Matching and decision for Vehicle tracking in road situation

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Abstract

We present in this communication the development of a multi-objects matching algorithm with ambiguity removal entering into the design of a dynamic perception system for intelligent vehicles. The originality of this system lies in the use of theories such as fuzzy mathematics and belief theory which allow the handling of inaccurate as well as uncertain information. Moreover, these theories allow both numeric and symbolic data fusion. We started from the hypothesis that we have some sensors providing redundant information in time. We develop in this article the problem of matching between the prediction (known objects) and the perception result (perceived objects). This make it possible to update a dynamic environment map for a vehicle. The belief theory will enable us to quantify association belief on each perceived and known objects. Some conflicts can appear in the case of object appearance or disappearance, or in the case of a bad perception or a confused situation. These conflicts are removed or solved using an assignment algorithm, giving a solution called the « best » and so ensuring the multi-objects tracking.

1 Introduction

Our research is focused on perception systems for vehicle in road situation. In order to increase driving safety, the European project Prometheus [1] has given the beginning of an answer about what could be the vehicle of the future: an interaction between the driver and the vehicle through a driving assistance system.

Our works is carrying on this project. We are particularly interested in data fusion algorithms giving to the driver some accurate but above all reliable and pertinent information in relation with the current situation. In order to deal with information reliability, we have designed a perception algorithm combining some tools dealing simultaneously with the inaccuracy and uncertainty of dynamic environment representation.

In this communication, we present a part of a perception system with the study of a multi-objects matching algorithm including ambiguity removal. Its goal is to associate perceived objects with known objects using fuzzy measures. This fuzzy data representation is introduced in a fuzzy estimator-predictor presented in [2]. The fuzzy measures (high level data) represent the perceived objects and the fuzzy prediction windows represent the known objects. The estimator allows the extraction of new data such as the number of objects in our environment with their associated inaccuracy and uncertainty. The predictor enables us to take into account the dynamics of each vehicle into our environment [3]. The matching algorithm makes it possible to switch from a multi-target detection to a multi-target tracking mode, which gives the possibility to take into account the appearance and disappearance of every object within our environment.

The management of these appearances and disappearances allows us to propagate virtual objects through their predictions. These virtual objects then have an uncertainty in time. When this uncertainty becomes too great, the object disappears. This object propagation reduces the effect of awkward events such as objects crossing, measure deterioration due to weather conditions, or temporary sensors degradation (information missing or false alarms).

Moreover, the multi-object tracking algorithm avoids some problems encountered by other algorithms of the same kind like the PDAF, which is not adjusted to targets crossing, the JPDAF, which takes into account a fixed number of targets and doesn't initialise new tracks, or the MHT that has combinatorial problems [4] [5].

We quickly present, in a first part, the basic notions, the general points and the disadvantage of the belief theory. Then we suggest a generalisation of the Dempster's combination rule [6] applied to our problem. These works are based on Michèle Rombaut research [7]. In second part, we will give an optimal solution to remove these conflicts and then obtain a new decision called « the best ». Afterwards, we will describe the way to build the initial mass set through a concordance operator between known and perceived objects. We will finish with some operating examples, a conclusion and our perspectives.

2 Belief theory for dynamic association

2.1 Generalities

Belief theory allows both to model and to use uncertain and inaccurate data, as well as qualitative and quantitative data, so as to keep a consistency and homogeneity with all concepts and tools develop in the remaining of the algorithm shown in [3]. This theory is well known to "*take into account what remains unknown and to represent perfectly what is already known*".

In a general framework, we can say that our problem consists in identifying an object designated by a generic variable X among a set of hypotheses *Y*i. One of these hypotheses is in a position to be the solution. In our case, we want to associate perceived objects X_i to known objects Y_j. Belief theory allows us to value the veracity of P_i propositions representing the matching of our different objects. These propositions can be simple as well as complex:

P_1 = " perceived object *X is* known object Y_i "

P_2 = " perceived object *X is* known object Y_i or Y_j "

We must then define a magnitude allowing the characterization of this truth. This magnitude is the elementary probabilistic mass $m_\Theta()$ defined on [0,1]. This mass is very close to the probabilistic mass, to the exception that we do not distribute this mass only on single elements but on all elements of the definition referential:

$$2^{\Theta} = \{ A / A \subseteq \Theta \} = \{\varnothing, Y_1, Y_2, ..., Y_n, Y_1 \cup Y_2, \ldots, \Theta\}.$$

This referential is build through the frame of discernment $\Theta = \{Y_1, Y_2, \cdots, Y_n\}$ gathering all admissible hypotheses, which must be exclusive ($Y_i \cap Y_j = \varnothing$, $\forall$ i $\neq$ j). This distribution is function of the knowledge about the source to model. The whole mass obtained is called « distribution of mass ». The sum of these masses is equal to 1 and the mass given to the impossible case $m(\varnothing)$ must be equal to 0.

2.2 The information combination

The combination of information coming from different sources have the advantages of increasing the information reliability and reducing the influence of failing information (inaccurate, uncertain, incomplete and conflicting). But to obtain this result, it is necessary to have complementary and/or redundant information.

The Dempster combination rule consists in obtaining a single mass distribution $m_\Theta()$ by combination of *n* elementary mass sets $m_\Theta^{Sj}()$ (which corresponding to the opinion of the source *j* about courant situation). We have thus an orthogonal sum noted: $m_\Theta = m_\Theta^{S1} \oplus \cdots \oplus m_\Theta^{Sn}$, $\oplus$ being the operator of the Dempster combination. Our new mass set is built with the conjunctions of focal element set on each source:

$$m_\Theta(A) = \sum_{A_i \cap B_j = A} m_\Theta^{S_1}(A_i) \cdot m_\Theta^{S_2}(B_j)$$

But it is possible to get an empty hypotheses conjunction ($A_i \cap B_j = \varnothing$) and by definition, we must have the empty mass $m_\Theta(\varnothing)=0$ and the masses sum on each proposition equal to 1. So it is necessary to re-allocate the mass affected to empty set on all other masses. For that, we need to re-normalise the final set of masses with the re-normalization coefficient K_Θ.

$$k_\Theta = \sum_{A_i \cap B_j = \varnothing} m_\Theta^{S_1}(A_i) \cdot m_\Theta^{S_2}(B_j) \qquad K_\Theta = \frac{1}{1 - k_\Theta}$$

In the framework of a processing an exhaustive frame of discernment, the combination of a great number of sources lead to a combinatorial explosion. This is the main drawback of this combination rule. On the other hand, it offers the advantage of being associative and commutative, which is not the case of the majority of the fusion operators.

2.3 Generalized combination and multi-objects association

In order to reduce this combinative complexity, we limit the reference frame of definition while adding as constraint that a perceived object can be connected with one and only one known object.

For example, for a detected object to associate among 3 known objects, we will have the following frame of discernment:

$\Theta = \{Y_1, Y_2, Y_3\}$ *with* Y_i *meaning X is in relation with* Y_i

From this frame of discernment, we build the referential of definition according to:

$$2^\Theta = \{*, Y_1, Y_2, Y_3, Y_1 \cup Y_2, Y_1 \cup Y_3, Y_2 \cup Y_3, \Theta\}$$

$$2^\Theta = \{*, Y_1, Y_2, Y_3, \overline{Y}_1, \overline{Y}_2, \overline{Y}_3, \Theta\}$$

with $\overline{Y}_1$, which mean X is not in relation with Y_i

In this referential of definition, we find the singleton hypotheses of the frame of discernment to which we added ignorance with the hypothesis Θ and the nothing with the hypotheses *. We obtain then a distribution of masses made up of the following masses:

$m_{i,j}(Y_j)$: mass associated with the proposition « X_i *is in relation with* Y_j. »

$m_{i,j}(\overline{Y}_j)$: mass associated with the proposition « X_i *is not in relation with* Y_j. »

$m_{i,j}(\Theta_{i,j})$: mass representing ignorance.

$m_{i,.}(*)$: mass representing the reject : « X_i *is in relation with nothing*. »

In this distribution of mass, the first index *i* indicates the processed perceived object and the second index *j* the known object. If an index is replaced by a dot, it means that the mass is applied to all objects perceived or known according to the location of this dot.

Moreover, if we use a combination in cascade, the mass $m_{i,.}(*)$ is not part of the initial mass set and appears only after the first combination. It replaces the conjunction of the combined masses $m_{i,j}(\overline{Y}_j)$.

By observing the behaviour of the combination in cascade with *n* mass sets, we unveiled a general behaviour which enables us to put in equation the final mass set according to the initial mass sets. We thus obtain an independence of our final masses in relation to the recurrence of the combination.

$$m_{i,.}(Y_j) = K_{i,.} \cdot m_{i,j}(Y_j) \cdot \prod_{\substack{k=1\cdots n \\ k \neq j}} (1 - m_{i,k}(Y_k))$$

$$m_{i,.}(*) = K_{i,.} \cdot \prod_{j=1\cdots n} m_{i,j}(\overline{Y}_j)$$

$$m_{i,.}(\Theta) = K_{i,.} \cdot \left(\prod_{j=1\cdots n} \left[m_{i,j}(\Theta_{i,j}) + m_{i,j}(\overline{Y}_j) \right] - \prod_{j=1\cdots n} m_{i,j}(\overline{Y}_j) \right)$$

with $K_{i,.} = \prod_{l=1\cdots n} K_{i,l}$

$K_{i,.}$ is the re-normalization of the n combinations, the product of the various re-normalization carried out during all the combinations.

$$m_{i,.}(\varnothing) = \left[\prod_{l=1\cdots n-1} K_{i,l}\right] \cdot m_{i,n}(Y_n) \cdot \sum_{k=1\cdots n-1}\left(m_{i,k}(Y_k) \cdot \left[\prod_{\substack{m=1\cdots n-1 \\ m\neq k}} A_m\right]\right)$$

with $A_m = m_{i,m}(\Theta_{i,m}) + m_{i,m}(\overline{Y}_m)$

$$K_{i,.} = \frac{1}{1 - m_{i,.}(\varnothing)}$$

$$K_{i,.} = \frac{1}{\prod_{j=1\cdots n}\left(1 - m_{i,j}(Y_j)\right) \cdot \left(1 + \sum_{j=1}^{n} \frac{m_{i,j}(Y_j)}{1 - m_{i,j}(Y_j)}\right)}$$

From each mass set, we build two matrices $\mathrm{M}_{i,.}^{cr}$ and $\mathrm{M}_{.,j}^{cr}$ which give the belief that a perceived object is associated with a known object and conversely. The sum of the elements of each column is equal to 1 due to re-normalisation. The resulting frames of discernment are:

$\Theta_{.,j} = \{Y_{1,j}, Y_{2,j}, \cdots, Y_{n,j}, Y_{*,j}\}$ and

$\Theta_{i,.} = \{X_{i,1}, X_{i,2}, \cdots, X_{i,m}, X_{i,*}\}$

We can interpret $Y_{*,1}$ as the relation «*no perceived object X_i is in relation with the known object Y_1*» and $Y_{1,*}$ as the relation «*the perceived object X_1 is not dependent with any known object*». In the first case we can deduce that an object has just disappeared and in the second case, that an object has just appeared. These objects, which appeared or disappeared, can also be false alarms.

The following step consists in establishing the best decision on association using these two matrices obtained previously. As we use a referential of definition built with singleton hypotheses, except Θ and *, the use of the mass redistribution function would not add any useful additional information. This redistribution would quite simply reinforce the fact that our perceived object is really in relation with a known object. This is why we use for our decision criterion the maximum of belief on each column of the two belief matrices.

$$d(Y_{i,.}) = \underset{j}{Max}[M_{i,j}^{Cr}]$$

This rule answers the question « *what is the known object Y_j in relation with the perceived object X_i* ». We have the same rule for the known objects:

$$d(X_{.,j}) = \underset{i}{Max}[M_{i,j}^{Cr}]$$

The problem is then to know how to process ambiguities. Ambiguity will intervene when an object, perceived or known, is in relation with two perceived or known objects, or if the first maximisation gives a decision on the relation between objects X_i and Y_j and the second maximisation gives a decision that is contradictory to the first, for example Y_j in relation to X_k, $i \neq k$.

The following step consists in obtaining a matrix in which all the objects will be classified without ambiguities and with a maximisation of the belief on the decision.

One wants to thus ensure that the decision taken is not " *good* " but " *the best* ". By the " *best* ", we mean that if we have a known object and some defective or frustrate sensor to perceive it, then we are unlikely to know what this object corresponds to, and therefore we have little chance to ensure that the association is good. But among all the available possibilities, we must certify that the decision is the " *best* " of all possible decisions.

It is thus necessary to find a way of combining the lines and the columns of the two matrices of beliefs in order to obtain a new general matrix representing final associations. The following chapter describes how to solve this problem while avoiding the study of all the combinations of the elements concerned by the conflict.

3 Conflicts Resolution

3.1 Affectation: a solution to resolve the conflicts

In order to use the most of possible information and to obtain an optimal decision with maximisation of the belief sum, we have decomposed the two belief matrix $\mathrm{M}_{.,j}^{cr}$ et $\mathrm{M}_{i,.}^{cr}$. Thus we obtain a more synthetic new structure combining all information at our disposal.

The decomposition of each of these matrices gives the sub-matrices A_1, A_2 and B_1, B_2. The two first represent the relations between the various objects and the two others represent the impossibility and unknown concepts on the relations linking the objects.

The two matrices B_1 and B_2 contain information on the appearance or disappearance of targets, or the expression of a conflict. By applying a conjunction to all the relations included in the matrices A_1 and A_2, we obtain a new more synthetic matrix that represents the relations between the n perceived objects and the m known objects. This matrix is homogeneous since we handle the same objects in the two matrices we combine.

We can interpret this new matrix as being a cost matrix connecting two sets of data. Our goal is now to find the best two to two assignment of n perceived objects with m known objects. Setting this matrix in graph form brings us back to a traditional problem of assignment, which is generally seen as a particular case of the transport problem without capacities [8]. It can also be seen as a problem of perfect coupling with minimum weight (or maximum) in a bipartite graph [9].

If the known objects are independent, the total belief on our coupling is the sum of the beliefs of each couple *perceived objects / known objects*. Our problem is thus an assignment problem on the bipartite graph *perceived objects / known objects*. An arc of this graph will indicate a possible assignment of a perceived object with a known object and will be valued by the corresponding belief. The required solution is thus a coupling of n arcs and maximum belief with a constraint of non-adjacency on the couples which means that a perceived object can be associated with one and only one known object and reciprocally. These algorithms of coupling have the advantage of generalising the assignment problems and being a part of a class of linear programs in integer numbers that admit a resolution algorithm with polynomial complexity in N and M (the number of arcs and the number of nodes of the graph) [10].

In our system, we used a traditional assignment algorithm called the Hungarian algorithm [11]. This algorithm relies on the fact that we do not change the problem by subtracting from a line (or a column) any number α. This property enables us to reveal admissible arcs representing the most probable relations.

The general principle of this Hungarian algorithm is the recurrence of the operations of *coupling*, of *search of improving chains, marking* and *modification of the admissible arcs* as long as we do not obtain a maximum coupling.

3.2 Maximisation of the decision belief

With this assignment algorithm, we have an optimal decision in the sense of the maximisation of the sum of belief. This algorithm is based on the processing of a square matrix, otherwise we add fictitious elements in order to have an exhaustive coupling, that is a coupling where each perceived object is affected to a known object. Each relation with a virtual object is valued with a belief equal to **0**.

For now, our decision is made of real or fictitious objects. In order to obtain our final decision, first we will take out these elements, then we will remove the assignments with a belief that is lower than the belief $m_{i,j}(*)$ associated with nothing. In fact, this second filtering enables us to use information on the unknown not used in the coupling algorithm. These two filterings are summarised by the following equation:

$$x_{i,j} = \begin{cases} 1 \quad if & m_{i,j}(Y_i) > \max\limits_{i \le |X|, j \le |Y|}(m_{\cdot,j}(*), m_{i,\cdot}(*)) \\ 0 & \text{else} \end{cases}$$

$x_{i,j}$ represents the relation between X_i and Y_j, this relation is validated if $x_{i,j}$=1 and is rejected if not.

3.3 Quantification of decision confidence

The cost that we will calculate from the sum of the beliefs will enable us to quantify the confidence we have in our decision. To say that a decision is " *the best* " is well but not sufficient. It is necessary to be able to quantify this concept of " *better* ". If in a case, the cost is 0.4 and in a second case, we have a cost of 0.9, we will be certain in both cases that the two decisions are the best (the two cases are obviously independent). However we will tend to give more confidence to the second decision because this one reflects a greater reliability on association.

This confidence can be obtained by using the cardinality of the coupling that gives us maximum confidence and by using the beliefs. The coefficient obtained represents the percentage of confidence we have in our decision. Knowing the cardinal of our association, we know that, if we have a maximum confidence on all associations (we do not have any unknown factor then), the cost associated on our decision will be equal to this cardinal. This means that the belief on each association is equal to **1**. Confidence we have in our decision is then:

$$\Psi = \frac{\sum_{i=1}^{X_n} \sum_{j=1}^{Y_m} C_{ij} \cdot x_{ij}}{\min(|X_n|, |Y_m|)}$$

C_{ij} represents the belief that object X_i is in relation to the object Y_j.

$x_{i,j}$ represents if the object X_i is associated with the object Y_j.. or not.

4 Generation of the sets of masses

One of the difficulties of this theory implementation is the creation of the set of the initial masses. To generate them, we must firstly use a distance measurement that quantifies the similarity between our perceived objects and our known objects, and secondly we need an operator that generates our mass set from our similarity index. According to the model of representation for the used information, this index is computed with the distance of Mahalanobis in the statistical representation framework or, in the possibilist framework, with the possibilist index or the index of Jacquard [12].

We studied an index of similarity from a representation of the objects by fuzzy quantities [3]. The support of a fuzzy quantity represents the inaccuracy around measurement and the height, its uncertainty.

The index of similarity (or agreement) quantifies, by a geometrical approach, the agreement between two fuzzy quantities that are symmetrical, asymmetrical, normalised or sub-normalised. We give here two versions of this distance measurement.

Similarity measurement for 1 dimension objects:

$$X_i \cap Y_j = \frac{(a+b) \cdot h(X_i \cap Y_j)}{2}$$

$$X_i = \frac{(c+d) \cdot h(X_i)}{2}$$

$$\Im_s = \frac{X_i \cap Y_j}{\underset{h(X_i)=1}{X_i}} = \frac{(a+b) \cdot h}{(c+d)}$$

Similarity measurement for 2 dimensions objects:

$$X_i \cap Y_j = \frac{h(X_i \cap Y_j)}{6} \cdot [c \cdot (2 \cdot b + b_2) + c_2 \cdot (2 \cdot b_2 + b)]$$

$$X_i = \frac{h(X_i)}{6} \cdot [i \cdot (2 \cdot a + a_2) + i_2 \cdot (2 \cdot a_2 + a)]$$

$$\Im_v = \frac{X_i \cap Y_j}{\underset{h(X_i)=1}{X_i}} = \frac{h(X_i \cap Y) \cdot [c \cdot (2 \cdot b + b_2) + c_2 \cdot (2 \cdot b_2 + b)]}{[i \cdot (2 \cdot a + a_2) + i_2 \cdot (2 \cdot a_2 + a)]}$$

This index quantifies the intersection between the known object and the perceived object. It is normalised by the projection of the fuzzy measurement whose certainty would be equal to 1. This index makes it well possible to take into account the uncertainty and the inaccuracy of the objects (perceived or known). The figures 4.1 and 4.2 show the behaviour of this agreement operator (2D) when we apply a translation motion on the perceived object on his axis and when we increase the uncertainty on the known object. We see the influence of a strong inaccuracy of our fuzzy measurement on the computation of the measurement of similarity. Indeed, we observe that the value of the agreement index is never equal to 1 even if the certainty of the perceived object and the certainty of the known object are both maximum.

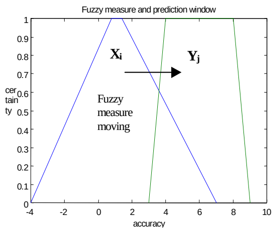

Fig 4.1 fuzzy quantities: perceived object and known object

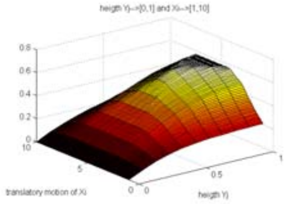

Fig 4.2 Concordance index behaviour

Using this index, we can generate our mass sets, but for that, it is necessary to find a suitable operator. Several works were already lead by [13], [14] or [15]. Often these operators are only well suited to particular cases. The most traditional operators are based either on exponential (adapted to classification), or on probabilities which makes it possible to handle the information modelled by the Gaussian. In our case, we want masses within [0,1] with the mass equal to **0** in the case of a total discordance and the mass equal to **1** for sources in total agreement. One wants also to be able to take into account the reliability of the information sources. This led us to develop this set of functions:

$$d_{i,j} = \pi \cdot (2 \cdot (1 - \Im_v) - 1)$$

$$m_{i,j}(Y_j) = \alpha_0 \cdot \left(1 - \left(\frac{\sin(\frac{d_{i,j}}{2}) + 1}{2}\right)\right)$$

$$m_{i,j}(\overline{Y}_j) = \alpha_0 \cdot \left(\frac{\sin(\frac{d_{i,j}}{2}) + 1}{2}\right)$$

$$m_{i,j}(\Theta_{i,j}) = 1 - \alpha_0$$

The coefficient α_0 represents the reliability of information sources.

The generated set of masses has the properties of the belief theory and reflects the initial beliefs on the hypotheses of the frame of discernment.

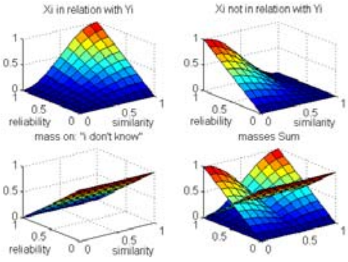

Fig 4.4 Generation of the mass set

5 Examples

In this part, we will show a representative example of the operating mode of this multi-object association algorithm with ambiguity removal. The purposes of this example will first to handle appearances and disappearances of objects or false alarm and secondly to maximise the sum of the beliefs on our association and thus to obtain " *the best* " decision.

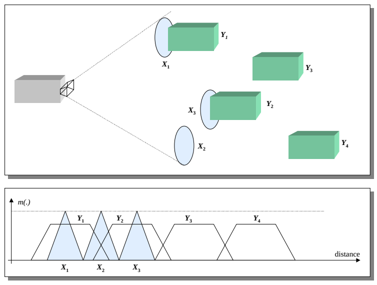

Fig 5.1 Our road situation corresponding to our scenario

This example simulates a scenario that includes three perceived objects and four known objects (fig 5.1). One object is a 1D information (distance). In the three perceived objects, we have a false alarm or the appearance of an object. In the four known objects, we have an object that probably has just disappeared. The sets of masses for each set of relations between a perceived object and the known objects are given below.

Set of mass associated to X_1

$$m_{1,1}(Y_1) = .8 \quad m_{1,2}(Y_2) = .5 \quad m_{1,3}(Y_3) = .1 \quad m_{1,4}(Y_4) = 0$$
$$m_{1,1}(\overline{Y}_1) = .1 \quad m_{1,2}(\overline{Y}_2) = .4 \quad m_{1,3}(\overline{Y}_3) = .8 \quad m_{1,4}(\overline{Y}_4) = .9$$
$$m_{1,1}(\Theta_{1,1}) = .1 \quad m_{1,2}(\Theta_{1,2}) = .1 \quad m_{1,3}(\Theta_{1,3}) = .1 \quad m_{1,4}(\Theta_{1,4}) = .1$$

Set of mass associated to X_2

$$m_{2,1}(Y_1) = .5 \quad m_{2,2}(Y_2) = .5 \quad m_{2,3}(Y_3) = .1 \quad m_{2,4}(Y_4) = 0$$
$$m_{2,1}(\overline{Y}_1) = .1 \quad m_{2,2}(\overline{Y}_2) = .1 \quad m_{2,3}(\overline{Y}_3) = .7 \quad m_{2,4}(\overline{Y}_4) = .9$$
$$m_{2,1}(\Theta_{2,1}) = .4 \quad m_{2,2}(\Theta_{2,2}) = .4 \quad m_{2,3}(\Theta_{2,3}) = .2 \quad m_{2,4}(\Theta_{2,4}) = .1$$

Set of mass associated to X_3

$$m_{3,1}(Y_1) = .4 \quad m_{3,2}(Y_2) = .8 \quad m_{3,3}(Y_3) = .1 \quad m_{3,4}(Y_4) = 0$$
$$m_{3,1}(\overline{Y}_1) = .1 \quad m_{3,2}(\overline{Y}_2) = .1 \quad m_{3,3}(\overline{Y}_3) = .6 \quad m_{3,4}(\overline{Y}_4) = .9$$
$$m_{3,1}(\Theta_{3,1}) = .5 \quad m_{3,2}(\Theta_{3,2}) = .1 \quad m_{3,3}(\Theta_{3,3}) = .3 \quad m_{3,4}(\Theta_{3,4}) = .1$$

From these sets of masses and by using the equations given in part 2.3 we obtain a new set of masses represented in two matrices of beliefs $\mathrm{M}_{i,.}^{cr}$ and $\mathrm{M}_{.,j}^{cr}$. The first gives the belief of the relations between the perceived objects and the known objects and the second the belief between the known objects and the perceived objects.

$\mathrm{M}_{i,.}^{cr}$	Y_1	Y_2	Y_3	Y_4	*	Θ
X_1	0.6545	0.1636	0.0182	0	0.0524	0.1113
X_2	0.3214	0.3214	0.0357	0	0.0090	0.3124
X_3	0.1154	0.6923	0.0192	0	0.0087	0.1644

$\mathrm{M}_{.,j}^{cr}$	X_1	X_2	X_3	*	Θ
Y_1	0.6000	0.1500	0.1000	0.0025	0.1475
Y_2	0.1429	0.1429	0.5714	0.0114	0.1314
Y_3	0.0833	0.0833	0.0833	0.3457	0.4043
Y_4	0	0	0	0.7290	0.2710

For each one of these matrices, we obtained a decision by using the maximum of belief. The first decision obtained on the matrix $\mathrm{M}_{i,.}^{cr}$ is:

X_1 is in relation to Y_1 X_2 is in relation to Y_1
X_2 is in relation to Y_2 X_3 is in relation to Y_2

And the second decision with the matrix gives us:

Y_1 is in relation to X_1 Y_2 is in relation to X_3
Y_3 is in relation to Θ Y_4 is in relation to *

We can deduce from the first decision we have a conflict on the object to associate with X_2. The second decision shows firstly a total ignorance on the association of the first three known objects, and secondly an association of the fourth object Y_4 with nothing.

In order to solve this conflict, we will use the algorithm of ambiguity lifting on the new matrix resulting from the combination of our two belief matrices.

$M_{i,j}^{cr}$	Y_1	Y_2	Y_3	Y_4
X_1	0.3927	0.0234	0.0015	0
X_2	0.0482	0.0459	0.0030	0
X_3	0.0115	0.3956	0.0016	0

In order to get a square matrix $M_{i,j}^{cr}$, we add a virtual perceived object X_4 with for each of its relations with the known objects a belief of 0. To reveal the admissible arcs in our matrix (given by $\overline{C}_{ij} = 1 - C_{ij}$), we use the cost matrix $\overline{M}_{i,j}^{cr}$ with a complement to 1. The result of this assignment algorithm gives the following association matrix:

$X_{i,j}$	Y_1	Y_2	Y_3	Y_4
X_1	**1**	0	0	0
X_2	0	0	**1**	0
X_3	0	**1**	0	0
X_4	0	0	0	**1**

By applying our filtering, We can immediately eliminate association with the virtual objects (criterion on the cardinality: $i \leq |X|$ *and* $j \leq |Y|$), in our case, X_4 is our virtual object. Then we will use information on the belief $m_{i,j}(*)$, that is the information on the fact that an object is affected with nothing, to filter the remainder of the objects.

We have $m_{2,3}(Y_3) < \max(m_{.,3}(*), m_{2,.}(*))$ thus X_2 is associated with " *nothing* ". We then obtain as a final decision that X_1 is associated with Y_1, X_2 is associated with nothing and X_3 is associated with Y_2. This enables us to build the assignment matrix that follows:

$X_{i,j}$	Y_1	Y_2	Y_3	Y_4	*
X_1	**1**	0	0	0	0
X_2	0	0	0	0	**1**
X_3	0	**1**	0	0	0
X_4	0	0	0	0	**1**

This association enters in the design of a wider algorithm enabling us to make multi-target tracking and dynamic environment cartography.

6 Conclusion and future works

This algorithm enables us to combine the opinions we have on relations between objects. We can take into account the inaccuracy and uncertainty on all measurements and predictions. We are also able, by using these fuzzy models of data, to generate sets of masses representative of the current situation. Moreover, this combination has the advantage of being associative and commutative, which is difficult to obtain with the majority of the data fusion operators. By generalising the Dempster combination rule, we also showed that it is possible to reduce the complexity of this combination and to make it independent of the recurrence.

With the assignment algorithm, we showed that we give a decision we can affirm to be optimal, which is the "*best*". We are also able to quantify the confidence we have on this decision. This algorithm must be integrated in a vehicle perception system in order to carry out the cartography of the dynamic environment of the vehicle in the purpose of characterising the current road situation. Also, the initialisation step of the algorithm is very simple and automatic, as it does not depend on constraining and hard to implement heuristic parameters, and does not suffer from constraints due to human intervention.

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