STABLE CLUSTERING ENSEMBLE BASED ON EVIDENCE THEORY

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ABSTRACT
As an unsupervised ensemble learning strategy, clustering ensemble combines multiple base clusterings into a high-quality one and has achieved successful applications in image analysis and data mining. However, extant clustering ensemble methods are ineffective to handle the data uncertainty in clustering consensus process, which may mislead to poor clustering ensemble results. To tackle the problem, we propose a stable clustering ensemble (SCE) method based on evidence theory (Dempster–Shafer theory) in this paper. Specifically, we construct a belief function of cluster membership to measure the uncertainty and stability of data instances in clustering ensemble and thereby implement the stable clustering ensemble algorithm. We test the proposed stable clustering ensemble method in the tasks of structural data clustering and image segmentation. The experimental results validate the proposed method is effective to process the uncertain data and produce high-quality data clusterings.

Index Terms—Clustering ensemble, stability, evidence theory

1. INTRODUCTION
Clustering ensemble methods have been proposed to combine multiple base clusterings into a single one, called the consensus, which aims at producing a more accurate and robust clustering of data [1]. Clustering ensemble has been successfully applied in the areas of image analysis [2], computer vision [3], multimedia [4] and data mining [5].

Although previous research works have achieved a great progress, extant clustering ensemble methods ignored the uncertainty of data in the clustering consensus process, which may mislead to poor clustering ensemble results. Specifically, if a data instance is partitioned unsteadily into different clusters in multiple base clusterings, it is indicated that the data instance is unstable in the consensus process and has much uncertainty in clustering ensemble. For the tasks of image analysis, the unstable pixels in the clustering ensemble correspond the uncertain regions of images. To improve the performances of data analysis, it is required to formulate and handle the data uncertainty in clustering ensemble.

To tackle the problem, we propose a stable clustering ensemble (SCE) method based on evidence theory (Dempster–Shafer theory) in this paper. In the proposed method, we construct a belief function of cluster membership to measure the uncertainty and stability of data instances in clustering ensemble and thereby implement the stable clustering ensemble algorithm. Based on the stability measure, we can divide data instances into two categories: cluster core and cluster halo [6]. Cluster core consists of the stable data instances with little uncertainty, which represent the structure of data distribution. In contrast, cluster halo contains the unstable data instances with high uncertainty, which denote the uncertain boundaries of clusters. To implement the stable clustering ensemble, we first detect the cluster core in clustering consensus as the certain fundamental clusters and then gradually distribute the uncertain data instances in the cluster halo into certain clusters. The main contributions of this paper are summarized below.

• Propose a measure of data stability in clustering ensemble based on evidence theory.
• Implement a stable clustering ensemble algorithm with stability measure to handle uncertain data in clustering consensus.

The rest of the paper is organized as follows. Section 2 briefly reviews the foundations of clustering ensemble and evidence theory. In Section 3, we introduce the proposed stable clustering ensemble method in detail, which include the stability measure with evidence theory and the clustering ensemble algorithm. In Section 4, the experiments are conducted to verify the proposed method. Section 5 concludes the paper work.

This work was supported by National Natural Science Foundation of China (Serial Nos. 61976134, 61991410, 61991415) and Natural Science Foundation of Shanghai (Serial No. 21ZR1423900).
2. BACKGROUND

Basics of Clustering Ensemble
Let \( X = \{x_1, \ldots, x_n\} \) be a data set and \( \Pi = \{\pi_1, \ldots, \pi_B\} \) denotes \( B \) base clusterings generated by multiple clustering procedures. Clustering ensemble aims at combining multiple base clusterings \( \Pi \) into an accurate and robust clustering. In general, clustering ensemble consists of two stages of generating base clusterings and assembling the consensus clustering result.

The diversity and quality of base clusterings are the key factors to affect the performances of clustering ensemble. Three kinds of strategies were investigated to generate diverse and accurate base clusterings, which include: 1) diverse parameter setting strategy [7] that uses one clustering algorithm with random cluster centers or cluster numbers; 2) diverse algorithm strategy [8] that uses various clustering algorithms to generate diverse base clusterings; 3) diverse feature strategy [9] that represents data clusterings in different feature spaces. For assembling the base clusterings, the methodologies can be categorized into following four kinds [10]. Feature based approach that transforms the clustering ensemble problem into the clustering of categorical data [11]. Direct approach, which is based on the relabeling process to find the best matched clustering [12]. Graph-based approach that utilizes the graph representation to solve the clustering ensemble problem [13]. Co-association approach that creates the pairwise correlation matrix among data instances to assemble base clusterings [14].

Preliminaries of Evidence Theory
Evidence theory, also referred to as Dempster–Shafer (D-S) theory or theory of belief functions [15, 16] is a theoretical framework for reasoning with partial and unreliable information. Let a variable \( w \) taking values in a finite set \( \Omega \), a mass function on \( \Omega \) is defined as a mapping from \( 2^\Omega \) to [0,1], satisfying the following condition

\[
\sum_{A \subseteq \Omega} m(A) = 1. 
\]

Each quantity \( m(A) \) can be interpreted as the probability that the evidence supports \( w \in A \). In particular, \( m(\Omega) \) is the probability that the evidence tells us nothing about \( w \), i.e., the unknown probability. A subset \( A \) of \( \Omega \) such that \( m(A) > 0 \) is called a focal set of \( m \). The mass function for which \( \Omega \) is the only focal set is said to be vacuous, it represents total ignorance. Given a mass function \( m \), belief \( \text{bel} \) and plausibility function \( \text{pl} \) are defined by

\[
\text{bel}(A) = \sum_{\phi \neq B \subseteq A} m(B), \quad (2)
\]

\[
\text{pl}(A) = \sum_{B \cap A \neq \phi} m(B). \quad (3)
\]

For all \( A \subseteq \Omega \), the quantities \( \text{bel}(A) \) and \( \text{pl}(A) \) denote the degree of total support in \( A \) and the degree that the evidence consistent with \( A \), respectively.

3. METHODS

3.1. Measuring data stability in clustering ensemble
Given \( B \) base clusterings for ensemble \( \Pi = \{\pi_1, \ldots, \pi_B\} \), the uncertainty of a data instance in the clustering ensemble is related to the stability of the data instance belonging to a cluster under the partitions of different base clusterings [17]. Suppose some data instances are partitioned into a cluster in one base clustering, but in other base clusterings, these data instances are distributed into different clusters. It is natural to consider that the hypothesis of these data instances belonging to the same cluster is inconclusive. Next we utilize the evidence theory to measure the stability of belongingness of data to clusters.

We define a discernment frame \( \Omega = \{c, \neg c\} \) to discern whether data belong to a cluster \( c \) or not \((\neg c)\). For a data instance \( x_i \) belonging to a cluster \( c^\pi_g \) in the base clustering \( \pi_g \), its mass function of cluster membership can be defined as

\[
m_i^\pi_g(A) = \begin{cases} 
\frac{\sum_{\pi \in \Pi} \sum_{x \in c^\pi_g} f(i, j, \pi)}{(B - 1) \cdot |c^\pi_g|}, & A = \{c^\pi_g\} \\
1 - \frac{\sum_{\pi \in \Pi} \sum_{x \in c^\pi_g} f(i, j, \pi)}{(B - 1) \cdot |c^\pi_g|}, & A = \Omega 
\end{cases} \quad (4)
\]

where \( |c^\pi_g| \) is the number of data instances in the cluster \( c^\pi_g \). \( f(i, j, \pi_t) \) is defined by

\[
f(i, j, \pi_t) = \begin{cases} 
1, x_i, x_j \text{ in the same cluster of } \pi_t \\
0, \text{ otherwise.} 
\end{cases} \quad (5)
\]

Through accumulating the evidences of data co-occurrence in a cluster from other \( B - 1 \) base clusterings, \( m_i^\pi_g(\{c^\pi_g\}) \) denotes the probability mass of the instance \( x_i \) certainly belonging to the cluster \( c^\pi_g \) in the base clustering \( \pi_g \), and \( m_i^\pi_g(\Omega) \) denotes the unknown mass (uncertainty).

Based on the mass function above, we can further formulate the pairwise relationship between data instances using Dempster’s combination rule. Suppose \( \Theta = \{s, \neg s\} \) is a frame of discernment, in which \( s \) denotes a pair of data instances belonging to the same cluster, and \( \neg s \) means that they belong to different clusters. For a pair of data instances \( x_i \) and \( x_j \), the belief and uncertainty about the hypothesis that \( x_i \) and \( x_j \) belong to the same cluster are defined below.

\[
\text{bel}_{ij}^{\pi_g}(\{s\}) = m_{ij}^\pi_g(\{c^\pi_g\}) \cdot m_{ij}^\pi_g(\{c^\pi_g\}),
\]

\[
m_{ij}^\pi_g(\Theta) = m_{ij}^\pi_g(\Omega) \cdot m_{ij}^\pi_g(\Omega). \quad (7)
\]
If \( x_i \) and \( x_j \) belong to different clusters, \( m_{ij}^\pi (\{ s \}) = 1 \), \( bel_{ij}^\pi (\{ s \}) \) and \( m_{ij}^\pi (\Theta) \) are both zero.

Considering all the base clustering, the average belief and uncertainty about that \( x_i \) and \( x_j \) belong to a same cluster are obtained by

\[
bel_{ij} (\{ s \}) = \frac{1}{B} \sum_{t=1}^{B} bel_{ij}^\pi (\{ s \}), \quad (8)
\]

\[
m_{ij} (\Theta) = \frac{1}{B} \sum_{t=1}^{B} m_{ij}^\pi (\Theta). \quad (9)
\]

In general, if a data instance is certainly assigned to a cluster by most base clusterings, we consider that the instance is stable in the clustering ensemble. Therefore, we measure the stability of a data instance \( x_i \) through accumulating its probability mass in all base clusterings.

\[
Stability(x_i) = \sum_{\pi_t \in \Pi} m_i^\pi (\{ c^{\pi_t} \}) \quad (10)
\]

Moreover, based on the stability measure, we can find the highly stable data instances from data sets to form the cluster core. Here we adopt the average stability degree of all data instances as the threshold to select the stable data in clustering ensemble.

### 3.2. Clustering ensemble with stable data

Based on the stability measure of data instances, we can implement the stable clustering ensemble to handle the uncertainty in clustering consensus. The process of the stable clustering ensemble consists of the following two stages.

1. In the first stage, we select the highly stable data instances as the cluster core and perform clustering of these stable data to capture the certain clusters of data distribution.

2. In the second stage, considering the remained unstable data as cluster halo, we distributed each data instance in the cluster halo into a cluster in the certain clustering obtained in the first stage.

For a selected stable instance \( x_i \) among \( n \) data instances, its belief and uncertainty of pairwise relationship belonging to the same cluster are defined in equation (8) and (9). We utilize the pairwise belief and uncertainty to form the feature vector \( R(i) \) of \( x_i \),

\[
R(i) = \{ bel_{i1}(\{ s \}), ..., bel_{in}(\{ s \}), m_{i1}(\Theta), ..., m_{in}(\Theta) \}.
\]

For each pair of stable instances from cluster core \( x_i, x_j \), we can measure their similarity using cosine measure and construct a evidential similarity matrix (ESM), in which each element is computed as

\[
ESM_{ij} = \frac{< R(i), R(j) >}{\sqrt{< R(i), R(i) >} \cdot < R(j), R(j) >}. \quad (12)
\]

**Algorithm 1 Clustering ensemble with stable data**

**Input:** Stability degrees \( stability(x_i) \) of data instances \( \{ x_1, ..., x_i, ..., x_n \} \) among \( B \) base clusterings;

**Output:** Clusters of data instances;

1: Average the stability degree as the threshold \( T \);
2: cluster core = \( \{ x_i | stability(x_i) > T, i = 1, ..., n \} \);
3: cluster halo = \( \{ x_i | stability(x_i) \leq T, i = 1, ..., n \} \);
4: Construct the similarity matrix \( ESM \) of cluster core;
5: Use HC algorithm on \( ESM \) to form clusters \( C \) of stable data;
6: while \( |cluster \_halo| > 0 \) do
7: Get a data instance from cluster halo, assign it into the nearest cluster in \( C \);
8: Update the cluster and remove the instance from the cluster halo;
9: end while
10: return the updated clusters.

Based on ESM of all the stable data instances, we simply utilize a hierarchical clustering (HC) algorithm [18] to form the clusters of cluster core. These clusters can be considered as the certain part of the final clustering result. For the remained unstable data instances in cluster halo, we assign each instance into its nearest cluster of stable data instances and update the cluster iteratively until all the unstable data instances are assigned. The process of the stable clustering ensemble is shown in Algorithm 1. Using the algorithm, we can obtain a clustering ensemble result \( P = \{ pc_1, ..., pc_k' \} \), in which \( pc_i \) denotes the \( i \)-th cluster and \( k' \) is the cluster number. If it is required to further merge the clusters in \( P \), we can adopt the pairwise similarity measure between clusters \( pc_i \) and \( pc_j \) and perform HC algorithm again to merge the clusters in \( P \).

### 4. EXPERIMENTS

In the experiments, we implement two tests to validate the superiority of the proposed stable clustering ensemble (SCE) method. The first test aims to verify the effectiveness of the SCE method for data clustering, we perform the SCE method on 10 structural data sets including both synthetic data sets (Flame, 2d-3c-no123, Aggregation, Chainlink, Wingnut) and UCI data sets (Ecoli, Segmentation, Glass, Knowledge Modeling, Yeast), and compare the clustering results with other 6 representative clustering ensemble methods, which include WTQ [19], WCT [19], CSM [19], MCLA [1], CSPA [1] and HGFB [20]. In the second test, we validate the ability of SCE for image analysis, we utilize the SCE method for image segmentation on Berkeley Segmentation Dataset and compare the segmentation results produced by other kinds of clustering methods.

For the experiment implementation, K-means algorithm is used to generate base clusterings and the cluster number of each base clustering is set as \( \sqrt{n} \), \( n \) is the data instance number. In the comparative experiments, we run each algorithm
10 times and present the average results. Besides, the decay factor parameter in the comparative methods WCT, WTQ are set to 0.9.

In the first test on structural data sets, we adopt the well-known criteria ARI [21] and NMI [1] to evaluate the clustering quality and generate 50 base clusterings for ensemble on each data set. Table 1 and 2 list the ARI and NMI evaluations of clustering results produced by all the comparative clustering ensemble methods. It is obvious that the proposed SCE method achieves the best performance.

### Table 1. ARI evaluations of comparative clustering methods

<table>
<thead>
<tr>
<th>Data sets</th>
<th>MCLA</th>
<th>HBGF</th>
<th>CSPA</th>
<th>CSM</th>
<th>WTQ</th>
<th>WCT</th>
<th>SCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flame</td>
<td>0.5190</td>
<td>0.4858</td>
<td>0.4514</td>
<td>0.7171</td>
<td>0.6511</td>
<td>0.8081</td>
<td><strong>0.8392</strong></td>
</tr>
<tr>
<td>2d-3c-no123</td>
<td>0.6045</td>
<td>0.5265</td>
<td>0.5547</td>
<td>0.8617</td>
<td>0.8132</td>
<td>0.9109</td>
<td><strong>0.9549</strong></td>
</tr>
<tr>
<td>Aggregation</td>
<td>0.5613</td>
<td>0.5173</td>
<td>0.528</td>
<td>0.9338</td>
<td>0.9563</td>
<td>0.971</td>
<td><strong>0.9920</strong></td>
</tr>
<tr>
<td>Chainlink</td>
<td>0.4276</td>
<td>0.2110</td>
<td>0.1962</td>
<td>0.1988</td>
<td>0.3100</td>
<td>0.3751</td>
<td><strong>0.4784</strong></td>
</tr>
<tr>
<td>Wingnut</td>
<td>0.8276</td>
<td>0.899</td>
<td>0.769</td>
<td>0.8807</td>
<td>0.8304</td>
<td>0.8501</td>
<td><strong>0.9543</strong></td>
</tr>
<tr>
<td>Glass</td>
<td>0.2250</td>
<td>0.1884</td>
<td>0.1523</td>
<td>0.2527</td>
<td>0.1968</td>
<td>0.2098</td>
<td><strong>0.2572</strong></td>
</tr>
<tr>
<td>Ecoli</td>
<td>0.3637</td>
<td>0.2941</td>
<td>0.2816</td>
<td>0.4318</td>
<td>0.3751</td>
<td>0.4784</td>
<td><strong>0.5939</strong></td>
</tr>
<tr>
<td>KM</td>
<td>0.1722</td>
<td>0.1863</td>
<td>0.1849</td>
<td>0.2654</td>
<td>0.2412</td>
<td>0.2574</td>
<td><strong>0.3006</strong></td>
</tr>
<tr>
<td>Yeast</td>
<td>0.0909</td>
<td>0.0695</td>
<td>0.0673</td>
<td>0.0997</td>
<td>0.1075</td>
<td>0.0919</td>
<td><strong>0.1490</strong></td>
</tr>
<tr>
<td>Segmentation</td>
<td>0.4210</td>
<td>0.4283</td>
<td>0.3958</td>
<td>0.4811</td>
<td>0.3577</td>
<td>0.3405</td>
<td><strong>0.4769</strong></td>
</tr>
</tbody>
</table>

Besides the structural data, we also test the SCE method on unstructural images. We perform the clustering ensemble methods on the images from Berkeley Segmentation Dataset for image segmentation. Comparing with traditional clustering ensemble methods, SCE can effectively fuse multiple clustering-based segmentation results and detect the uncertain regions based on the stability measure. To illustrate this, we generate 20 segmentations using Chan-Vese method [22] with random initial contours. The stability of each pixel in an image is calculated by equation (10), Fig.1 shows the uncertain image regions (marked by gray color) that consist of the unstable pixels in multiple segmentation results.

### Table 2. NMI evaluations of comparative clustering methods

<table>
<thead>
<tr>
<th>Data sets</th>
<th>MCLA</th>
<th>HBGF</th>
<th>CSPA</th>
<th>CSM</th>
<th>WTQ</th>
<th>WCT</th>
<th>SCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flame</td>
<td>0.4784</td>
<td>0.4599</td>
<td>0.4295</td>
<td>0.6389</td>
<td>0.5638</td>
<td>0.7284</td>
<td><strong>0.7833</strong></td>
</tr>
<tr>
<td>2d-3c-no123</td>
<td>0.6337</td>
<td>0.5849</td>
<td>0.6253</td>
<td>0.8608</td>
<td>0.7944</td>
<td>0.8947</td>
<td><strong>0.9575</strong></td>
</tr>
<tr>
<td>Aggregation</td>
<td>0.7681</td>
<td>0.7246</td>
<td>0.7290</td>
<td>0.9666</td>
<td>0.9566</td>
<td>0.9803</td>
<td><strong>0.9884</strong></td>
</tr>
<tr>
<td>Chainlink</td>
<td>0.2891</td>
<td>0.3455</td>
<td>0.3082</td>
<td>0.4318</td>
<td>0.4769</td>
<td><strong>0.5939</strong></td>
<td>0.5025</td>
</tr>
<tr>
<td>Wingnut</td>
<td>0.7697</td>
<td>0.8463</td>
<td>0.6673</td>
<td>0.8267</td>
<td>0.7445</td>
<td>0.7656</td>
<td><strong>0.9478</strong></td>
</tr>
<tr>
<td>Glass</td>
<td>0.3498</td>
<td>0.3160</td>
<td>0.2836</td>
<td>0.3807</td>
<td>0.3570</td>
<td>0.3606</td>
<td><strong>0.3822</strong></td>
</tr>
<tr>
<td>Ecoli</td>
<td>0.5465</td>
<td>0.4938</td>
<td>0.4921</td>
<td>0.5822</td>
<td>0.5763</td>
<td>0.5680</td>
<td><strong>0.7130</strong></td>
</tr>
<tr>
<td>KM</td>
<td>0.2700</td>
<td>0.2956</td>
<td>0.2981</td>
<td>0.3704</td>
<td>0.3733</td>
<td>0.3528</td>
<td><strong>0.4142</strong></td>
</tr>
<tr>
<td>Yeast</td>
<td>0.2023</td>
<td>0.1794</td>
<td>0.1725</td>
<td>0.2092</td>
<td>0.2280</td>
<td>0.2121</td>
<td><strong>0.2507</strong></td>
</tr>
<tr>
<td>Segmentation</td>
<td>0.5397</td>
<td>0.5398</td>
<td>0.5095</td>
<td>0.6059</td>
<td>0.5882</td>
<td>0.5336</td>
<td><strong>0.6418</strong></td>
</tr>
</tbody>
</table>

### Table 3. Evaluations of image segmentation results

<table>
<thead>
<tr>
<th>Methods</th>
<th>PRI ↑</th>
<th>VOI ↓</th>
<th>GCE ↓</th>
<th>BDE ↓</th>
</tr>
</thead>
<tbody>
<tr>
<td>WTQ</td>
<td>71.10</td>
<td>3.42</td>
<td>43.44</td>
<td>15.49</td>
</tr>
<tr>
<td>WCT</td>
<td>71.38</td>
<td>3.40</td>
<td>42.88</td>
<td>15.36</td>
</tr>
<tr>
<td>CSM</td>
<td>71.34</td>
<td>3.40</td>
<td>42.83</td>
<td>15.57</td>
</tr>
<tr>
<td>MCLA</td>
<td>72.05</td>
<td>3.43</td>
<td>43.72</td>
<td>14.73</td>
</tr>
<tr>
<td>CSPA</td>
<td>71.10</td>
<td>3.54</td>
<td>45.35</td>
<td>15.47</td>
</tr>
<tr>
<td>HBGF</td>
<td>71.64</td>
<td>3.49</td>
<td>44.43</td>
<td>15.11</td>
</tr>
<tr>
<td>SCE</td>
<td><strong>75.81</strong></td>
<td><strong>3.07</strong></td>
<td><strong>36.95</strong></td>
<td><strong>13.81</strong></td>
</tr>
</tbody>
</table>

Fig. 1. Example of image segmentation based on SCE, (b-c) two different segmentation results, (d) the ensemble segmentation in which uncertain regions are marked by gray color.

Fig. 2. Segmentation results based on different clustering ensemble methods.

Utilizing different clustering ensemble methods for image segmentation, we evaluate the clustering-based segmentation results by the measurements of BDE [23], PRI [24], VOI [25] and GCE [26]. To accelerate the clustering-based segmentation, we compress image pixels to superpixels [27] for clustering ensemble. Moreover, referring to [28], we initialize the cluster number from 2 to 6 for each image and determine the optimal cluster number to form the final image segmentation according to the highest PRI. Table 3 lists the detailed evaluations and Fig.2 presents some comparative segmentation results. We can find that our method produces more precise image segmentations than other clustering methods.

### 5. CONCLUSION

To tackle the drawback of handling the data uncertainty in clustering ensemble, we propose a stability measure of data in clustering ensemble based on evidence theory and thereby implement a stable clustering ensemble (SCE) method with the stability measure. The experiments of structural data clustering and image segmentation validate the superiority of the proposed SCE method. Our future work will focus on the acceleration of the stability computation.
6. REFERENCES


