Computational Statistics. Chapter 1: Continuous optimization. Solution of exercises

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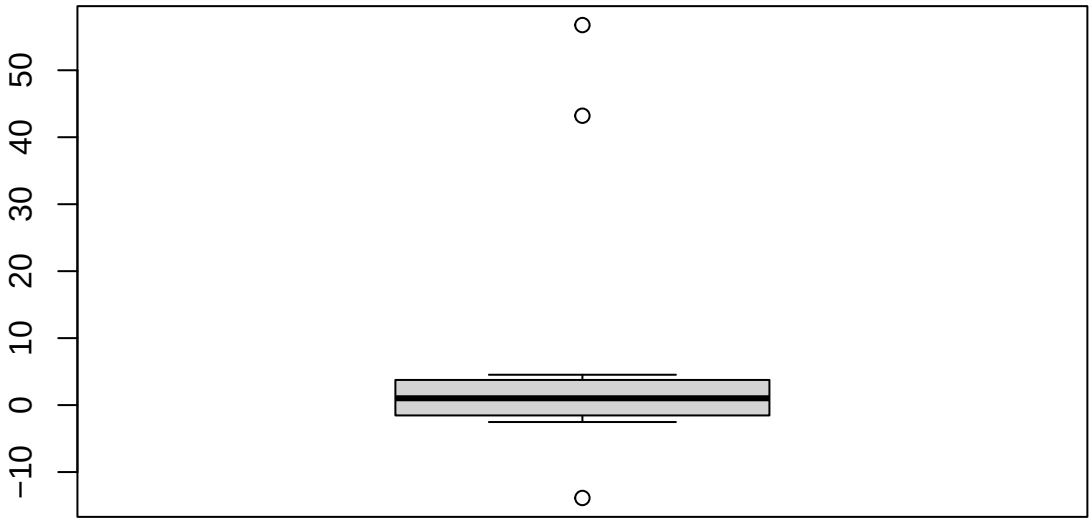
8/18/2021

Exercise 1

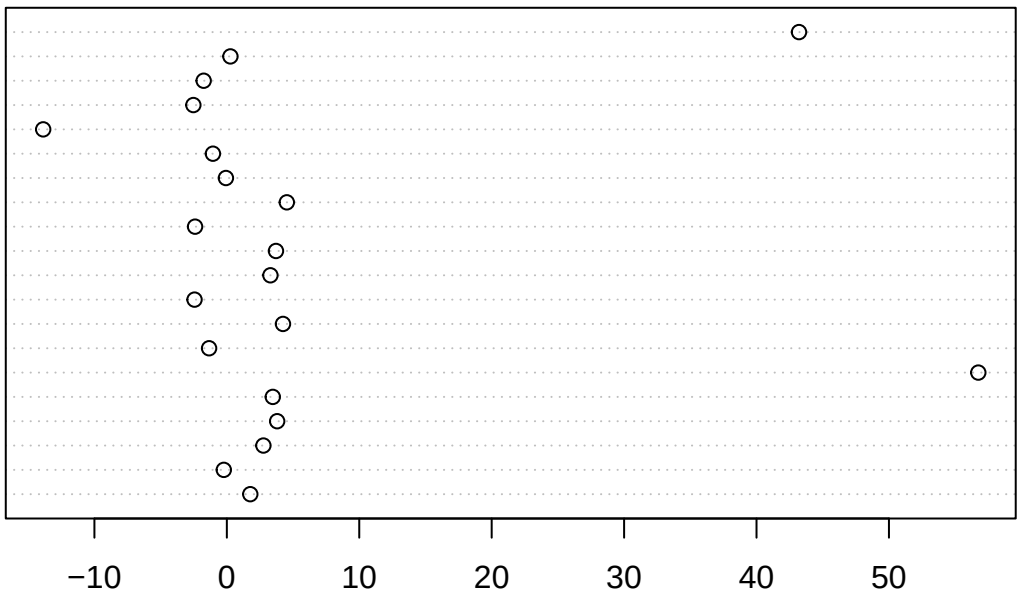
Question a

```
x<-c(1.77,-0.23,2.76,3.80,3.47,56.75,-1.34,4.24,-2.44,
     3.29,3.71,-2.40,4.53,-0.07,-1.05,-13.87,-2.53,
     -1.75,0.27,43.21)
n<- length(x)

boxplot(x)
```




```
dotchart(x)
```



Question b

We first write a function to compute the log-likelihood:

```
loglik <- function(theta,x) return(sum(log(dcauchy(x,location=theta))))
```

We compute the log-likelihood for different values of θ:

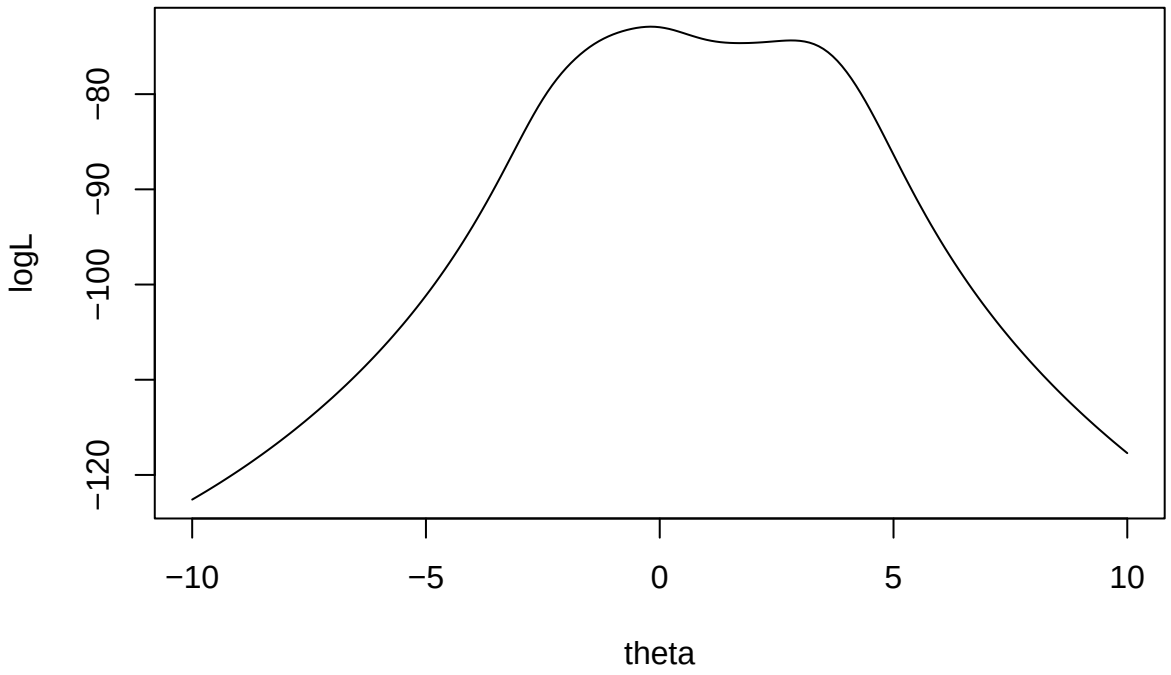
```
theta<- seq(-10,10,0.1)
N<-length(theta)
logL<-rep(0,N)
for(i in 1:N) logL[i]<- loglik(theta[i],x)
```

We can get the same result much faster without a loop, thanks to function `sapply`:

```
logL<-sapply(theta,loglik,x)
```

Finally, we plot the result:

```
plot(theta,logL,type="l")
```



We observe that the likelihood has 2 modes.

Question c

We first need to compute the score function (first derivative of the log-likelihood). We have

$$L(\theta) = \frac{1}{\pi^n} \prod_{i=1}^{n} \frac{1}{(x_i - \theta)^2 + 1}$$

$$\ell(\theta) = -\sum_{i=1}^{n} \log[(x_i - \theta)^2 + 1] - n \log \pi$$

$$\ell'(\theta) = 2 \sum_{i=1}^{n} \frac{x_i - \theta}{(x_i - \theta)^2 + 1}$$

We can then write the R function:

```
dloglik <- function(theta,x) return(2*sum((x-theta)/((x-theta)^2+1)))
```

This is a function that encodes the bisection method:

```
bisection <-function(fun,dfun,a,b,epsi,...){
    theta<-(a+b)/2
    delta<-1
    while(delta>epsi){
        theta0<-theta
        if(dfun(a,x)*dfun(theta0,...)<=0) b<-theta0 else a<-theta0
        theta<-(a+b)/2
        delta<-abs(theta-theta0)/abs(theta0)
        print(c(a,b,delta))
    }
    return(list(objective=fun(theta,...),optimum=theta))
}
```

We run it on the data and plot the result:

```
opt<-bisection(loglik,dloglik,-1,3,1e-6,x)
```

```
## [1] -1  1  1
## [1]  -1   0 Inf
## [1] -0.5  0.0  0.5
## [1] -0.25  0.00  0.50
## [1] -0.250 -0.125  0.500
## [1] -0.2500000 -0.1875000  0.1666667
## [1] -0.21875000 -0.18750000  0.07142857
## [1] -0.20312500 -0.18750000  0.03846154
## [1] -0.1953125 -0.1875000  0.0200000
## [1] -0.19531250 -0.19140625  0.01020408
## [1] -0.193359375 -0.191406250  0.005050505
## [1] -0.192382812 -0.191406250  0.002538071
## [1] -0.192382812 -0.191894531  0.001272265
## [1] -0.192382812 -0.192138672  0.000635324
## [1] -0.1923828125 -0.1922607422  0.0003174603
## [1] -0.1923217773 -0.1922607422  0.0001586798
## [1] -1.922913e-01 -1.922607e-01  7.935248e-05
## [1] -1.922913e-01 -1.922760e-01  3.967939e-05
## [1] -1.922913e-01 -1.922836e-01  1.983891e-05
## [1] -1.922874e-01 -1.922836e-01  9.919257e-06
## [1] -1.922874e-01 -1.922855e-01  4.959678e-06
## [1] -1.922874e-01 -1.922865e-01  2.479827e-06
## [1] -1.922870e-01 -1.922865e-01  1.239910e-06
## [1] -1.922867e-01 -1.922865e-01  6.199559e-07
```

```
plot(theta,logL,type="l")
points(opt$optimum,opt$objective)
```