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Exercises on Lecture 1

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1. An urn contains 90 balls, of which 30 are white, and 60 are either black or yellow. A ball is going to be drawn from the urn. Represent the uncertainty about the outcome of this experiment using a random set and the corresponding mass function on a suitable frame of discernment. Compute the corresponding belief, plausibility and commonality functions.

Solution: The frame can be denoted as $\Theta = \{w, b, y\}$ for the three colors. Let Ω_w be the set of balls known to be white, $\Omega_{b,y}$ the set of balls that are black or yellow, $\Omega = \Omega_w \cup \Omega_{b,y}$. The chance to get a white ball is $1/3$ and the chance to get a ball that is either black or yellow is $2/3$. We thus have a probability measure P on Ω such that $P(\Omega_w) = 1/3$ and $P(\Omega_{b,y}) = 2/3$. Now, we can define the following multi-valued mapping

$$\begin{aligned} \overline{X} : \Omega &\to \mathcal{P}(\Theta) \\ \omega &\mapsto \begin{cases} \{w\} & \text{if } \omega \in \Omega_w \\ \{b, y\} & \text{if } \omega \in \Omega_{b,y} \end{cases} \end{aligned}$$

The corresponding mass function is

$$\begin{aligned} m(\{w\}) &= P(\{\omega : \overline{X}(\omega) = \{w\}\}) = P(\Omega_w) = 1/3 \\ m(\{b, y\}) &= P(\{\omega : \overline{X}(\omega) = \{b, y\}\}) = P(\Omega_{b,y}) = 2/3. \end{aligned}$$

The corresponding belief, plausibility and commonality functions are given in the following table:

A	$\varnothing$	$\{w\}$	$\{b\}$	$\{w, b\}$	$\{y\}$	$\{w, y\}$	$\{b, y\}$	$\{w, b, y\}$
$Bel_m(A)$	0	1/3	0	1/3	0	1/3	2/3	1
$Pl_m(A)$	0	1/3	2/3	1	2/3	1	2/3	1
$Q_m(A)$	1	1/3	2/3	0	2/3	0	2/3	0

2. Let $\Theta = \{a, b, c\}$ and f the following function from $\mathcal{P}(\Theta)$ to $[0, 1]$:

A	$\varnothing$	$\{a\}$	$\{b\}$	$\{a, b\}$	$\{c\}$	$\{a, c\}$	$\{b, c\}$	$\{a, b, c\}$
$f(A)$	0	0.5	0.2	0.8	0	0.5	0.5	1

Is f a belief function?

Solution: Let us assume that f is a belief function, and let us compute the corresponding mass function. We have

$$m(\{a\}) = 0.5 \quad \text{and} \quad m(\{b\}) = 0.2,$$

so

$$m(\{a, b\}) = 0.8 - 0.5 - 0.2 = 0.1.$$

Similarly, $m(\{c\}) = 0$, so $m(\{a, c\}) = 0.5 - 0.5 - 0 = 0$, and

$$m(\{b, c\}) = 0.5 - 0.2 - 0 = 0.3.$$

Finally,

$$m(\{a,b,c\}) = 1 - 0.5 - 0.2 - 0.1 - 0.3 = -0.1.$$

This mass is negative, so f is not a belief function.

3. An expert has given the following contour function on $\Theta = \{a, b, c, d, e, f\}$:

θ	a	b	c	d	e	f
$\text{pl}(\theta)$	0.1	0.3	0.5	1	0.7	0.3

Compute the corresponding mass function, assuming that it is consonant.

Solution: The elements of Θ are ordered by decreasing plausibility as

$$1 = pl(d) > pl(e) > pl(c) > pl(b) = pl(f) > pl(a),$$

so the focal sets are $\{d\}$, $\{d, e\}$, $\{d, e, c\}$, $\{d, e, c, b, f\}$ and Θ. We have

$$\begin{aligned} m(\{d\}) &= 1 - 0.7 = 0.3 \\ m(\{d,e\}) &= 0.7 - 0.5 = 0.2 \\ m(\{d,e,c\}) &= 0.5 - 0.3 = 0.2 \\ m(\{d,e,c,b,f\}) &= 0.3 - 0.1 = 0.2 \\ m(\Theta) &= 0.1. \end{aligned}$$

All other subsets have mass zero.

4. Let m_1 and m_2 be two mass functions on $\Theta = \{a, b, c\}$ defined as follows:

$$m_1(\{a\}) = 0.4 \quad m_1(\{a,c\}) = 0.1 \quad m_1(\{b,c\}) = 0.5,$$

$$m_2(\{c\}) = 0.2 \quad m_2(\{b,c\}) = 0.3 \quad m_2(\Theta) = 0.5.$$

Compute the combined mass function $m_{12} = m_1 \oplus m_2$. (Present the calculations in a two-way table). What is the degree of conflict between m_1 and m_2?

Solution:

		m_2		
		$\{b\}, 0.2$	$\{b,c\}, 0.3$	$\{a,b,c\}, 0.5$
m_1	$\{a\}, 0.1$	$\varnothing, 0.02$	$\varnothing, 0.03$	$\{a\}, 0.05$
	$\{a,c\}, 0.6$	$\varnothing, 0.12$	$\{c\}, 0.18$	$\{a,c\}, 0.3$
	$\{b,c\}, 0.3$	$\{b\}, 0.06$	$\{b,c\}, 0.09$	$\{b,c\}, 0.15$

The degree of conflict is

$$\kappa = 0.02 + 0.03 + 0.12 = 0.17$$

and the combined mass function is

$$\begin{aligned} m_{12}(\{a\}) &= 0.05/0.83 \approx 0.060 \\ m_{12}(\{b\}) &= 0.06/0.83 \approx 0.072 \\ m_{12}(\{c\}) &= 0.18/0.83 \approx 0.22 \\ m_{12}(\{a,c\}) &= 0.3/0.83 \approx 0.36 \\ m_{12}(\{b,c\}) &= 0.24/0.83 \approx 0.29. \end{aligned}$$