Evidential Logistic Regression for Binary SVM Classifier Calibration

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Labex MS2T
ANR-11-IDEX-0004-02

ANR-NFSC PRETIV
ANR-11-IS03-0001

Outlines

1. **Calibration of scores**
   a) Calibration methods
   b) Logistic regression
2. **Evidential logistic regression**
   a) Likelihood-based belief function
   b) Evidential calibration
3. **Experimental evaluation**
   a) Experimental setup
   b) Classification results

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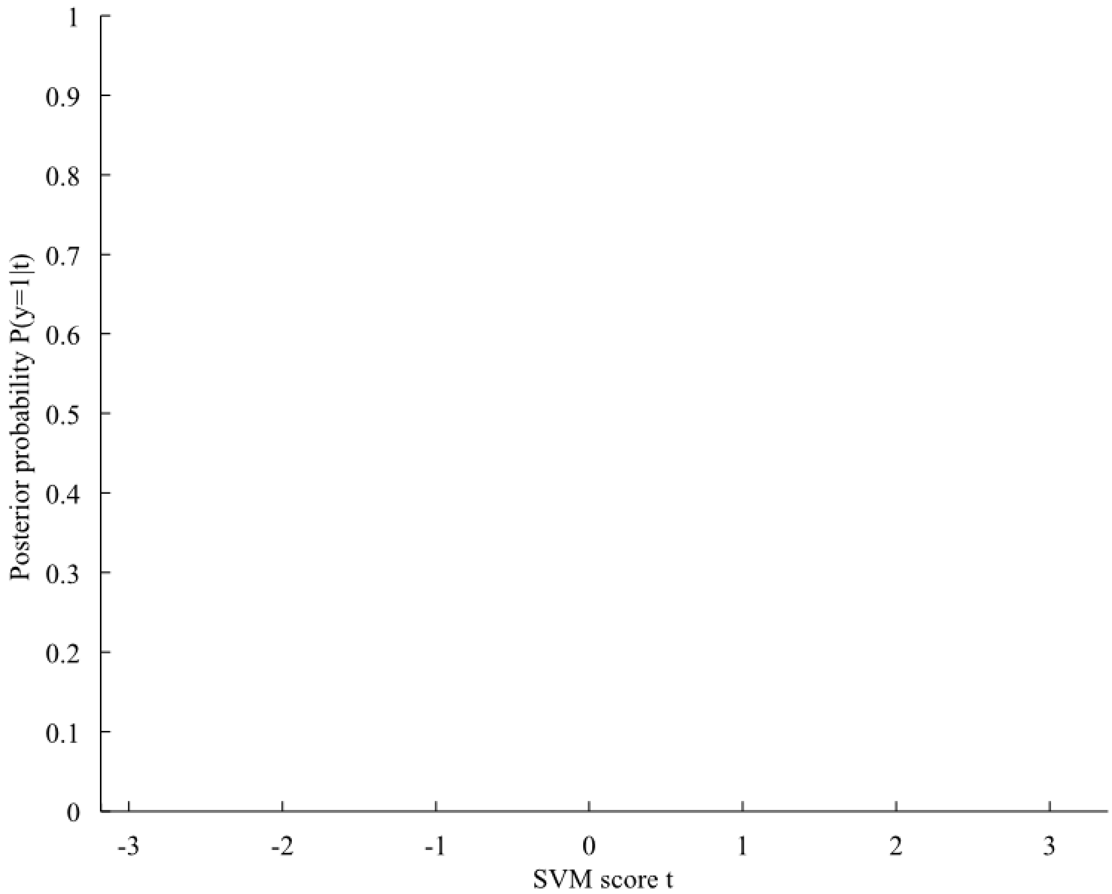
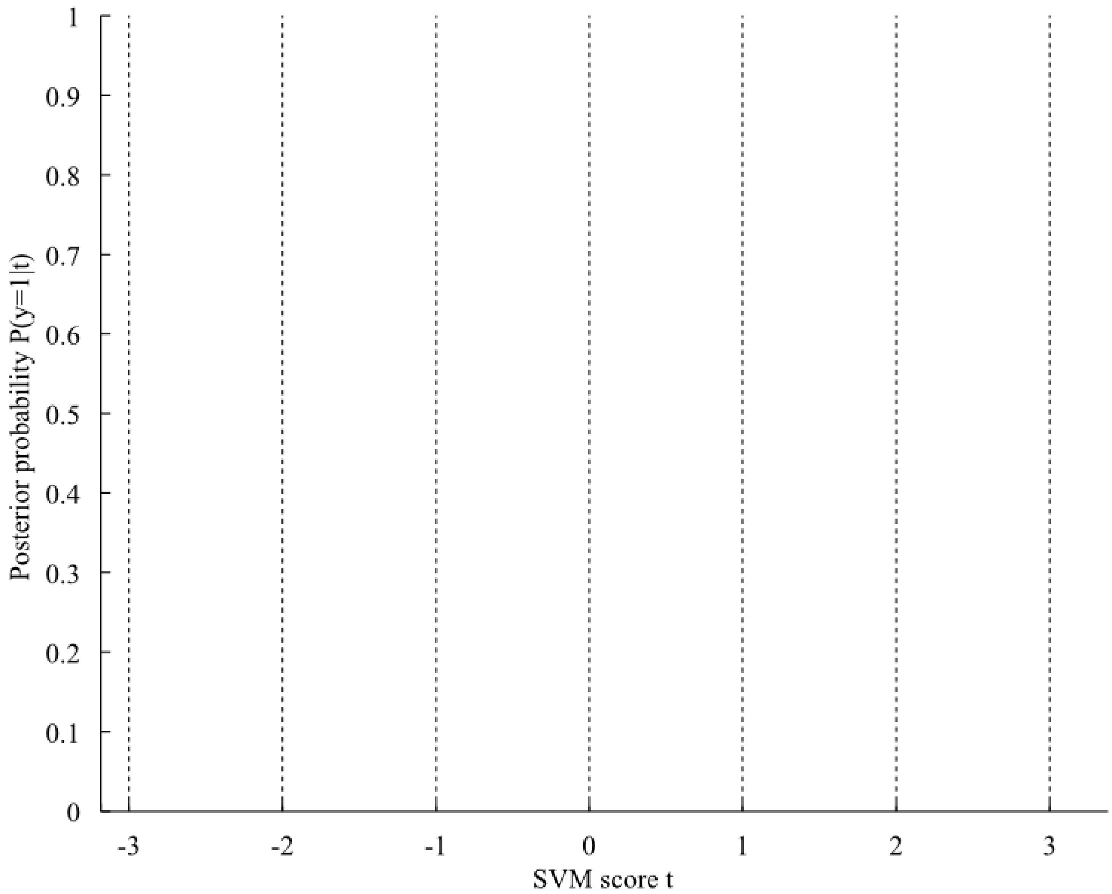
Calibration SVM scores



Calibration SVM scores



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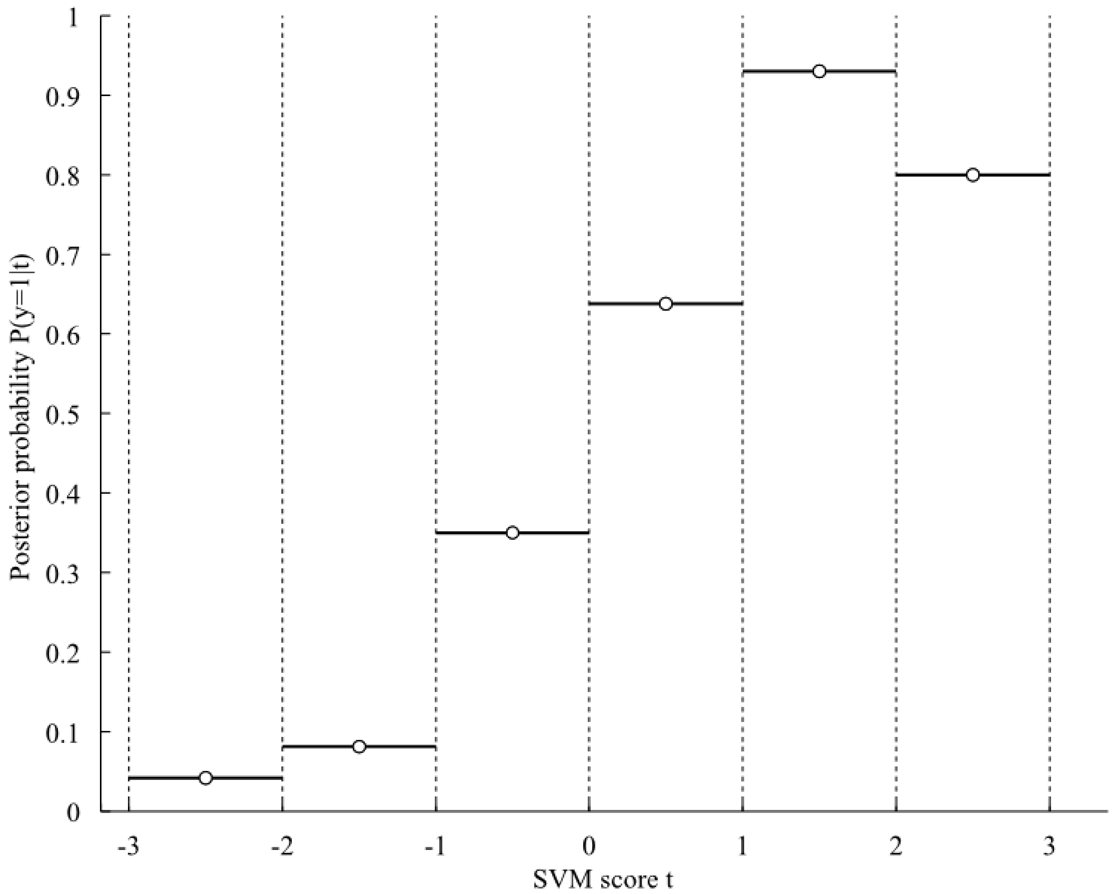

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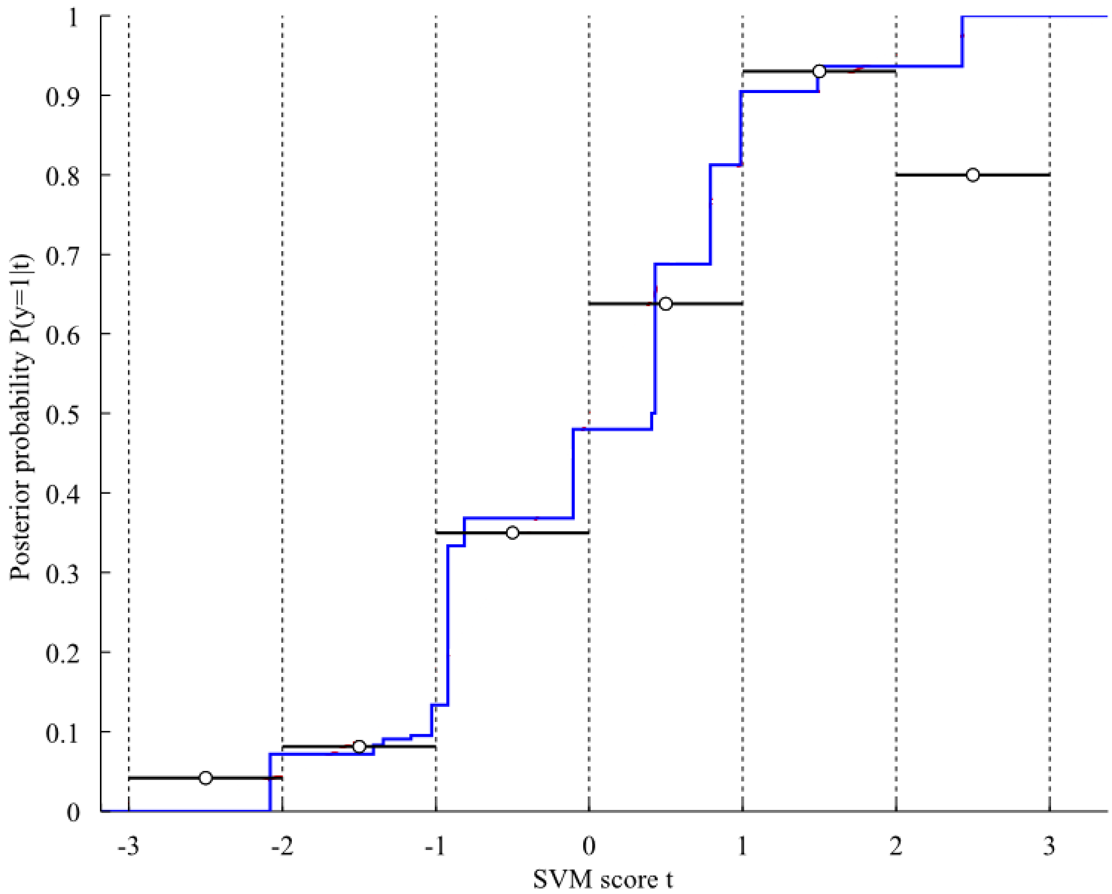

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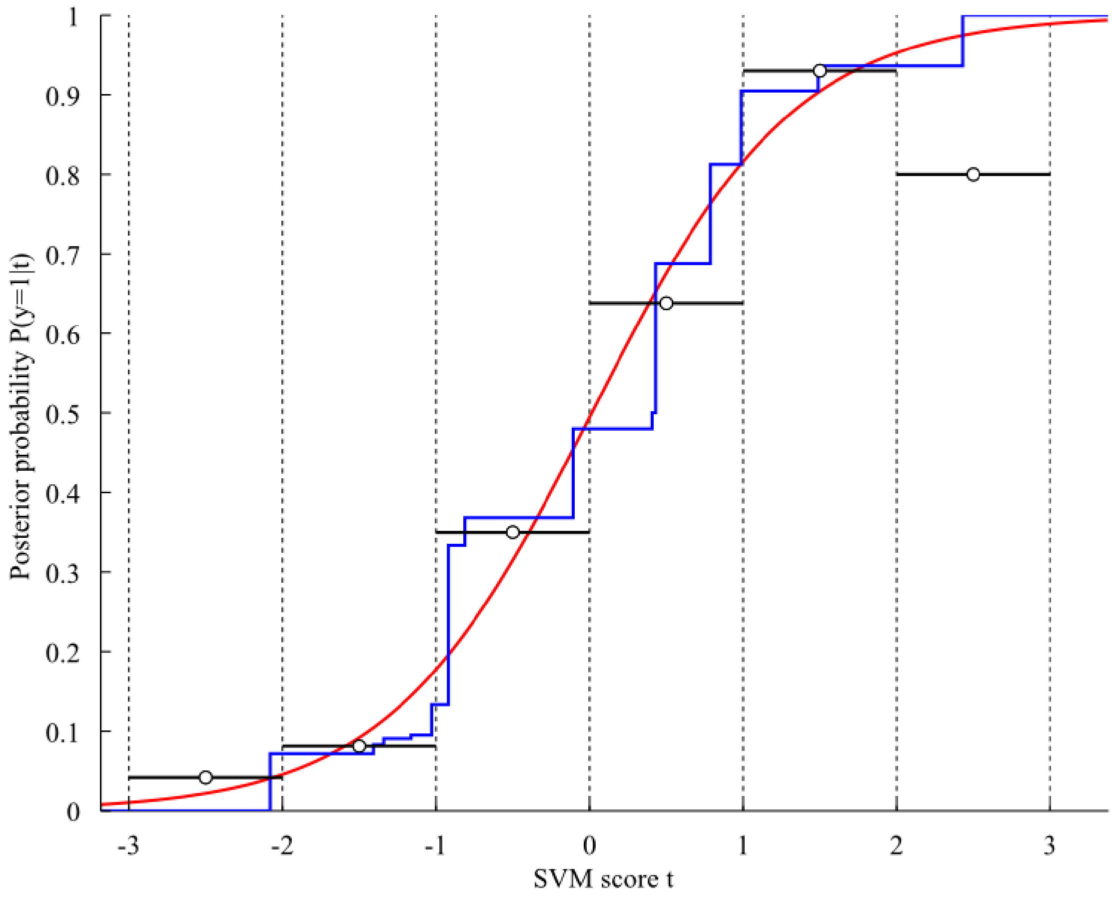

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Logistic regression

Sigmoid function:

$$P(y=1|s) \approx h_s(\theta) = \frac{1}{1+\exp(a+bs)}$$

Unknown parameters:

$$\theta = (a,b) \in \mathbb{R}^2$$

Likelihood function:

$$L_x(\theta) = \prod_{k=1}^{n} p_k^{y_k}(1-p_k)^{1-y_k} \text{ with } p_k = \frac{1}{1+\exp(a+bx_k)}$$

[1] Platt, J.C.: Probabilistic outputs for support vector machines and comparisons to regularized likelihood methods. Advances in Large-Margin Classifiers. pp. 61-74. MIT Press (1999)

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Statistical inference

- Observed data: $X \in \mathbb{X}$
- Unknown parameter: $\theta \in \Theta$
- Density function: $f_\theta(x)$
- Likelihood function: $L_x: \theta \mapsto f_\theta(x)$

Normalized likelihood contour function:

$$pl_x^\Theta(\theta) = \frac{L_x(\theta)}{\sup_{\theta' \in \Theta} L_x(\theta')}, \forall \theta \in \Theta$$

Consonant plausibility function:

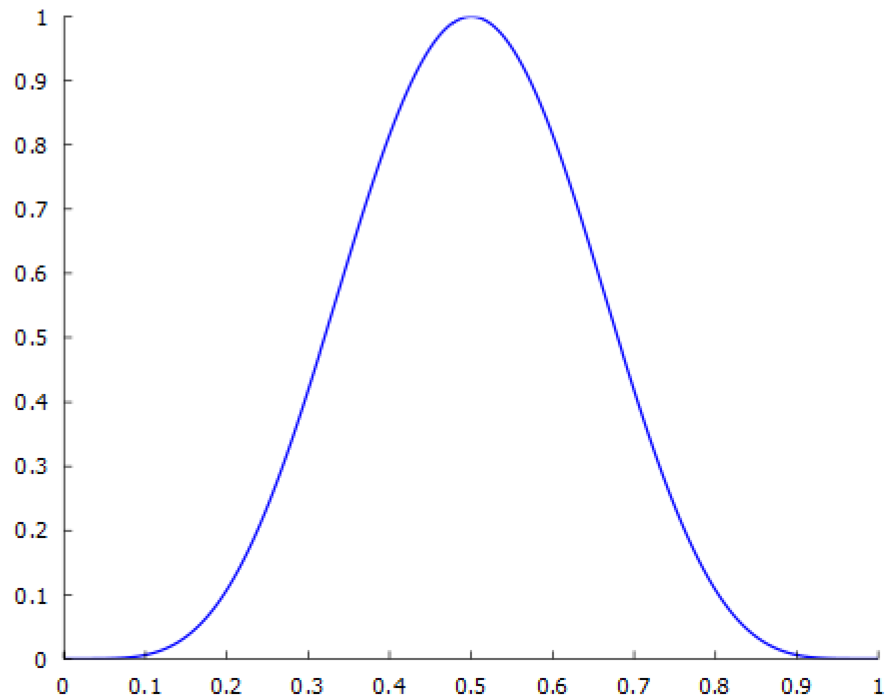
$$Pl_x^\Theta(A) = \sup_{\theta \in A} pl_x^\Theta(\theta), \forall A \subseteq \Omega$$

[2] Denoeux, T.: Likelihood-based belief function: justification and some extensions to low-quality data. International Journal of Approximate Reasoning 55(7), 1535-1547 (2014)

Binary variable

Bernoulli distribution: $\mathcal{B}(\theta)$

Likelihood function: $L(\theta) = \theta^{X}(1-\theta)^{N-X}$



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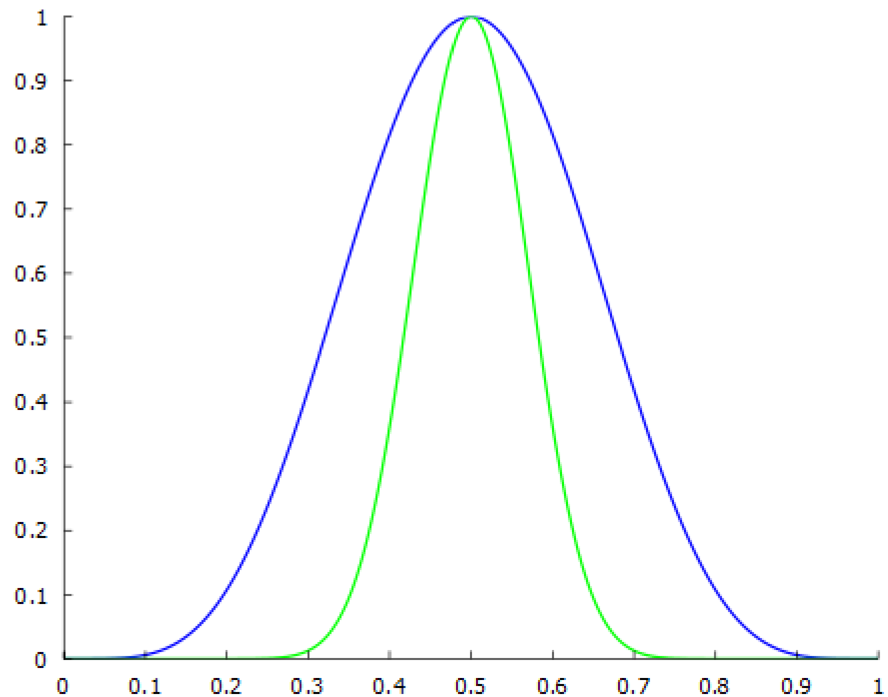

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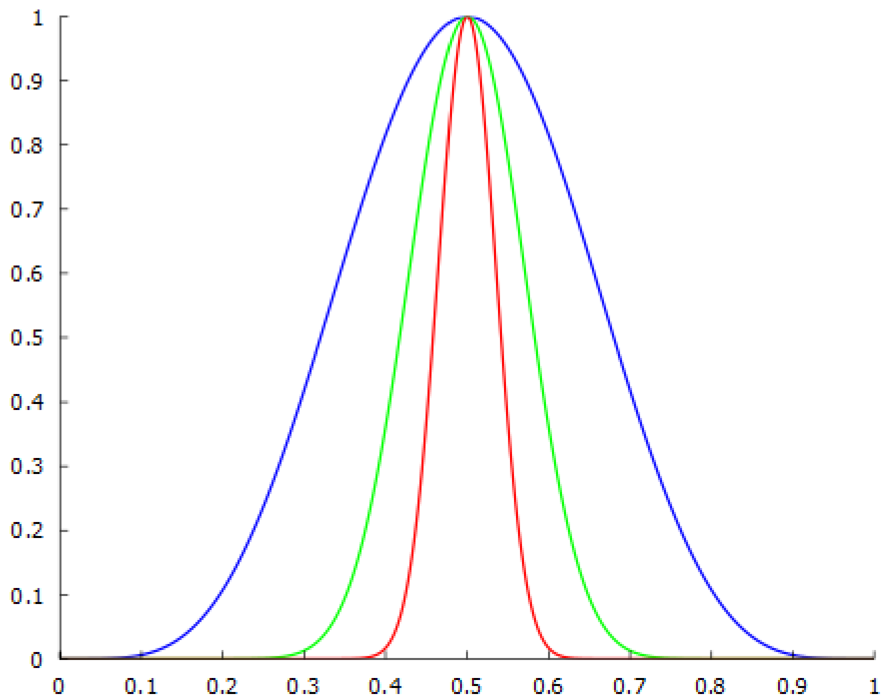


Forecasting

Let $Y \in \mathbb{Y}$ be a random variable with a Bernoulli distribution $\mathcal{B}(\theta)$.

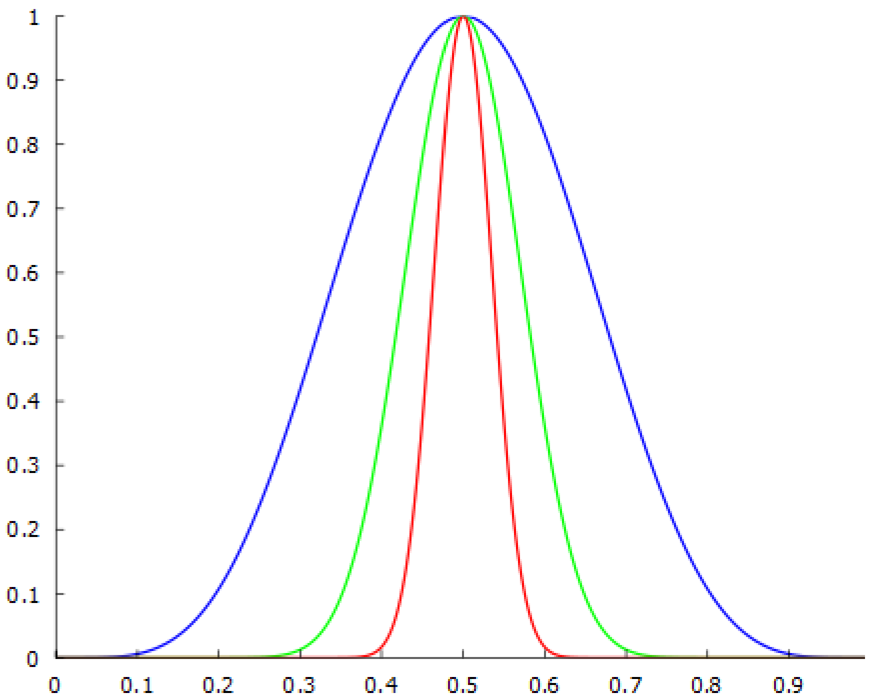
Let pl_x^{Θ} represents the knowledge about $\theta \in \Theta$.

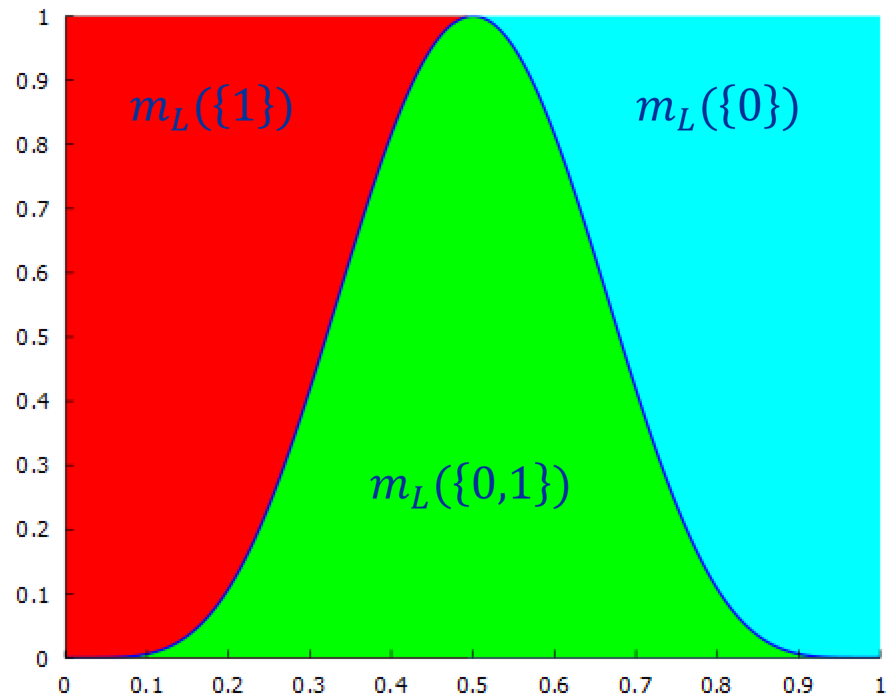
The belief function on Y is then given by

$$Bel_x^{\mathbb{Y}}(\{1\}) = \hat{\theta} - \int_0^{\hat{\theta}} pl_x^{\Theta}(u)du$$

$$Pl_x^{\mathbb{Y}}(\{1\}) = \hat{\theta} + \int_{\hat{\theta}}^{1} pl_x^{\Theta}(v)dv$$

Forecasting





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Plausibility function: Pl_x^{Θ}

Evidential logistic regression

Bernoulli distribution $\mathcal{B}(\omega)$, with $\omega = h_s(\theta)$.

$$pl_{x,s}^{\Omega}(\omega) = \begin{cases} 0 & \text{if } \omega \in \{0,1\} \\ Pl_x^{\Theta}(h_s^{-1}(\omega)) & \text{otherwise} \end{cases}$$

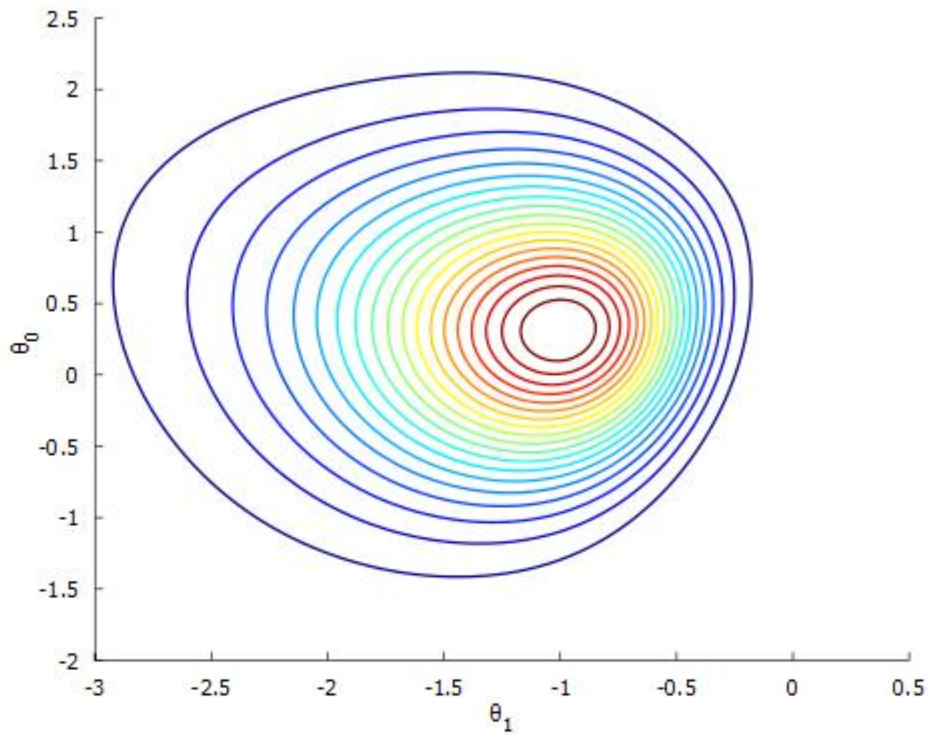
where

$$h_s^{-1}(\omega) = \left\{(a,b) \in \Theta \middle| a = \ln\left(\omega^{-1} - 1\right) - bs\right\}$$

which yields

$$pl_{x,s}^{\Omega}(\omega) = \sup_{b \in \mathbb{R}} pl_x^{\Theta}\left(\ln\left(\omega^{-1} - 1\right) - bs, b\right)$$

Evidential logistic regression



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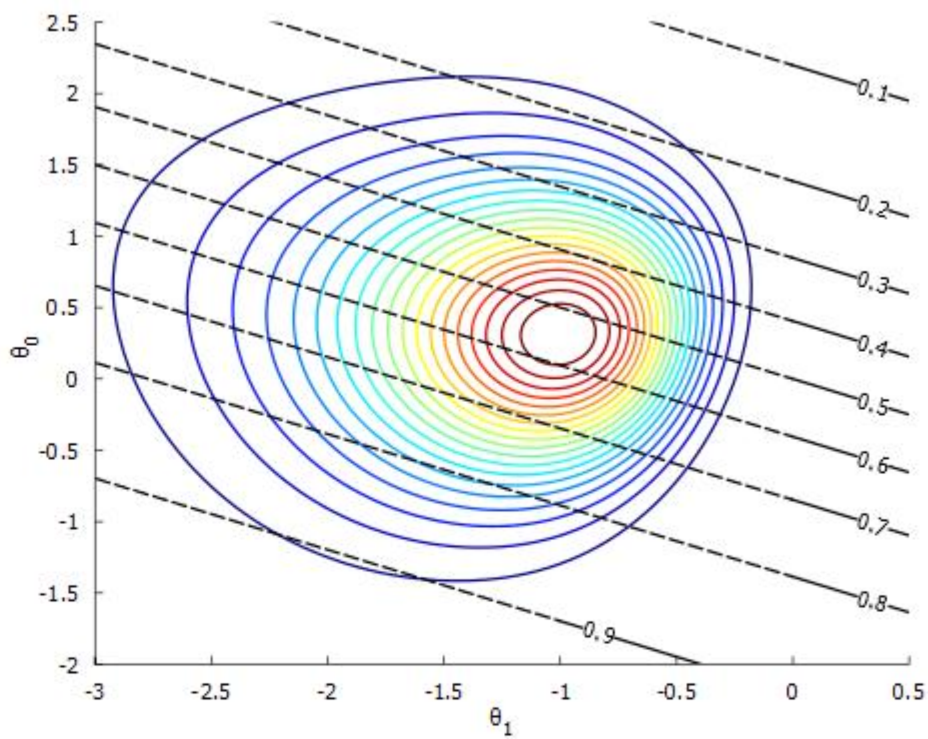

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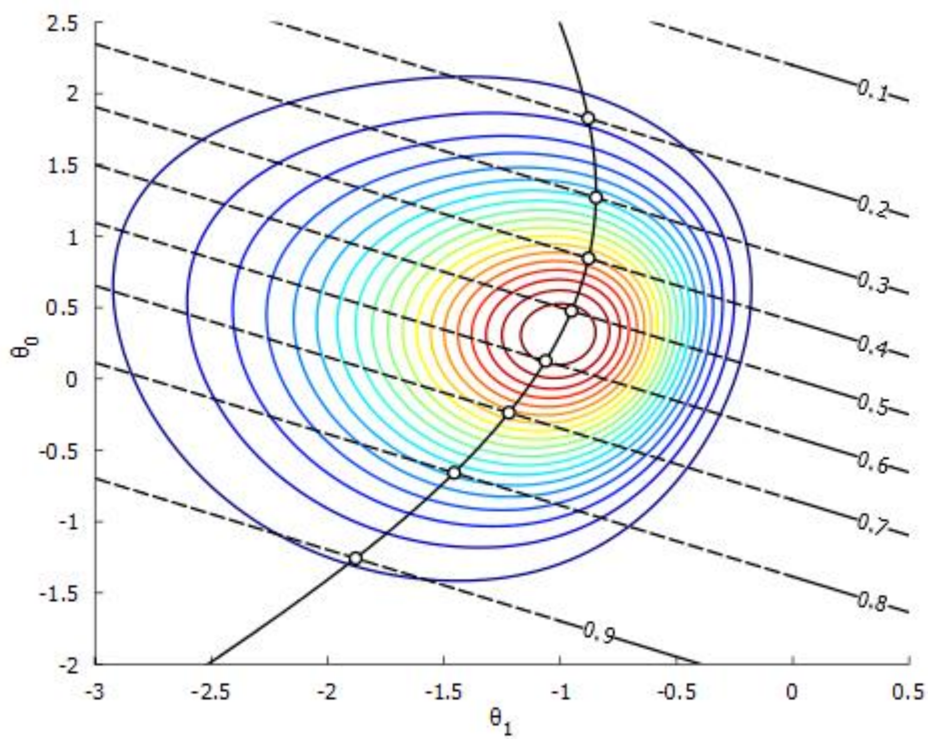

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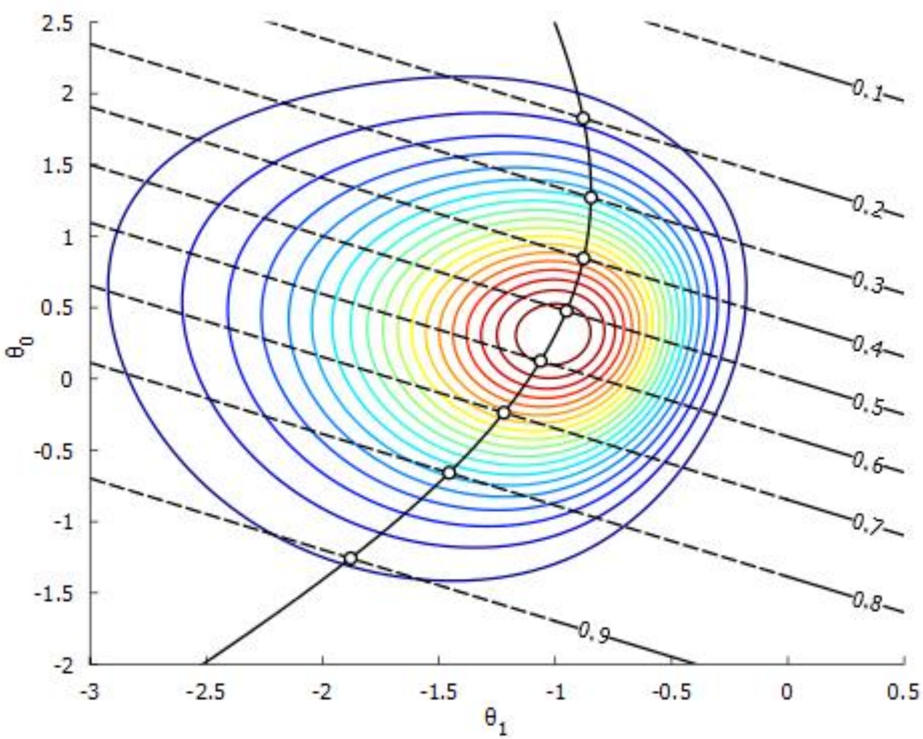

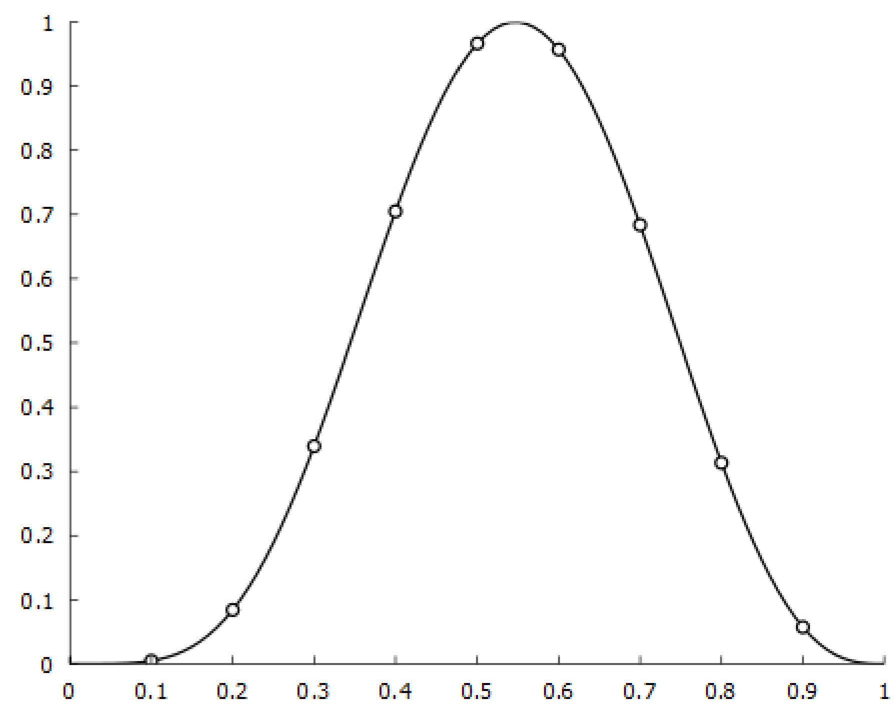

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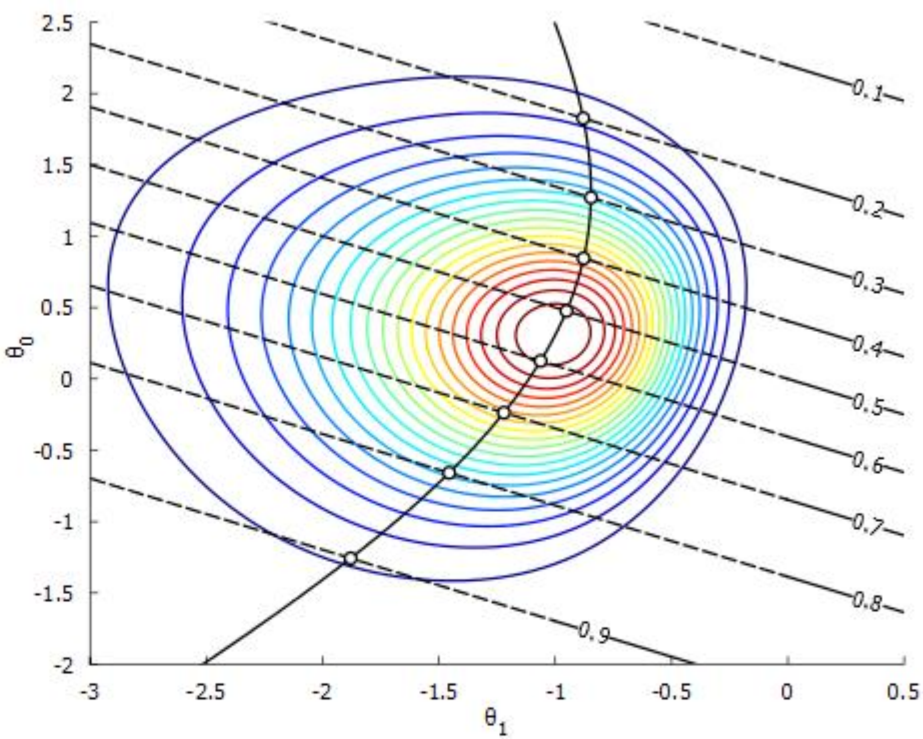

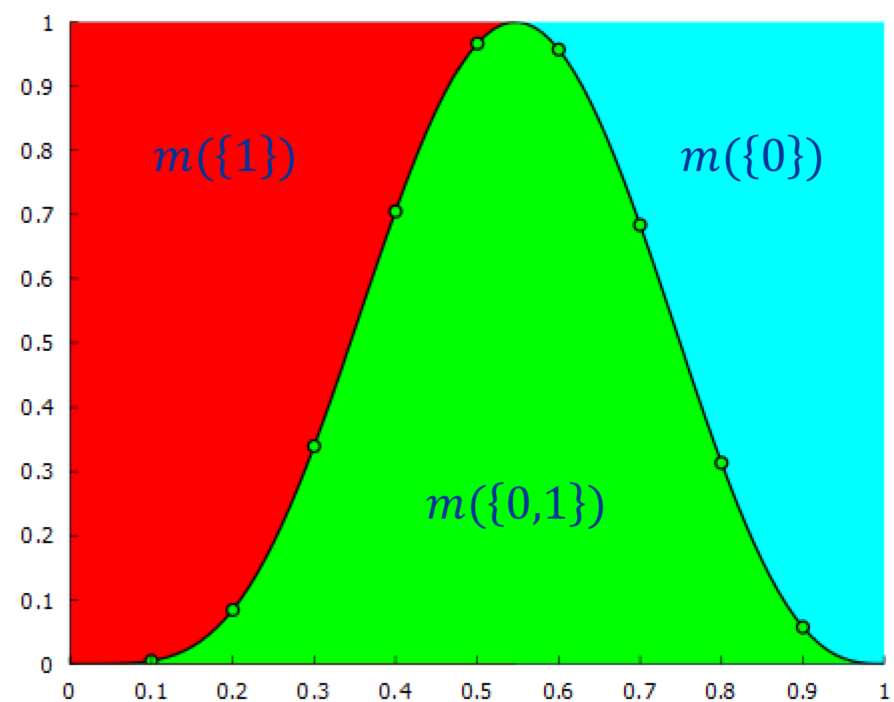

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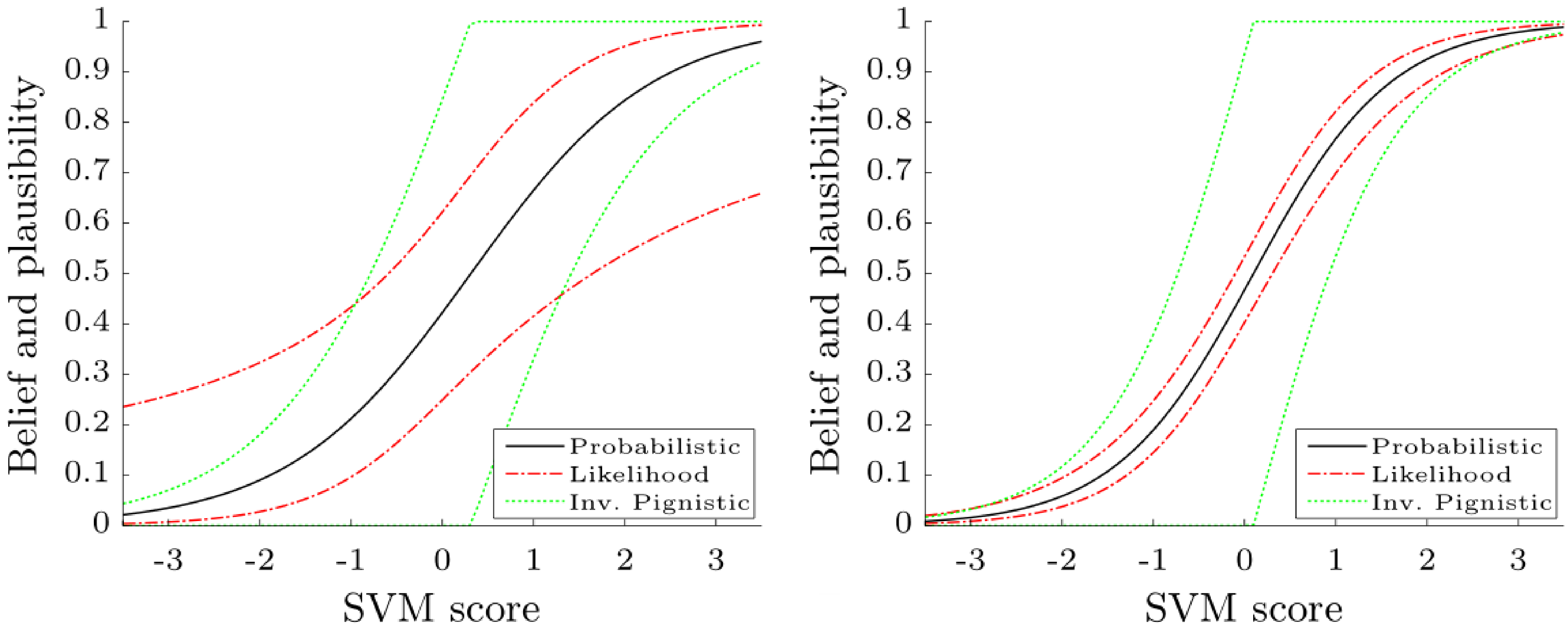

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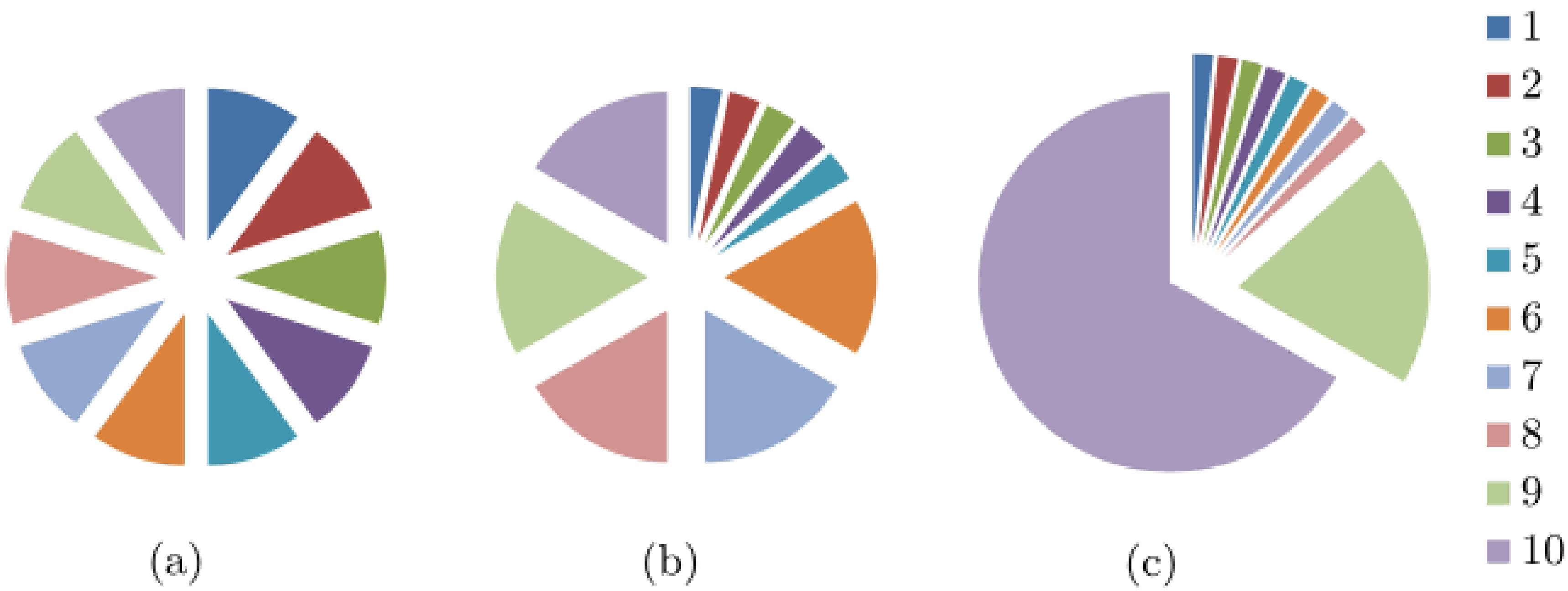
Training data



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Classification results

	Adult #train=600, #test=16 281				Australian #train=300, #test=390				Diabetes #train=300, #test=468		
Scenario	(a)	(b)	(c)		(a)	(b)	(c)		(a)	(b)	(c)
Probabilistic	83,24	82,70	80,90		**85,13**	**85,90**	85,90		**78,42**	**77,14**	53,42
Inv. Pignistic	**83,32**	82,79	81,02		**85,13**	**85,90**	**86,41**		**78,63**	**77,14**	54,70
Likelihood	**83,29**	**83,03**	**81,65**		**85,13**	**86,67**	**88,46**		**79,06**	**77,35**	**68,16**

Conclusion

- **The amount of training data is taken into account**
- **Reach better results when several classifiers are trained with unbalanced amounts of data**
- **Calibration can be done with the outputs of any classifier**
- **Future works:**
  - Calibration using other methods: binning, isotonic
  - Extension to multi-class classification using binary decomposition (one-vs-one, one-vs-all)

Thank you!