# Map Matching algorithm using interval analysis and Dempster-Shafer theory

Ghalia Nassreddine, Fahed Abdallah, Thierry Denœux

***Abstract*— The goal of map matching methods is to compute an accurate position of a vehicle from an initial estimated position using a digital road network data. In this paper, a new map matching method based on Dempster-Shafer theory and interval analysis is presented. The core idea of this method is the use of Dempster-Shafer theory for modeling partial information on model and measurement uncertainties and for managing multiple hypothesis situations. This technique proves to be relevant to treat junction roads situations or parallel roads. The results on simulated data show the usefulness of the proposed method.**

**Keywords: Map matching, Dempster-Shafer theory, data fusion, state estimation, interval analysis, multiple hypothesis technique.**

## I. Introduction

Vehicle navigation has become a major focus of many applications. In the vehicle navigation system, a device is generally used to determine the geometric position of the vehicle. The most common geometric positioning devices used for land vehicle navigation are deduced reckoning (DR) motion sensors, global navigation satellite systems such as the global positioning system (GPS), and integrated navigation systems such as the integrated of GPS/DR system. The integrated GPS/DR system combines GPS data with DR data using a data fusion method to compute a more accurate estimation of the vehicle position. The common data fusion method used in such system is the Extended Kalman Filter (EKF). Often, classical data fusion methods using stochastic filters like EKF are strongly affected by measurement errors like bias and drift, or even by conflicts between the sources of information. Moreover, these approaches require the specification of accurate state space and measurement error models, which may be difficult in real applications. Sometimes, it may be more convenient to use a *bounded error approach* (BEE) based on interval computation and only assume the model and measurement errors to be bounded [6][7]. The major implementation problem of the BEE method is to determine correctly the bounds of the noises. Indeed, if these bounds are underestimated, the contractor may lead to no solution. On the contrary, if the bounds are overestimated, the estimated boxes can be very large (the estimates are then very pessimistic)[7]. In [1], a new particle filter method for state estimation where particles are no longer precise data, but interval vectors also referred to as *boxes* is presented. In this method box particles are propagated and combined using interval tools and probability theory. The main advantage of this approach is that it requires a significantly smaller set of particles than the normal PF algorithm, thereby reducing the computational cost. Unfortunately, GPS and the integrated GPS/DR system fail to provide the actual location of a vehicle on a given road segment. This is due to the various error sources that affect such systems. The availability of higher accuracy digital spatial road network data should make it possible to account for these errors and allow the actual vehicle position on a given road to be determined. This technique is often called map matching (MM). A formal definition of MM can be found in [5] [14]. A number of different algorithms have been proposed for map matching in different applications. The multiple hypothesis technique (MHT) keeps track of several positions of the vehicle simultaneously and selects eventually which candidate is the best. Recently, some works used the multi-hypothesis technique in MM algorithms in order to manage some situations of Map Matching problems like junction situations and parallel roads [10][8]. In [8], a belief map matching method, noted BMM, is presented. The main idea of this method is the use of a bound error method for combining sensors data and belief function theory for managing some situations of Map matching problems like junction situations. In this paper, we propose a new MM method based on interval analysis and Dempster-Shafer (DS) theory. It uses a new data fusion strategy, taking advantage of DS theory by using mass functions which assign a belief masses to a *finite* number of focal sets, chosen to be axis-aligned boxes. Such mass functions can be seen as “generalized boxes” composed of a collection of boxes with associated weights. Focal sets are propagated in the system equations using tools from interval arithmetics and constraint satisfaction techniques [6]. This is associated with a given map data, under the DS framework, in order to select a set of candidate roads. The best candidate road is eventually chosen using a decision rule of DS theory [3].

This paper is organized as follows. In Sections II and III we present the background on interval analysis and DS theory, respectively. The proposed map matching method, is introduced in Section IV. In Section V, we show the results of the application of the proposed method to dynamic vehicle localization. Finally, in Section VI, we conclude and discuss the main contributions of the paper.

## II. Interval analysis

In this section we briefly introduce some notions of interval analysis [6] [1]. A real interval, denoted $[x]$, is defined

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[1]A more detailed background on some interval analysis tools used in this paper can be found on http://www.hds.utc.fr/∼fabdalla

as a closed and connected subset of $\mathbb{R}$: $[x] = [\underline{x}, \overline{x}] = \{x \in \mathbb{R}/\underline{x} \leq x \leq \overline{x}\}$, where $\underline{x}$ and $\overline{x}$ are the lower and upper bound of $[x]$. Set-theoretic operations such as intersection or union can be applied to intervals. The four classical arithmetic operations can be extended to intervals. For any such binary operator $\diamond \in \{+, -, *, \backslash\}$, the interval $[x] \diamond [y]$ is defined for any intervals $[x]$ and $[y]$ as $[x] \diamond [y] = [\{x \diamond y \in \mathbb{R} | x \in [x], y \in [y]\}]$.

A box $[\mathbf{x}]$ of $\mathbb{R}^{n_x}$ is defined as a Cartesian product of $n_x$ intervals: $[\mathbf{x}] = [x_1] \times [x_2] \cdots \times [x_{n_x}] = \times_{i=1}^{n_x} [x_i]$. The set of $n$-dimensional interval real vectors will be denoted $\mathbb{IR}^n$. The notions recalled above may be easily extended to boxes. In general, the image of a box $[\mathbf{x}] \in \mathbb{R}^n$ by a function $\mathbf{f}$ is not a box. An inclusion function $[\mathbf{f}]$ defined as: $\forall [\mathbf{x}] \in \mathbb{R}^n, \mathbf{f}([\mathbf{x}]) \subset [\mathbf{f}]([\mathbf{x}])$, computes a box containing $\mathbf{f}([\mathbf{x}])$. This function should be calculated such that the box enclosing $\mathbf{f}([\mathbf{x}])$ is optimal. Different algorithms exist in order to reduce the size of boxes enclosing $\mathbf{f}([\mathbf{x}])$. In this paper we use the *Waltz algorithm* [6] which is based on the propagation of primitive constraints[2]. This method is independent of the non linearity of the constraints and give accurate results when the system present a great redundancy of data and equations [7] [8] [6].

## III. Dempster-Shafer theory

In this section, we introduce the main concepts of DS theory. Let $\Omega$ denote a finite set of mutually exclusive and exhaustive hypotheses, called the frame of discernment. A *belief structure* (BS) is a mass function $m$ from $2^\Omega$ to $[0,1]$, verifying: $\sum_{A \subseteq \Omega} m(A) = 1$. Every subset $A$ of $\Omega$ such that $m(A) > 0$ is called a *focal element* of $m$. We note $\mathcal{F}(m)$ the set of all focal elements of $m$. A BS $m$ such that $m(\emptyset) = 0$ is said to be normal. A categorical mass function is a mass function that satisfies: $m(A) = 1$ for some $A \subset \Omega$, $A \neq \Omega$ and $m(B) = 0, \ \forall B \subseteq \Omega,$ and $B \neq A$. The belief function on $\Omega$ which have $m(\Omega) = 1$ is the vacuous belief function. In the following, all BSs will be assumed to be normal, unless otherwise specified. In most presentations of DS theory, $\Omega$ is assumed to be finite. However, the theory remains basically unchanged if $\Omega$ is infinite (even uncountable), as long as the number of focal sets remains finite. If $\Omega = \mathbb{R}$, the focal sets are usually assumed to be intervals [9]. In the multidimensional case where $\Omega = \mathbb{R}^n$, this approach can be extended by assuming focal sets to be $n$-dimensional boxes.

Assume that a source of information provides a mass function m, and we have a degree of confidence $1-\alpha \in [0,1]$ in the reliability of that source. Then, $m$ can be discounted by a factor $\alpha$, resulting in the following discounted mass function [12]:

$$^{\alpha}m(A) = \begin{cases} \alpha \, m(A) & \text{if } A \subset \Omega, \\ 1 - \alpha(1 - m(\Omega)) & \text{if } A = \Omega. \end{cases}$$

Two different, and independent mass functions $m_1$ and $m_2$ defined on the same frame of discernment $\Omega$ can be

[2]A primitive constraint is a constraint involving a single operator (such as $+, -, *$ or $\backslash$) or a single function (such as $\cos$, $\sin$ or $\sinh$).

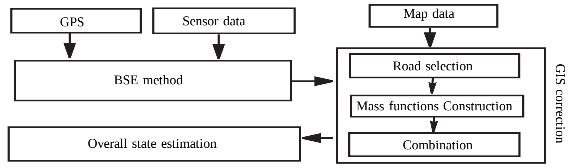


Fig. 1. The basic steps of the BSMM algorithm.

combined by the *conjunctive rule* [13] defined as:

$$\forall A \subseteq \Omega, m_{12}(A) = m_1 \cap m_2(A) = \sum_{B \cap C = A} m_1(B) m_2(C). \tag{1}$$

The conjunctive combination followed by a normalization step is known as *Dempster's rule* of combination [13]. It is denoted by $\oplus$.

Let $m$ be a mass function on $\Omega$ after combining all available items of evidence. Assume that we have to select an element of $\Omega$. In the DS theory, different rules of decision have been proposed. We could select the element with maximum belief, highest plausibility or highest *pignistic probability* [13]. For a mass function $m$, the pignistic probability function, noted $betp$, is given by

$$betp(\omega) = \sum_{\{A \subseteq \Omega | \omega \in A\}} \frac{m(A)}{(1 - m(\emptyset))|A|}, \quad \forall \omega \in \Omega, \tag{2}$$

where $|A|$ is the cardinality of $A$. The pignistic probability function is thus obtained from $m$ by distributing equally each normalized mass $m(A)/(1 - m(\emptyset))$ among the elements of $A$.

Let us now consider the case where we have two variables $X$ et $Y$ defined on frames of discernment $\Omega$ and $\Theta$. Assume that $X$ and $Y$ are linked by a multi-valued mapping $\rho : \Omega \to 2^\Theta$, such that if $X = \omega$, then we know that $Y \in \rho(\omega)$. This mapping can be extended to $2^\Omega$ as follows:

$$\rho(A) = \begin{cases} \bigcup_{\omega \in A} \rho(\omega), & \text{if } A \subseteq \Omega, A \neq \emptyset, \\ 0 & \text{if } A = \emptyset. \end{cases} \tag{3}$$

Let us further assume that we have a mass function $m^\Omega$ on $\Omega$ representing our state of knowledge about $X$. A mass function $m^\Theta$ on $\Theta$ can be built by transferring each mass $m^\Omega(A)$ to $\rho(A)$ [4]. Formally, $m^\Theta$ is then defined as follows:

$$m^\Theta(B) = \sum_{\{A \subseteq \Omega | \rho(A) = B\}} m^\Omega(A), \quad \forall B \subseteq \Theta.$$

This may also be noted

$$m^\Theta(B) = \sum_{A \subseteq \Omega} M_\rho(B, A) m^\Omega(A), \quad \forall B \subseteq \Theta, \tag{4}$$

with

$$M_\rho(B, A) = \begin{cases} 1 & \text{if } B = \rho(A), \\ 0 & \text{otherwise.} \end{cases}$$

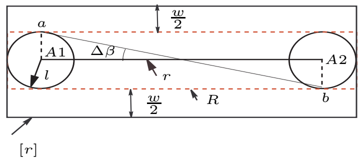

Fig. 2. Rectangular road $[r]$ constructed using map data, geometrical errors and the road width. The road $r$ is represented by two node points $(A1, A2)$.

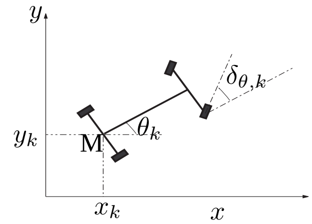

Fig. 3. Definition of the frames.

## IV. Belief State Map Matching Method

### A. Introduction

The goal of MM method is to compute an accurate estimation of the vehicle position using a digital map data. This is done by selecting first the road on which the vehicle is moved then by computing an accurate position of the vehicle on the selected road. In this paper we introduce a new MM method called *Belief State Map Matching* (BSMM), taking advantage of interval analysis and DS theory. Figure 1, shows the main steps of the BSMM method. As shown in this figure, GPS data and dead reckoning sensor data are integrated via a belief state estimation strategy (BSE), where all variables are represented by general mass functions with interval focal elements. Such mass functions can be seen as *generalized boxes* composed of a collection of boxes with associated masses. In the BSE, interval focal elements are propagated via the state space model using interval tools and a resulting mass function on the state is deduced. From the resulting state mass function, a set of candidate roads (CRs) will be computed from an existing two dimensional geographical information system (GIS-2D). By handling interval knowledge of the state mass function and rectangular representations of the roads, a new "on-road" mass function can be computed. Finally, using a decision rule of DS theory, an estimation of the vehicle position can be computed. We will first describe, in section IV-B, the geometry of the available rectangular road map. The state model of the vehicle used in this paper is presented in section IV-C. In section IV-D, the construction of mass functions will be presented. The scenarios of the BSMM method will be shown in section IV-E.

### B. Road map representation

There are several ways to represent digital spatial road network data. In this paper, we use the planar model in which a road $r$ is represented by a finite sequence of points $(A^0, A^1, \cdots, A^{nA})$, where $\{A^i\}_{i=1}^{nA} \in \mathbb{R}^2$. The points $A^0$ and $A^{nA}$ are referred to nodes while $(A^1, A^2, \cdots, A^{nA-1})$ are referred to vertices or shape points. As done in [8], rectangular roads will be constructed from GIS data as shown in Figure 2. By considering the positional error, we assume that the node points of road $r$ can be anywhere within a circle of radius $l$. The shape error of $r$ is thus represented by $\Delta\beta$. As a result, the road $r$ can be considered anywhere within a rectangle $R$ as shown in Figure 2. The rectangular road representation $[r]$ of road $r$ can be constructed by adjusting the width of $R$ using the predefined road width $w$.

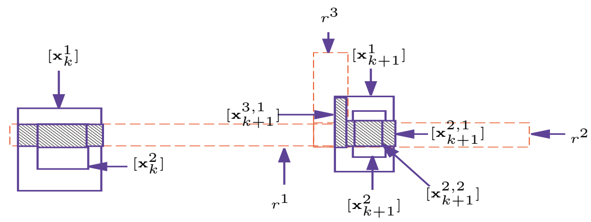

Fig. 4. Example of state mass function $m^{\mathbf{x}_k}$ with two CRs $r^1$ and $r^2$.

### C. Dynamic State Space Model

Consider a car-like vehicle with front-wheel drive. The vehicle position is represented by the Cartesian coordinates $(x_k, y_k)$ of the point M attached to the center of the rear axle as shown in Figure 3. The $x-$axis is aligned with the longitudinal axis of the car. The heading angle is denoted $\theta_k$. The state $\mathbf{x}_k = (x_k, y_k, \theta_k)^T$ is calculated at each time step $k$ using the following discrete representation:

$$\begin{cases} x_{k+1} = & x_k + \delta_{S,k}\cos(\theta_k + \frac{\delta_{\theta,k}}{2}) \\ y_{k+1} = & y_k + \delta_{S,k}\sin(\theta_k + \frac{\delta_{\theta,k}}{2}) \\ \theta_{k+1} = & \theta_k + \delta_{\theta,k}, \end{cases} \tag{5}$$

where $\delta_{S,k}$ is the elementary linear displacement and $\delta_{\theta,k}$ is the measure of the elementary rotation given by an ABS sensor and a gyrometer, respectively. The observation of the position at time step $k$, $\mathbf{z}_k = (x_{GPS,k}, y_{GPS,k})$, is given by a Global Position System (GPS). The *longitude*, *latitude* estimated point of the GPS is converted to a Cartesian local frame and the error boundary of the position characterized by $\sigma_x$ and $\sigma_y$, is obtained thanks to the weaves *GST NMEA* [1].

### D. Mass Function Construction

*1) Road map mass functions:* Given a state mass function $m^{\mathbf{x}_{k+1}}$ and a rectangular road map, a set $R_{k+1}$ of CRs can be selected such that any rectangular road $[r]$ in $R_{k+1}$ verifies: $\exists i / [\mathbf{x}^i_{k+1}] \cap [r] \neq \emptyset$, where $[\mathbf{x}^i_{k+1}]$ is a box focal elements of $m^{\mathbf{x}_{k+1}}$. Thus, the associated mass function at time step $k+1$, noted $m^{R_{k+1}}$, can be computed by exploring conjointly $m^{\mathbf{x}_{k+1}}$, topology criterion and similarity criterion. Hereafter we will explain different computation that may be involved in the calculation of $m^{R_{k+1}}$.

- Topology criterion: using the topology of the map and a mass function $m^{R_k}$, a new mass function on $R_{k+1}$, noted $m_1^{R_{k+1}}$ can be computed. Let $r^i$ be a road of $R_k$. Suppose that the vehicle leaves road $r^i$ at time step $k+1$

and let $S^i$ be the set of roads directly linked to $r^i$ and which have an intersection with the focal elements of $m^{\mathbf{x}_{k+1}}$. Let $\rho$ be a multi valued function defined from $R_k$ to $2^{R_{k+1}}$ by:

$$\rho(r^i) = S^i \quad (6)$$

where $\rho(r^i)$ denotes the possible roads at time $k+1$, given that the vehicle was on road $r^i$ at time step $k$. Using $\rho$ and equation (4), $m_1^{R_{k+1}}$ can be computed.

**Example 1** *Consider the case of Figure 4. At time step $k$, the state mass function $m^{\mathbf{x}_k}$ is represented by two focal elements $[\mathbf{x}_k^1]$ and $[\mathbf{x}_k^2]$. This gives rise to $m^{\mathbf{x}_{k+1}}$, at time step $k+1$, with two focal elements $[\mathbf{x}_{k+1}^1]$ and $[\mathbf{x}_{k+1}^2]$. From $m^{\mathbf{x}_{k+1}}$, two CRs, $r^2$ and $r^3$, can be selected and thus $R_{k+1} = \{r^2, r^3\}$. From the fact that road $r^1$ is linked to $r^2$ and $r^3$, a mass function $m_1^{R_{k+1}}$ can be computed from $m^{R_k}$ using the following multi-valued function $\rho$ defined from $R_k$ to $2^{R_{k+1}}$ such that $\rho(r^1) = \{r^2, r^3\}$. The mass function $m_1^{R_{k+1}}$ is then given by: $m_1^{R_{k+1}}(\{r^2, r^3\}) = m^{R_k}(\{r^1\}) = 1$.*

A more detailed example on the construction of a mass function on $R_{k+1}$ using the topology of the map, can be found in [8].

- Similarity criterion: Using a measure of similarity between the rectangular roads in $R_{k+1}$ and the state mass function, a mass function $m_2^{R_{k+1}}$ on $R_{k+1}$ can be calculated. This similarity is characterized by the area of the intersection between the state boxes and the rectangular roads. For each CR $r^i$, a geometrical likelihood $L^i$ of the road given a state mass function can be computed: $L^i = max_j \frac{|[\mathbf{x}_{k+1}^{i,j}]|}{|[\mathbf{x}_{k+1}^{j}]|}$, where $[\mathbf{x}_{k+1}^j]$ is the $j^{th}$ focal elements of $m^{\mathbf{x}_{k+1}}$ and $[\mathbf{x}_{k+1}^{i,j}]$ is the minimal box englobing $[\mathbf{x}_{k+1}^j] \cap [r^i]$. Using $L^i$, a mass function $m_i$ can be computed as follows [2]:

$$\begin{cases} m_i(\{r^i\}) = & 0 \\ m_i(\overline{\{r^i\}}) = & \alpha_i(1-L^i) \\ m_i(R_{k+1}) = & 1-\alpha_i(1-L^i) \end{cases} \quad (7)$$

where $\overline{\{r^i\}}$ is the complement of $\{r^i\}$ in $R_{k+1}$ and $\alpha_i$ is a discounting coefficient associated with road $r^i$. The mass function $m_2^{R_{k+1}}$ is the combination of all $m_i$ using:

$$m_2^{R_{k+1}} = \circledcirc_i m_i. \quad (8)$$

- Exploring $m^{\mathbf{x}_{k+1}}$: using the state mass function, a mass function on $R_{k+1}$, noted $m_s^{R_{k+1}}$, can be computed. Let $\rho_s$ be a multi valued function defined from $\mathbb{IR}^2$ to $2^{R_{k+1}}$ by

$$\rho_s([\mathbf{x}_{k+1}^i]) = \{r^j/(r^j \in R_{k+1} \text{ and } [r^j] \cap [\mathbf{x}_{k+1}^i] \neq \emptyset)\} \quad (9)$$

Using $\rho_s$, $m^{\mathbf{x}_{k+1}}$ and equation (4), $m_s^{R_{k+1}}$ can be computed. $m_s^{R_{k+1}}$ quantifies the part of belief on $R_{k+1}$ given by the state, independently of the topology and similarity criteria.

**Example 2** *Consider again the case of Figure 4. As it can be seen in Figure 4 the associated multi-valued function $\rho_s$ is given by: $\rho_s([\mathbf{x}_{k+1}^1]) = \{r^2, r^3\}$ and $\rho_s([\mathbf{x}_{k+1}^2]) = \{r^2\}$. Thereby, $m_s^{R_{k+1}}$ is given by: $m_s^{R_{k+1}}(\{r^2\}) = m^{\mathbf{x}_{k+1}}([\mathbf{x}_{k+1}^2])$ and $m_s^{R_{k+1}}(\{r^2, r^3\}) = m^{\mathbf{x}_{k+1}}([\mathbf{x}_{k+1}^1])$.*

The final mass function $m^{R_{k+1}}$ can be computed using the conjunctive rule of combinations:

$$m^{R_{k+1}} = m_1^{R_{k+1}} \circledcirc m_2^{R_{k+1}} \circledcirc m_s^{R_{k+1}}. \quad (10)$$

*2) State mass functions:* For each CR, $r^i \in R_{k+1}$ a state mass function $m_i^{\mathbf{x}_{k+1}}$ can be computed using $m^{\mathbf{x}_{k+1}}$ and a multi valued function $\rho^i$: $\mathbb{IR}^2 \to \mathbb{IR}^2$

$$\rho^i([\mathbf{x}_{k+1}^j]) = [\mathbf{x}_{k+1}^{i,j}] \quad (11)$$

where $[\mathbf{x}_{k+1}^{i,j}]$ is the minimal box englobing $[\mathbf{x}_{k+1}^j] \cap [r^i]$. It is considered as the $j^{th}$ focal element of $m_i^{\mathbf{x}_{k+1}}$. Thereby, according to (4) $m_i^{\mathbf{x}_{k+1}}([\mathbf{x}_{k+1}^{i,j}]) = m^{\mathbf{x}_{k+1}}([\mathbf{x}_{k+1}^j])$.

**Example 3** *Consider the case of Figure 4. As shown in this figure, $R_{k+1} = \{r^2, 2^3\}$, thereby, two state mass functions $m_2^{\mathbf{x}_{k+1}}$ and $m_3^{\mathbf{x}_{k+1}}$ should be computed using (11) and (4): $m_2^{\mathbf{x}_{k+1}}([\mathbf{x}_{k+1}^{2,1}]) = m^{\mathbf{x}_{k+1}}([\mathbf{x}_{k+1}^1])$, $m_2^{\mathbf{x}_{k+1}}([\mathbf{x}_{k+1}^{2,2}] = m^{\mathbf{x}_{k+1}}([\mathbf{x}_{k+1}^2])$ and $m_3^{\mathbf{x}_{k+1}}([\mathbf{x}_{k+1}^{3,1}]) = 1$, where $[\mathbf{x}_{k+1}^{i,j}]$ is the minimal box englobing $[\mathbf{x}_{k+1}^j] \cap [r^i]$.*

### E. Sketch of the BMM method

*1) Initialization:* At time step $k = 0$, a mass function $m^{\mathbf{x}_0}$ on the state can be constructed with $p$ focal elements. A possible solution is to use a triangular possibility distribution around the GPS measurement thanks to standard deviations $\sigma_x$ and $\sigma_y$ estimated in real time by the GPS receiver. The $\alpha-$cuts of this distribution are interval focal elements of $m^{\mathbf{x}_0}$. From $m^{\mathbf{x}_0}$ and the rectangular road map, a set $R_0$ of CRs is selected. Let $n_{R_0}$ be the cardinal of $R_0$. As there is no prior information on the vehicle position at time step $k = 0$, $m_1^{R_0}$ should be initialized as a vacuous mass function on $R_0$. The mass function $m_2^{R_0}$ is calculated using (7) and (8). $m_s^{R_0}$ can be calculated using (11) and (4). The final mass function $m^{R_0}$ is thus the result of the combination of $m_1^{R_0}$ , $m_2^{R_0}$ and $m_s^{R_0}$ according to (10). For each CR $r^i \in R_0$, a state mass function $m_i^{\mathbf{x}_0}$ is calculated as explained in IV-D.2.

*2) Prediction :* Mass functions $m_k^{\mathbf{S}}$ and $m_k^{\Theta}$, on $\delta_{s,k}$ and $\delta_{\theta,k}$, can be constructed using a triangular possibility distribution, built around ABS sensor and gyrometer respectively. Note that $\sigma_s$ and $\sigma_\theta$ are estimated thanks to specific static tests. According to system (5), the mass functions $m_i^{\mathbf{x}_k}$, $m_k^{\mathbf{S}}$ and $m_k^{\Theta}$ can be combined in order to give rise to a predicted mass function $m_i^{\mathbf{x}_{k+1/k}}$ with general focal element $[\mathbf{x}_{k+1/k}^{i,j}]$. Note here that, as inclusion functions are used, we may obtain non-optimized predicted focal elements.

*3) GPS correction:* Based on triangular possibility distribution, a mass function $m_{k+1}^{\mathbf{z}}$ may be constructed at time step $k+1$ on the GPS measurement vector $\mathbf{z}_{k+1}$. $m^{\mathbf{z}_{k+1}}$ and $m_i^{\mathbf{x}_{k+1/k}}$ are then combined conjunctively and give rise to

$m_i^{\mathbf{x}_{k+1}}$. The *Waltz algorithm* may be used here in order to contract the non-optimized focal elements. Note that the belief mass assigned to $[\mathbf{x}^i_{k+1}]$ will be the product of the masses assigned to $[\mathbf{x}_k^{i,j}]$, $[\delta_{\mathbf{s},\mathbf{k}}{}^j]$, $[\delta_{\theta,\mathbf{k}}{}^j]$ and $[\mathbf{z}^j_{k+1}]$ [15]. Thereby, $m_i^{\mathbf{x}_{k+1}}([\mathbf{x}^{i,j}_{k+1}]) = m_i^{\mathbf{x}_k}([\mathbf{x}^{i,j}_k]) \cdot m_k^{\mathbf{S}}([\delta_{\mathbf{s},\mathbf{k}}{}^j]) \cdot m_k^{\Theta}([\delta_{\theta,\mathbf{k}}{}^j]) \cdot m^{\mathbf{z}}_{k+1}([\mathbf{z}^j_{k+1}])$.

**Algorithm 1** Belief State Map Matching algorithm

$k \leftarrow 0$ and from the GPS measurement create a state mass function $m^{\mathbf{x}_0}$ with $p$ focal elements.
Construct rectangular roads $[r^i]$
Compute $R_k = \{r^i/[r^i] \cap [\mathbf{x}^j_k] \neq \emptyset\}$ for $j = 1 \cdots p$
$m_1^{R_k} \leftarrow$ vacuous mass function on $R_k$
Construct $m_2^{R_k}$ using (7) and (8)
Compute $m_s^{R_{k+1}}$ using (9) and (4)
$m^{R_{k+1}} \leftarrow m_1^{R_{k+1}} \textcircled{\cap} m_2^{R_{k+1}} \textcircled{\cap} m_s^{R_{k+1}}$
Compute $\{m_i^{\mathbf{x}_k}\}_{i=1}^{n_{R_k}}$ using (11) and (4)
**loop**
  Build $m_k^{\mathbf{S}}$, $m_k^{\Theta}$ and $m^{\mathbf{z}_{k+1}}$ with $p$ focal elements from sensors data
  **for** $i = 1$ to $n_{R_k}$ **do**
    **for** $j = 1$ to $p$ **do**
      Calculate $[\mathbf{x}^{i,j}_{k+1/k}]$ using $[\delta^j_{S,k}]$, $[\delta^j_{\theta,k}]$ and (5)
      $[\mathbf{x}^{i,j}_{k+1}] \leftarrow [\mathbf{x}^{i,j}_{k+1/k}] \cap [\mathbf{z}^j_{k+1}]$
      $[\mathbf{x}^{i,j}_{k+1}] \leftarrow$ Waltz$([\mathbf{x}^{i,j}_k], [\mathbf{x}^{i,j}_{k+1}], [\delta^j_{s,k}], [\delta^j_{\theta,k}],\ (5))$
      $m_i^{\mathbf{x}_{k+1}}([\mathbf{x}^{i,j}_{k+1}]) \leftarrow m_i^{\mathbf{x}_k}([\mathbf{x}^{i,j}_k]) \cdot m_k^{\mathbf{S}}([\delta_{\mathbf{s},\mathbf{k}}{}^j]) \cdot m_k^{\Theta}([\delta_{\theta,\mathbf{k}}{}^j]) \cdot m^{\mathbf{z}}_{k+1}([\mathbf{z}^j_{k+1}])$.
      **if** the distance between the center of $[\mathbf{x}^{i,j}_k]$ and a node or a shape point of the road $r^i$ is less then $\delta_{S,k}$ **then**
        $R_{k+1} \leftarrow R_{k+1} \cup \{r^l/(r^l$ linked to $r^i, [r^l] \cap [\mathbf{x}^{i,j}_{k+1}] \neq \emptyset)\}$
        Compute $m_1^{R_{k+1}}$ using (4) and (6)
      **end if**
      Compute $m_i^{\mathbf{x}_{k+1}}$ using (11) and (4)
    **end for**
    Compute $m_2^{R_{k+1}}$ using (7) and (8)
    Compute $m_s^{R_{k+1}}$ using (9) and (4)
  **end for**
  $m^{R_{k+1}} \leftarrow m_1^{R_{k+1}} \textcircled{\cap} m_2^{R_{k+1}} \textcircled{\cap} m_s^{R_{k+1}}$
  Chose the best road from $m^{R_{k+1}}$ using a decision rule of DS theory
  The estimate position on the best road is computed using (12)
  $k \leftarrow k + 1$
**end loop**

*4) GIS correction:* Regarding junctions situations, two cases should be considered. Let $d^i$ be the distance from the center of $[\mathbf{x}^{i,j}_k]$ to a node or shape point of $r^i$ and $\delta_{S,k}$ be the elementary movement given by the rear wheels ABS sensors, then :

- If $d^i$ is less then $\delta_{S,k}$, then it is possible that the vehicle leaves road $r^i$ (see figure 4). For this reason, the set $R_k$ of CRs must be changed to a set $R_{k+1} = R_k \cup S_k$ where $S_k$ is the set of all roads directly linked to $r^i$ and which have an intersection with the focal elements of $m_i^{\mathbf{x}_{k+1}}$: $S_k = \{r^l/(r^l$ is linked to $r^i$ and $[r^l] \cap [\mathbf{x}^{i,j}_{k+1}] \neq \emptyset)\}$. The associated mass function, $m_1^{R_{k+1}}$ computed from the topology of the map is then calculated.
- If $d^i$ is higher than $\delta_{S,k}$, then $R_{k+1} = R_k$.

The mass function $m_2^{R_{k+1}}$ resulting from the similarity criterion is computed using (7) and (8). $m_s^{R_{k+1}}$ can be calculated using (11) and (4). The final mass function $m^{R_{k+1}}$ is thus computed using (10). For each CR $r^i \in R_{k+1}$, a state mass function $m_i^{\mathbf{x}_{k+1}}$ is calculated as explained in IV-D.2.

*5) Overall estimation:* First the best road of $R_{k+1}$ should be selected using $m^{R_{k+1}}$ and a decision rule of DS theory as the maximum plausibility or the maximum pignistic probability. Then, an estimation of the vehicle position on the selected road should be calculated. This is done using the state mass function associated to the selected road. Let $r^i$ be the best road of $R_{k+1}$ and $m_i^{\mathbf{x}_{k+1}}$ be the associated state mass function. The estimation vehicle position on $r^i$ is given by

$$\widehat{\mathbf{x}}_{k+1} = \sum_j m_i^{\mathbf{x}_{k+1}}([\mathbf{x}^{i,j}_{k+1}]) \cdot c_j \tag{12}$$

where $c_j$ is the center of $[\mathbf{x}^{i,j}_{k+1}]$. The BSMM algorithm is presented in Algorithm 1.

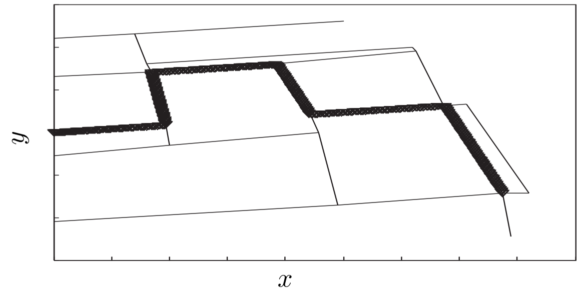


Fig. 5. Simulated road map.

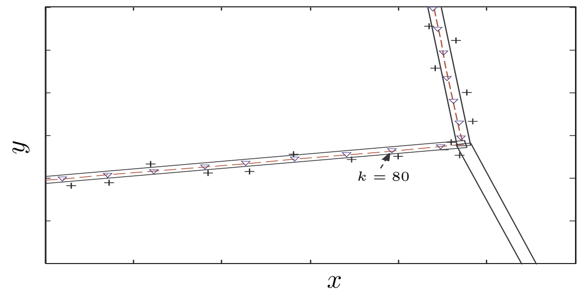


Fig. 6. Result of BSMM method.

## V. Application

In this Section, we present the results of applying the BSMM algorithm on simulated Data. The vehicle position, the heading, the elementary movement and the elementary rotation were generated using the Matlab simulink toolbox. The GPS measurement noise was supposed to be white with $\sigma_x = 7$ m and $\sigma_y = 9$ m. The noise in the input data (elementary movement and elementary rotation) was supposed to be white with $\sigma_s = 1/4$ m and $\sigma_\theta = 0.002$ degrees. In this application, we assumed that the road parameters are $l = 1$ m and $w = 6$ m. The number of focal elements of $m^{\mathbf{x}_k}$, $m^{\mathbf{S}_k}$, $m^{\Theta_k}$ and $m^{\mathbf{z}_k}$ is fixed to $p = 3$.

The simulated map is showed in Figure 5. The vehicle trajectory is plotted by bold lines and the roads are represented by solid black lines. Figure 6 shows the result of the BSMM method near the first junction of the simulated map. The GPS positions are plotted by $(+)$ points and the estimated position of the BSMM method are represented by $(\nabla)$ points. The real trajectory of the vehicle is plotted by dashed lines. Table I

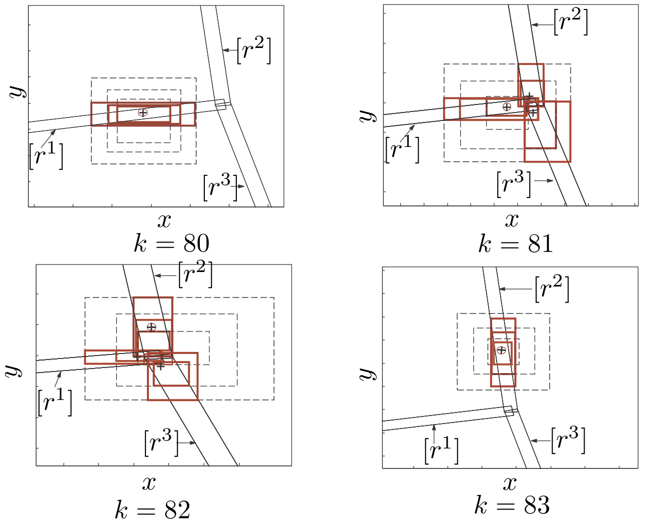


Fig. 7. Results in the neighborhood of roads junction.

| | GPS | BSE | BMM | BSMM | Quddus |
|---|---|---|---|---|---|
| MSE on $x$ (m) | 0.174 | 0.130 | 0.125 | 0.090 | 0.127 |
| MSE on $y$ (m) | 0.227 | 0.161 | 0.164 | 0.140 | 0.163 |

TABLE I

MEAN SQUARE ERROR OF GPS, BMM, BSE, BSMM AND QUDDUS METHOD

shows a comparison between the results of BMM [8], BSE method, BSMM methods and the Quddus method developed in [11] where probabilistic approach is used for integrating sensors data and for selecting the most likely road on which vehicle is moved. As illustrated in this table the result of BSMM method are more accurate. Indeed, The BSMM method combine the result of the BSE method with the map data using DS theory in order to compute a more accurate estimation of the vehicle position. The main advantage of the BSMM method compared to BMM is the use of BSE method in order to combine all sensors data. The BSE method uses belief structures composed of a finite number of axis-aligned boxes with associated masses for representing measurement uncertainties. Such belief structures can model partial information on model and measurement uncertainties, more accurately than the bounded error approach alone, which is used in the BMM method form combining sensors data.

Figure 7 shows how the BSMM method may manage the several hypotheses caused by a junction roads situation. As shown in the figure, at time step $k = 80$, the vehicle is moved on road $r^1$ and $R_k = \{r^1\}$. At time step $k = 81$, there is three possible CRs and $R_k = \{r^1, r^2, r^3\}$. The dashed rectangles represented the GPS mass function. The estimated positions corresponding to all CRs are represented by $(+)$ points. The overall estimated position is chosen between the previous estimated positions and is circled on the figures. The state mass function associated to CRs are represented by bold solid rectangles. Although the BSMM method can provide an state estimate on each CR, and gives an overall estimation position on the best road. As can be seen, at time step $k = 83$, only one CR is kept and the other are eliminated by the similarity criterion. As a conclusion, the BSMM method is able to save all possible positions for eventual correction and choose the most likely one by using a decision rule of DS theory. In this application we select the road with the highest pignistic probability.

## VI. CONCLUSION

In this paper, a new method for map matching and state estimate has been presented. This method used a belief state estimation method in which model and measurement uncertainties are represented by belief structures composed of a finite number of axis-aligned boxes with associated masses. The output of the belief state estimation method are used under the Dempster-Shafer framework with a rectangular road map in order to select a set of candidate roads and compute an accurate estimation of the vehicle position. This method seems to be adequate to deal with some crucial situations of the map matching problems like multi hypothesis scenarios on junctions. Also, it uses belief structures composed of a finite number of axis-aligned boxes with associated masses for representing measurement uncertainties which can model partial information on model and measurement uncertainties, more accurately than the bounded error approach alone. The implementation of this method is quite simple using geometrical properties of boxes and rectangular roads map. Results on simulated data have demonstrated the effectiveness of the proposed method.

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