State Recognition in Discrete Dynamical Systems Using Petri Nets and Evidence Theory

Michèle Rombaut[1], Iman Jarkass[2], and Thierry Denœux[2]

[1] LM2S Université de Technologie de Troyes
IUT de Troyes, France
[2] UMR CNRS 6599 HeuDiaSyC
Université de Technologie de Compiègne, France
Thierry.Denoeux@hds.utc.fr

Abstract. A method is proposed for determining the state of a dynamical system modeled by a Petri net, using observations of its inputs. The initial state of the system may be totally or partially unknown, and sensor reports may be uncertain. In previous work, a belief Petri net model using the formalism of evidence theory was defined, and the resolution of the system was done heuristically by adapting the classical evolution equations of Petri nets. In this paper, a more principled approach based on the Transferable Belief Model is adopted, leading to simpler computations. An example taken from an intelligent vehicle application illustrates the method throughout the paper.

1 Preliminaries

1.1 Introduction

The problem addressed in this paper concerns the determination of the state of a dynamical system, using sensor reports. The sequential evolution of the system is modeled by a simple class of Petri nets [1–3] composed of a set $P = \{p_1, \ldots, p_n\}$ of places, a set $T = \{t_1, \ldots, t_q\}$ of transitions or logical propositions, and a set $F \subseteq (P \times T) \cup (T \times P)$ of arcs connecting a place to a transition, or a transition to a place (each transition is assumed to have only one output place, and each place has at most one output transition). At each time step, one of the n places is marked by a token. A move of the token from place p to place p' occurs if there is a transition t with input place p and output place p', and if this transition has truth value 1.

This formalism may be used to model the behavior of certain physical systems, the state X_k of the system at each time step k being described by the marking of the Petri net at time k. In the type of applications considered here, the initial state of the system and/or the truth value of the transitions are only partially known, and the goal is to determine as accurately as possible the actual system state.

Our approach is illustrated throughout this paper by a typical example taken from an intelligent vehicle application [4].

1.2 Scenario Description

The goal is to detect and characterize an overtaking maneuver on a highway composed of two one-way lanes, the dynamic motion of the vehicle being observed by proprioceptive sensors such as accelerometers, steering wheel angle sensors and braking sensors. An experimental vehicle (EV) goes on the right lane of a highway. It catches a target vehicle (TV) going on the same lane with a lower speed. The EV is beginning an overtaking of the TV. It begins to go left for lane changing, then goes straight forward. When TV is overtaken, it goes right to the right lane.

The recognition of the phases of such a maneuver can be used, for instance, to run a lateral target detection sensor when EV is on the left lane, the obtained information being useless in the other phases of the maneuver. The overtaking maneuver can also be stopped during its execution, for instance if EV establishes that it is impossible to overtake TV before the exit he wants to take. Note that no temporal information is used, because the duration of each phase depends on the context of the maneuver (speed, length of TV, etc.) and cannot be easily evaluated.

A Petri net for this problem is shown in Figure 1. It is composed of 5 places and 4 transitions, the interpretations of which are given in Table 1. In this model, the maneuver is described as a sequence of token positions. The token is initially in place p_1, and ends up in place p_5. It is removed from a place p and added to place p' if there is a transition t connecting these two places, which has truth value equal to 1. These truth values are deduced from numerical measurements provided by the sensors, which requires some form of numerical to symbolic conversion [5]. For instance a report such as "*lateral speed = 0,1 m/s*" is transformed into a degree of confidence in the proposition "*positive lateral speed and positive steering wheels angle*". Furthermore, measurement errors and the possibility of sensor faults are additional sources of uncertainty.

Note that some transitions have the same expression, for instance t_2 and t_4.

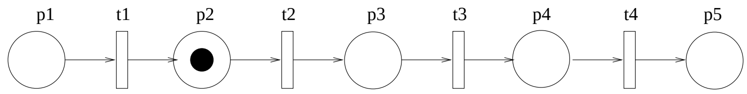

Fig. 1. Petri net for the overtaking maneuver. Places are drawn as circles, and transitions as boxes. The token is in place $p2$.

1.3 Formalization

The state of a Petri net at a given time step k is defined by a marking m^k assigning an integer to each place. The marks take values in $\{0, 1\}$ when the net

Table 1. Interpretation of places and transition in the overtaking maneuver example.

p_1	Initial state
p_2	Left lane change
p_3	Overtaking
p_4	Right lane change
p_5	Final state
t_1	positive lateral speed and positive steering wheels angle
t_2	small lateral speed and positive or small longitudinal acceleration
t_3	negative lateral speed and negative steering wheels angle
t_4	small lateral speed and positive or small longitudinal acceleration

is a state machine [3]. The marking at time k may then be represented by a vector $M^k = [m^k(p_1), \ldots, m^k(p_n)]^t \in \{0,1\}^n$. For instance, we may have:

$$M^k = [0\ 1\ 0\ 0\ 0]^T,$$

meaning that the EV is moving to the left lane. Equivalently, the state of the system at time k may be described by a variable X^k taking values in P. We then have

$$X^k = p_i \Leftrightarrow M_i^k = 1.$$

Similarly, the truth values of the transitions at time k may be described by a vector $R^k \in \{0,1\}^q$, such as:

$$R^k = [0\ 1\ 0\ 1]^T,$$

meaning that the lateral speed is very small, and the longitudinal speed is positive.

The evolution of the net depends on its structure and on the validity of the transitions. Let R denote the vector of truth values of the transitions. If R is known, the marking at time $k+1$ is completely determined by the marking at time k. We can then define a transition function

$$f : P \times \{0,1\}^q \mapsto P$$

such that $f(p, R) = p'$ if $R_i = 1$ for some transition t_i connecting p to p', and $f(p, R) = p$ otherwise. For instance, with $R = [0\ 1\ 0\ 1]^T$, we have $f(p_2, R) = p_3$ and $f(p_3, R) = p_3$. The states X^k and X^{k+1} of the system at times k and $k+1$ are therefore related by the following equation:

$$X^{k+1} = f(X^k, R^k).$$

Although the above model does not include any form of uncertainty, it is interesting to give some of the above notions a probabilistic interpretation, which will become useful in later developments. Since $m^k(p_i) = 1$ means that the token is for sure in place p_i, $m^k(p_i)$ may be interpreted as the probability, taking value in $\{0,1\}$, that $X^k = p_i$. In the same way, let p be an input place of transition t_i, and p' be its output place. Then, R_i^k may be interpreted as the conditional probability that $X^{k+1} = p'$, given that $X^{k+1} = p$.

2 Modeling Uncertain Knowledge of State

The belief Petri net model [1] was introduced to deal with situations in which the goal is to identify the state of a dynamical system at time k with partial or no a priori knowledge of its initial state, and/or when the transitions are uncertain. The structure of the net is assumed to be known, and is the same as considered in the previous section. Alternative approaches based on fuzzy sets and possibility theory are described in [6].

2.1 The Belief Petri Net Model

Belief Petri nets use the Transferable Belief Model (TBM) [7], a subjectivist interpretation of Dempster-Shafer theory, to quantify one's belief concerning the state of the system at each time step. More precisely, one's belief regarding the value of variable X^k is described by a basic belief assignment (BBA) m^k, i.e., a function from 2^P to $[0,1]$ verifying

$$\sum_{A \subseteq P} m^k(A) = 1.$$

For any set A of places, $m^k(A)$ is interpreted as the portion of a unit mass of belief, that one is willing to commit to the hypothesis that $X^k \in A$. By analogy with the standard Petri net model described above, we may define an extended marking vector $M^k = [m^k(A_1), \ldots, m^k(A_{2^n})]^T \in [0,1]^{2^n}$, where $A_1, \ldots, A_{2^n}$ are the 2^n subsets of places. In this model, the marking thus takes the form of a distribution of mass on the power set of P.

2.2 Computation of Beliefs

Let us now assume one's belief regarding X^k to be quantified by a BBA m^k, and let R^k denote the vector of truth values of the transitions at time k, which is considered to be known. A BBA m^{k+1} describing one's belief concerning the state X^{k+1} of the system at the next time step may be computed by transferring the mass $m^k(A)$ to the set

$$f(A, R_k) = \bigcup_{p \in A} f(p, R_k), \tag{1}$$

for all $A \subseteq P$. We thus have

$$m^{k+1}(B) = \sum_{\{A \subseteq P | f(A, R^k) = B\}} m^k(A) \tag{2}$$

Example 1 *In the overtaking example, assume that*

$$m^k(\{p_1, p_2\}) = 0.7$$
$$m^k(\{p_1, p_2, p_3\}) = 0.2$$
$$m^k(P) = 0.1,$$

and $R^k = [0\ 1\ 0\ 1]^T$. *Using Eq. (1), we have:*

$$\begin{aligned} f(\{p_1, p_2\}, R^k) &= \{p_1, p_3\} \\ f(\{p_1, p_2, p_3\}, R^k) &= \{p_1, p_3\} \\ f(P, R^k) &= \{p_1, p_3, p_5\}. \end{aligned}$$

Hence, Eq. (2) yields

$$\begin{aligned} m^{k+1}(\{p_1, p_3\}) &= m^k(\{p_1, p_2\}) + m^k(\{p_1, p_2, p_3\}) = 0.9 \\ m^{k+1}(\{p_1, p_3, p_5\}) &= m^k(P) = 0.1 \end{aligned}$$

Note that the mass of belief is concentrated on smaller subsets at time $k+1$. This phenomenon is general, since

$$|f(A, R^k)| \leq |A| \quad \forall A \subseteq P.$$

An immediate consequence is that m^{k+1} is more informative than m^k in terms of non specificity [8], i.e.,

$$N(m^{k+1}) \leq N(m^k)$$

where $N(m)$ is the nonspecificity of m, defined by:

$$N(m) = \sum_{A \subseteq P, A \neq \emptyset} m(A) \log_2(|A|).$$

The difference $N(m^k) - N(m^{k+1})$ may then be interpreted as a measure of the information gained by observing the system at time step k (and contained in R^k).

3 Uncertain Knowledge of Transitions

In the class of applications considered in this paper, the truth values of the transitions are usually deduced from sensor measurements. Because of the limited precision and reliability of sensors, the truth values of the transitions are usually not known with certainty. In the belief Petri net model, this uncertainty is represented by belief functions describing one's uncertain knowledge of the truth value of each transition. More precisely, let m_i^k denote the BBA regarding the truth value of transition t_i at time k. Its frame of discernment is $\Omega = \{0, 1\}$. By analogy with the previous notations, the belief masses $m_i^k(A)$ for $i = 1, \ldots, q$ and $A \subseteq \Omega$ may be presented in a matrix R^k of size $(q, 3)$, such that

$$R^k_{i,1} = m_i^k(\{0\}) \quad R^k_{i,2} = m_i^k(\{1\}) \quad R^k_{i,3} = m_i^k(\{0,1\}) \quad \forall i \in \{1, \ldots, q\}.$$

The problem is now to combine one's knowledge of the state of system at time k, represented by m^k, with one's knowledge of the transitions at k, represented by the m_i^k for $i = 1, \ldots, q$, to arrive at a BBA m^{k+1} quantifying one's belief regarding the state of the system at time $k+1$. This problem may be solved in the TBM framework by applying the Generalized Bayes Theorem (GBT) [9], in the following way.

First of all, let us remark that one's beliefs about t_i at time k may be translated into conditional beliefs about X^{k+1}, given X^k. Let t_i be a transition with input place p and output place p'. If $X^k = p$, then $X^{k+1} = p$ if $t_i = 0$, and $X^{k+1} = p'$ otherwise. Hence, one's beliefs about X^{k+1}, conditionally on X^k being equal to p, may be deduced from m_i^k. If $m^{k+1|k}(A|\{p\})$ denotes the mass of belief assigned to the hypothesis $X^{k+1} \in A \subseteq P$, given that $X^k = p$, we then have:

$$m^{k+1|k}(\{p\}|\{p\}) = m_i^k(\{0\}) \tag{3}$$

$$m^{k+1|k}(\{p'\}|\{p\}) = m_i^k(\{1\}) \tag{4}$$

$$m^{k+1|k}(\{p, p'\}|\{p\}) = m_i^k(\{0, 1\}). \tag{5}$$

Thus, we know how to express our beliefs on X^{k+1}, given that $X^k = p$. What are now our beliefs on X^{k+1}, if we only know that $X^k \in A \subseteq P$? The disjunctive rule of combination [9] provides an answer to this question. In general, the disjunctive sum of two BBA's m and m' defined on the same frame of discernment P is defined as:

$$(m \cup m')(A) = \sum_{\{B,C \subseteq P | B \cup C = A\}} m(B)m'(C) \quad \forall A \subseteq P.$$

This rule is appropriate to combine information coming from distinct sources, of which at least one is known to tell the truth. Here, $m^{k+1|k}(\cdot|A)$ may be obtained as the disjunctive sum of $m^{k+1|k}(\cdot|\{p\})$, for all $p \in A$:

$$m^{k+1|k}(\cdot|A) = \bigcup_{p \in A} m^{k+1|k}(\cdot|\{p\}) \quad \forall A \subseteq P. \tag{6}$$

Finally, the GBT allows to combine one's beliefs regarding X^k, with one's beliefs regarding X^{k+1}, conditionally on X^k, as

$$m^{k+1}(A) = \sum_{B \subseteq P} m^k(B)m^{k+1|k}(A|B) \quad \forall A \subseteq P. \tag{7}$$

Example 2 *In the overtaking maneuver example, assume m^k to be defined as in Example 1, and R^k to be:*

$$R^k = \begin{bmatrix} 0.0 & 0.9 & 0.1 \\ 0.3 & 0.0 & 0.7 \\ 0.0 & 0.7 & 0.3 \\ 0.3 & 0.0 & 0.7 \end{bmatrix}$$

These numbers may be translated into the following conditional belief numbers:

$$m^{k+1|k}(\{p_1\}|\{p_1\}) = m_1^k(\{0\}) = 0.9$$

$$m^{k+1|k}(\{p_1,p_2\}|\{p_1\}) = m_1^k(\{0,1\}) = 0.1$$
$$m^{k+1|k}(\{p_3\}|\{p_2\}) = m_2^k(\{1\}) = 0.3$$
$$m^{k+1|k}(\{p_2,p_3\}|\{p_2\}) = m_2^k(\{0,1\}) = 0.7$$
$$m^{k+1|k}(\{p_3\}|\{p_3\}) = m_3^k(\{0\}) = 0.7$$
$$m^{k+1|k}(\{p_3,p_4\}|\{p_3\}) = m_3^k(\{0,1\}) = 0.3$$
$$m^{k+1|k}(\{p_5\}|\{p_4\}) = m_4^k(\{1\}) = 0,3$$
$$m^{k+1|k}(\{p_4,p_5\}|\{p_4\}) = m_4^k(\{0,1\}) = 0.7$$

Additionally, as there is no transition from p_5, we have:

$$m^{k+1|k}(\{p_5\}|\{p_5\}) = 1.$$

Since m^k has three focal elements: $\{p_1,p_2\}$, $\{p_1,p_2,p_3\}$ and P, we must compute three conditional BBA's $m^{k+1|k}(\cdot|\{p_1,p_2\})$, $m^{k+1|k}(\cdot|\{p_1,p_2,p_3\})$ and $m^{k+1|k}(\cdot|P)$. From Eq. (6), we have:

$$m^{k+1|k}(\cdot|\{p_1,p_2\}) = m^{k+1|k}(\cdot|\{p_1\}) \cup m^{k+1|k}(\cdot|\{p_2\}),$$

which leads to:

$$\begin{aligned}
m^{k+1|k}(\{p_1,p_3\}|\{p_1,p_2\}) &= m^{k+1|k}(\{p_1\}|\{p_1\})m^{k+1|k}(\{p_3\}|\{p_2\}) \\
&= 0.9 \times 0.3 = 0.27 \\
m^{k+1|k}(\{p_1,p_2,p_3\}|\{p_1,p_2\}) &= m^{k+1|k}(\{p_1,p_2\}|\{p_1\}).m^{k+1|k}(\{p_3\}|\{p_2\}) + \\
&\quad m^{k+1|k}(\{p_1\}|\{p_1\})m^{k+1|k}(\{p_2,p_3\}|\{p_2\}) + \\
&\quad m^{k+1|k}(\{p_1,p_2\}|\{p_1\})m^{k+1|k}(\{p_2,p_3\}|\{p_2\}) \\
&= 0.1 \times 0.3 + 0.9 \times 0.7 + 0.1 \times 0.7 = 0.73
\end{aligned}$$

Similarly,

$$m^{k+1|k}(\cdot|\{p_1,p_2,p_3\}) = m^{k+1|k}(\cdot|\{p_1\}) \cup m^{k+1|k}(\cdot|\{p_2\}) \cup m^{k+1|k}(\cdot|\{p_3\})$$

leads to:

$$m^{k+1|k}(\{p_1,p_3\}|\{p_1,p_2,p_3\}) = 0.189$$
$$m^{k+1|k}(\{p_1,p_2,p_3\}|\{p_1,p_2,p_3\}) = 0.511$$
$$m^{k+1|k}(\{p_1,p_3,p_4\}|\{p_1,p_2,p_3\}) = 0.081$$
$$m^{k+1|k}(\{p_1,p_2,p_3,p_4\}|\{p_1,p_2,p_3\}) = 0.219.$$

Finally,

$$m^{k+1|k}(\cdot|P) = \bigcup_{i=1}^{5} m^{k+1|k}(\cdot|\{p_i\})$$

and we have

$$m^{k+1|k}(\{p_1,p_3,p_5\}|P) = 0.0567$$
$$m^{k+1|k}(\{p_1,p_2,p_3,p_5\}|P) = 0.1533$$
$$m^{k+1|k}(\{p_1,p_3,p_4,p_5\}|P) = 0.2133$$
$$m^{k+1|k}(P|P) = 0.5766$$

The BBA quantifying one's beliefs about the state at $k+1$ may finally be obtained by applying Eq. (7). For example, we have:

$$\begin{aligned} m^{k+1}(\{p_1,p_3\}) &= m^k(\{p_1,p_2\})m^{k+1|k}(\{p_1,p_3\}|\{p_1,p_2\}) + \\ &\quad m^k(\{p_1,p_2,p_3\})m^{k+1|k}(\{p_1,p_3\}|\{p_1,p_2,p_3\}) \\ &= 0.7 \times 0.27 + 0.2 \times 0.189 = 0.2268 \end{aligned}$$

Similarly, we obtain:

$$\begin{aligned} m^{k+1}(\{p_1,p_2,p_3\}) &= 0.6132 \\ m^{k+1}(\{p_1,p_3,p_4\}) &= 0.0162 \\ m^{k+1}(\{p_1,p_2,p_3,p_4\}) &= 0.0438 \\ m^{k+1}(\{p_1,p_3,p_5\}) &= 0.00567 \\ m^{k+1}(\{p_1,p_2,p_3,p_5\}) &= 0.01533 \\ m^{k+1}(\{p_1,p_3,p_4,p_5\}) &= 0.02133 \\ m^{k+1}(P) &= 0.05766 \end{aligned}$$

4 Simulation Results

Some simulated tests have been made, such as represented in Figure 2. In this example, the initial state is known and corresponds to place p_1. The belief masses m_i^k associated to the transitions are represented in Figure 3.

These results show that, when the truth value of a transition is not sure, the mark changes from one place p to the next p' through the proposition $\{p,p'\}$ composed of the two places (see, e.g., the transition from place 2 to place 3 at time step 4). This phenomenon can be seen as some form of "gradual" transition.

5 Conclusion

In this paper, a method has been proposed to recognize the state of a sequential system modeled by a Petri net, using uncertain observations. This method is based on the Transferable Belief Model, in which belief functions are used to represent imperfect knowledge. This approach allows to deal with both partial (or total) ignorance of the initial system state, and limited precision and reliability of sensor data. The use of the Generalized Bayes Theorem allows to drastically reduce the amount of computing time in this method, as compared to previous approaches [1].

The application of the proposed framework to the development of a driving assistance system is currently underway. Real measurements have been performed on an experimental vehicle, and the current problem is to transform numerical data into truth values of logical propositions associated to the transitions. The combined use of fuzzy logic and evidence theory might be a useful approach to this problem.

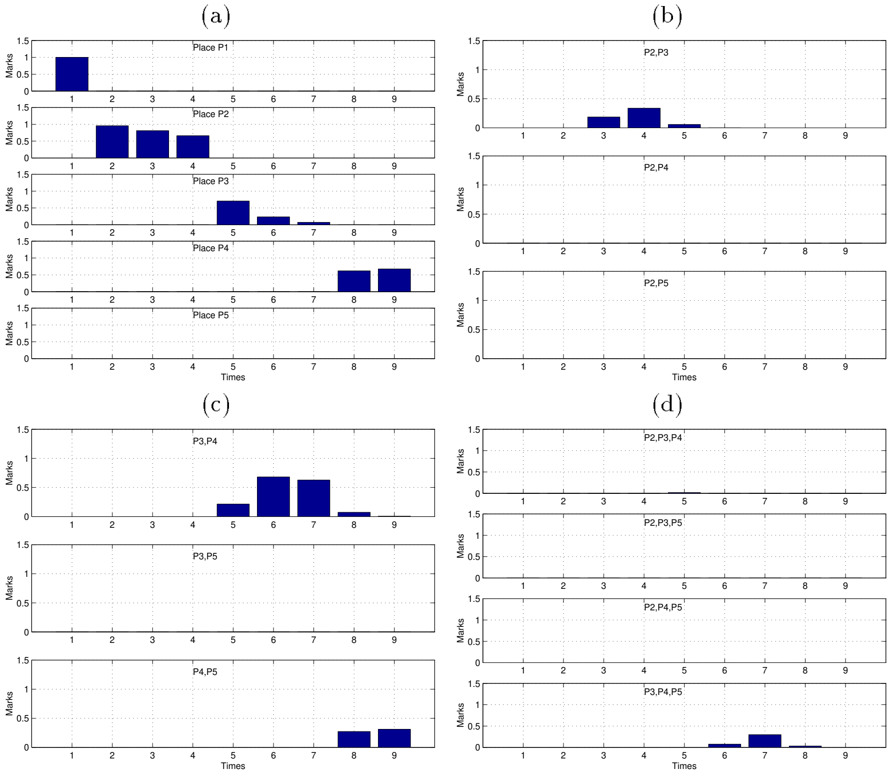

Fig. 2. Simulated example, showing the belief masses $m^k(A)$ as a function of k for nine consecutive time steps. (a): $A = \{p_i\}$, $i = 1, 5$; (b): $A = \{p_2, p_3\}$, $\{p_2, p_4\}$, $\{p_2, p_5\}$. (c): $A = \{p_3, p_4\}$, $\{p_3, p_5\}$, $\{p_4, p_5\}$; (d): $A = \{p_2, p_3, p_4\}$, $\{p_2, p_3, p_5\}$, $\{p_2, p_4, p_5\}$,$\{p_3, p_4, p_5\}$. The corresponding belief masses m_i^k associated to the transitions are shown in Figure 3.

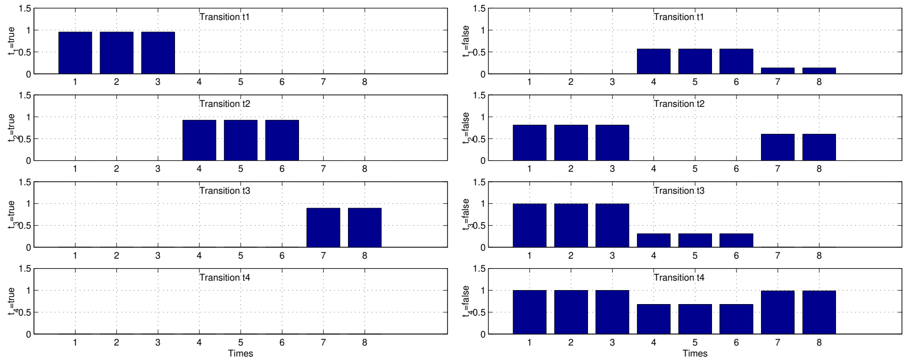

Fig. 3. Belief masses $m_i^k(\{1\})$ (left) and $m_i^k(\{0\})$ (right) associated to each of the four transitions, for nine consecutive time steps.

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