Workshop on Belief Functions
Lecture 1 – Exercises

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1 Representation of Evidence

1. An urn contains 90 balls, of which 30 are white, and 60 are either black or yellow. A ball is going to be drawn from the urn. Represent the uncertainty about the outcome of this experiment using a mass function on a suitable frame. Compute the corresponding belief and plausibility functions.

2. Let Bel be a belief function on Ω and let Pl be the corresponding plausibility function. Show that

$$Bel(A \cup B) \geq Bel(A) + Bel(B) - Bel(A \cap B)$$

and

$$Pl(A \cap B) \leq Pl(A) + Pl(B) - Pl(A \cup B),$$

for all $A, B \subseteq \Omega$.

3. Let m be the mass function on $\Omega = \{a, b, c\}$ defined by:

$$m(\{a\}) = 0.2 \quad m(\{a, b\}) = 0.5 \quad m(\Omega) = 0.3.$$

Compute $Bel(A)$ and $Pl(A)$ for all $A \subseteq \Omega$. Which special properties do these functions verify?

4. Let us consider the following plausibility function on $\Omega = \{a, b, c\}$:

A	$\emptyset$	$\{a\}$	$\{b\}$	$\{a, b\}$	$\{c\}$	$\{a, c\}$	$\{b, c\}$	$\{a, b, c\}$
$Pl(A)$	0	0.5	0.55	1	0.5	0.8	0.7	1

   (a) Compute the corresponding belief function.

   (b) Is the corresponding mass function consonant? Give its expression.

(c) Give a compatible probability measure, and the corresponding allocation of probability.

5. Let π be the following possibility distribution on $\Omega = \{a, b, c, d, e, f\}$:

ω	a	b	c	d	e	f
$\pi(\omega)$	0.1	0.3	0.5	1	0.7	0.3

Compute the corresponding mass function.

6. Let m be a consonant mass function on a frame Ω and let Bel and Pl be the corresponding belief and plausibility functions. Show that, for any subset A of Ω, $Bel(A) > 0 \Rightarrow Pl(A) = 1$.

2 Operations on Belief Functions

1. Let m_1 and m_2 be two mass functions on $\Omega = \{a, b, c, d\}$ defined as follows

$$m_1(\{a\}) = 0.3 \quad m_1(\{a, c\}) = 0.5 \quad m_1(\{b, c, d\}) = 0.2$$

and

$$m_2(\{b, c\}) = 0.4 \quad m_2(\{a, c, d\}) = 0.5 \quad m_2(\{d\}) = 0.1.$$

Compute the combined mass functions using different combination operators.

2. Let $\Omega = \{a, b\}$, and let m and m' be the following mass functions on Ω,

$$m = \{a\}^{\alpha} \oplus \{b\}^{\beta}, \quad m' = \{a\}^{\alpha'} \oplus \{b\}^{\beta'},$$

where A^w denotes the simple mass function m such that $m(A) = 1 - w$ and $m(\Omega) = w$.

(a) Compute m and m'.

(b) Compute $m \oplus m'$.

3. Let m be a mass function on Ω and B a non-empty subset of Ω.

(a) Express the conditional belief function $Bel(.|B)$ as a function of Bel.

(b) What does this formula become when Bel is a probability measure?

4. Let m_1 and m_2 be two consonant mass functions, and let Pl_1 and Pl_2 be the corresponding plausibility measures.

(a) Show that $Pl_1 \vee Pl_2 = \max(Pl_1, Pl_2)$ is a plausibility measure.

(b) What are the properties of this operator?

(c) Using a counterexample, show that $Pl_1 \vee Pl_2$ may not be a plausibility measure when m_1 and m_2 are not consonant.

5. Let $\Theta = \{\theta_1, \theta_2, \theta_3\}$ and $\Omega = \{a, b, c\}$ be two frames of discernment. We consider the following mass function on $\Omega \times \Theta$:

$$m^{\Omega\times\Theta}(\{(a,\theta_1)\}) = 0.2 \quad m^{\Omega\times\Theta}(\Omega \times \{\theta_2\}) = 0.3$$

$$m^{\Omega\times\Theta}(\{b\} \times \Theta\}) = 0.4 \quad m^{\Omega\times\Theta}(\{(a,\theta_1),(b,\theta_2),(c,\theta_3)\}) = 0.1$$

(a) Compute $m^{\Omega\times\Theta\downarrow\Omega}$ and its vacuous extension on $\Omega \times \Theta$.

(b) Compute $m^{\Omega\times\Theta\downarrow\Theta}$ and its vacuous extension on $\Omega \times \Theta$.