainty degree of the expert.

We may also propose a different goal programming model allowing us to construct BFs while tolerating inconsistency of some preference relations if it is needed. This is done by relaxing, in the formulated problem, the constraints derived from these relations. So, we introduce slack variables in the constraints to be relaxed so that we accept, in some situations, to violate them. Hence, the problem is formulated as follows:

Model 4
$\text{Min} \ \sum_{i=1}^{j} (\delta_i^+ + \delta_i^-) + \eta_{AB} + \varphi_{AB} + \varphi'_{AB}$
s.t.
$UM(m) - \delta_1^+ + \delta_1^- = G_1$
$\vdots$
$UM(m) - \delta_j^+ + \delta_j^- = G_j$
$bel(A) - bel(B) + \eta_{AB} \geq \varepsilon \ \ \forall \ A \cdot > B$
$bel(A) - bel(B) \leq \varepsilon + \varphi_{AB} \ \ \forall \ A \sim B$
$bel(A) - bel(B) + \varphi'_{AB} \geq \varepsilon \ \forall \ A \sim B$
$\sum_{A \in \mathcal{F}(m)} m(A) = 1; \ m(A) \geq 0 \ \ \forall A \subseteq \Omega;$
$m(\emptyset) = 0; \ \delta_i^+, \delta_j^-, \eta_{AB}, \varphi_{AB}, \varphi'_{AB} \geq 0;$
$i \in \{1, \cdots, j\}$

Notice that this model allows the optimization of several UMs. N(m) may also be considered in the objective function as it was in Model 3.

5 Conclusion

A new method for constructing BFs from elicited expert opinions expressed in terms of qualitative preference relations has been defined. It consists in transforming the preference relations provided by the expert into constraints of an optimization problem involving one or several uncertainty measures. Mono-objective and multiobjective optimization techniques were used to solve such constrained optimization problem. The BFs generated are the least informative ones. Further work is under way to extend our method to combine multi-expert qualitative opinions.

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References

[1] Aouni, B., Ben Abdelaziz F. and Martel J.M. (2003). Decision-maker's preferences modeling in the stochastic goal programming. European Journal of Operational Research (EJOR), volume 162, pages 610-618.

[2] Ayyub, B.M. (2001). Elicitation of Expert Opinions for Uncertainty and Risks. CRC Press, Florida.

[3] Ben Yaghlane, A., Denoeux T. and Mellouli K. (2006). Constructing Belief Functions from Qualitative Expert Opinions. Proceeding of ICTTA'06, Damascus, Syria.

[4] Bryson, N. and Mobolurin A. (1999). A process for generating quantitative belief functions. EJOR, volume 115, pages 624-633.

[5] Collette, Y. and Siarry P. (2003). Multiobjective Optimization. Springer-Verlag, Germany.

[6] Dubois D., Prade H. and Smets Ph. (2001). New Semantics for Quantitative Possibility Theory. 2nd International Symposium on Imprecise Probabilities and Their Applications, Ithaca, New York.

[7] George, T. and Pal N. (1996). Quantification of conflict in Dempster-Shafer framework: A new approach. International Journal of General Systems, volume 24, issue 4, pages 407-423.

[8] Klir, G.J. and Wierman M.J. (1998). Uncertainty-Based Information: Elements of Generalized Information Theory. Heidelberg; New York: Physica-Verlag.

[9] Pal, N., Bezdek J. and Hemasinha R. (1992). Uncertainty Measures for Evidential Reasoning I: A Review. International Journal of Approximate Reasoning (IJAR), volume 7, issues 3-4, pages 165-183.

[10] Pal, N., Bezdek J. and Hemasinha R. (1993). Uncertainty Measures for Evidential Reasoning II: New measure of total Uncertainty. IJAR, volume 8, issue 1, pages 1-16.

[11] Parsons, S. (1994). Some qualitative approaches to applying the Dempster-Shafer theory. Information and Decision Technologies, volume 19, pages 321-337.

[12] Shafer, G. (1976). A Mathematical Theory of Evidence. Princeton University Press.

[13] Smets, Ph. (1993). Belief functions: the disjunctive rule of combination and the generalized Bayesian theorem. International Journal of Approximate Reasoning, volume 9, pages 1-35.

[14] Smets, Ph. and Kennes R. (1994). The Transferable Belief Model. Artificial Intelligence, volume 66, pages 191-234.

[15] Vincke, Ph. (2001). Preferences and Numbers. In Colorni A., Paruccini M., and Roy B., editors, *A-MCD-A: Multiple Criteria Decision Aiding*, pages 343-354.

[16] Wong, S.K.M. and Lingras P.J. (1994). Representation of Qualitative User Preference by Quantitative Belief Functions. IEEE Transactions on Knowledge and Data Engineering, volume 6, issue 1, pages 72-78.